Since \( \mathbf{A} \times \mathbf{B} \) is a vector perpendicular to \( \mathbf{A} \) and to \( \mathbf{B} \), we can use (4.19) to find a vector perpendicular to two given vectors.

**Example 4.** Find a vector perpendicular to both \( \mathbf{A} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} \) and \( \mathbf{B} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} \).

\[
\mathbf{A} \times \mathbf{B} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & 1 & -1 \\
1 & 3 & -2
\end{vmatrix} = \mathbf{i}(-2 + 3) - \mathbf{j}(-4 + 1) + \mathbf{k}(6 - 1)
= \mathbf{i} + 3\mathbf{j} + 5\mathbf{k}.
\]

**Problems, Section 4**

9. Let \( \mathbf{A} = 2\mathbf{i} + 3\mathbf{j} \) and \( \mathbf{B} = 4\mathbf{i} - 4\mathbf{j} \). Show graphically, and find algebraically, the vectors \(-\mathbf{A}, 3\mathbf{B}, \mathbf{A} - \mathbf{B}, \mathbf{B} + 2\mathbf{A}, \frac{1}{2}(\mathbf{A} + \mathbf{B})\).

10. If \( \mathbf{A} + \mathbf{B} = 4\mathbf{j} - \mathbf{i} \) and \( \mathbf{A} - \mathbf{B} = \mathbf{i} + 3\mathbf{j} \), find \( \mathbf{A} \) and \( \mathbf{B} \) algebraically. Show by a diagram how to find \( \mathbf{A} \) and \( \mathbf{B} \) geometrically.

11. Let \( 3\mathbf{i} - \mathbf{j} + 4\mathbf{k}, 7\mathbf{j} - 2\mathbf{k}, \mathbf{i} - 3\mathbf{j} + \mathbf{k} \) be three vectors with tails at the origin. Then their heads determine three points \( \mathbf{A}, \mathbf{B}, \mathbf{C} \) in space which form a triangle. Find vectors representing the sides \( \mathbf{AB}, \mathbf{BC}, \mathbf{CA} \) in that order and direction (for example, \( \mathbf{A} \) to \( \mathbf{B} \), not \( \mathbf{B} \) to \( \mathbf{A} \)) and show that the sum of these vectors is zero.

12. Find the angle between the vectors \( \mathbf{A} = -2\mathbf{i} + \mathbf{j} - 2\mathbf{k} \) and \( \mathbf{B} = 2\mathbf{i} - 2\mathbf{j} \).

13. If \( \mathbf{A} = 4\mathbf{i} - 3\mathbf{k} \) and \( \mathbf{B} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} \), find the scalar projection of \( \mathbf{A} \) on \( \mathbf{B} \), the scalar projection of \( \mathbf{B} \) on \( \mathbf{A} \), and the cosine of the angle between \( \mathbf{A} \) and \( \mathbf{B} \).

14. Find the angles between (a) the space diagonals of a cube; (b) a space diagonal and an edge; (c) a space diagonal and a diagonal of a face.

15. Let \( \mathbf{A} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k} \). (a) Find a unit vector in the same direction as \( \mathbf{A} \). Hint: Divide \( \mathbf{A} \) by \( |\mathbf{A}| \). (b) Find a vector in the same direction as \( \mathbf{A} \) but of magnitude 12. (c) Find a vector perpendicular to \( \mathbf{A} \). Hint: There are many such vectors; you are to find one of them. (d) Find a unit vector perpendicular to \( \mathbf{A} \). See hint in (a).

16. Find a unit vector in the same direction as the vector \( \mathbf{A} = 4\mathbf{i} - 2\mathbf{j} + 4\mathbf{k} \), and another unit vector in the same direction as \( \mathbf{B} = -4\mathbf{i} + 3\mathbf{k} \). Show that the vector sum of these unit vectors bisects the angle between \( \mathbf{A} \) and \( \mathbf{B} \). Hint: Sketch the rhombus having the two unit vectors as adjacent sides.

17. Find three vectors (none of them parallel to a coordinate axis) which have lengths and directions such that they could be made into a right triangle.

18. Show that \( 2\mathbf{i} - \mathbf{j} + 4\mathbf{k} \) and \( 5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} \) are orthogonal (perpendicular). Find a third vector perpendicular to both.

19. Find a vector perpendicular to both \( \mathbf{i} - 3\mathbf{j} + 2\mathbf{k} \) and \( 5\mathbf{i} - \mathbf{j} - 4\mathbf{k} \).

20. Find a vector perpendicular to both \( \mathbf{i} + \mathbf{j} \) and \( \mathbf{i} - 2\mathbf{k} \).

21. Show that \( \mathbf{B}|\mathbf{A}| + \mathbf{A}|\mathbf{B}| \) and \( \mathbf{A}|\mathbf{B}| - \mathbf{B}|\mathbf{A}| \) are orthogonal.

22. Square \( \mathbf{A} + \mathbf{B} \); interpret your result geometrically. Hint: Your answer is a law
Example 7. Find the cosine of the angle between the planes of Example 6.

The angle between the planes is the same as the angle between the normals to the planes. Thus our problem is to find the angle between the vectors \( \mathbf{A} = i - 2j + 3k \) and \( \mathbf{B} = 2i + j - k \). Since \( \mathbf{A} \cdot \mathbf{B} = |\mathbf{A}| |\mathbf{B}| \cos \theta \), we have \( -3 = \sqrt{14} \sqrt{6} \cos \theta \), and so \( \cos \theta = -\sqrt{3/28} \). This gives the obtuse angle between the planes; the corresponding acute angle is \( \pi - \theta \), or \( \arccos \sqrt{3/28} \).

Problems, Section 5

In Problems 1 to 5, all lines are in the \((x, y)\) plane.

1. Write the equation of the straight line through \((2, -3)\) with slope \(3/4\), in the parametric form \( \mathbf{r} = \mathbf{r}_0 + \mathbf{A}t \).

2. Find the slope of the line whose parametric equation is \( \mathbf{r} = (1 - j) + (2i + 3j)t \).

3. Write, in parametric form [as in Problem 1], the equation of the straight line that joins \((1, -2)\) and \((3, 0)\).

4. Write, in parametric form, the equation of the straight line that is perpendicular to \( \mathbf{r} = (2i + 4j) + (i - 2j)t \) and goes through \((1, 0)\).

5. Write, in parametric form, the equation of the \(y\) axis.

Find the symmetric equations (5.6) or (5.7) and the parametric equations (5.8) of a line, and/or the equation (5.10) of the plane satisfying the following given conditions.

6. Line through \((1, -1, -5)\) and \((2, -3, -3)\).

7. Line through \((2, 3, 4)\) and \((5, 1, -2)\).

8. Line through \((0, -2, 4)\) and \((3, -2, -1)\).

9. Line through \((-1, 3, 7)\) and \((-1, -2, 7)\).

10. Line through \((3, 4, -1)\) and parallel to \(2i - 3j + 6k\).

11. Line through \((4, -1, 3)\) and parallel to \(1 - 2k\).

12. Line through \((5, -4, 2)\) and parallel to the line \( r = 1 - j + (5i - 2j + k)t \).

13. Line through \((3, 0, -5)\) and parallel to the line \( r = (2, 1, -5) + (0, -3, 1)t \).

14. Plane containing the triangle \( ABC \) of Problem 4.11.

15. Plane through the origin and the points in Problem 8.

16. Plane through the point and perpendicular to the line in Problem 12.

17. Plane through the point and perpendicular to the line in Problem 13.

18. Plane containing the two parallel lines in Problem 12.


20. Plane containing the three points \((0, 1, 1)\), \((2, 1, 3)\), and \((4, 2, 1)\).

In Problems 21 to 23, find the angle between the given planes.
24. Find a point on both the planes (that is, on their line of intersection) in Problem 21. Find a vector parallel to the line of intersection. Write the equations of the line of intersection of the planes. Find the distance from the origin to the line.

25. As in Problem 24, find the equations of the line of intersection of the planes in Problem 22. Find the distance from the point (2, 1, -1) to the line.

26. As in Problem 24, find the equations of the line of intersection of the planes in Problem 23. Find the distance from the point (1, 0, 0) to the line.

27. Find the equation of the plane through (2, 3, -2) and perpendicular to both planes in Problem 21.

28. Find the equation of the plane through (-4, -1, 2) and perpendicular to both planes in Problem 22.

29. Find a point on the plane 2x - y - z = 13. Find the distance from (7, 1, -2) to the plane.

30. Find the distance from the origin to the plane 3x - 2y - 6z = 7.

31. Find the distance from (-2, 4, 5) to the plane 2x + 6y - 3z = 10.

32. Find the distance from (3, -1, 2) to the plane 5x - y - z = 4.

33. Find the perpendicular distance between the two parallel lines in Problem 12.

34. Find the distance (perpendicular is understood) between the two parallel lines in Problem 13.

35. Find the distance from (2, 5, 1) to the line in Problem 10.

36. Find the distance from (3, 2, 5) to the line in Problem 11.

37. Determine whether the lines

\[
\frac{x - 1}{2} = \frac{y + 3}{1} = \frac{z - 4}{-3} \quad \text{and} \quad \frac{x + 3}{4} = \frac{y + 4}{1} = \frac{8 - z}{4}
\]

intersect. Two suggestions: (1) Can you find the intersection point, if any? (2) Consider the distance between the lines.

38. Find the angle between the lines in Problem 37.

In Problems 39 and 40, show that the given lines intersect and find the acute angle between them.

39. \( r = 2i + k + (3i - k)t_1 \) and \( r = 7i + 2k + (2i - j + k)t_2 \).

40. \( r = (5, -2, 0) + (1, -1, -1)t_1 \) and \( r = (4, -4, -1) + (0, 3, 2)t_2 \).

In Problems 41 to 44, find the distance between the two given lines.

41. \( r = (4, 3, -1) + (1, 1, 1)t \) and \( r = (4, -1, 1) + (1, -2, -1)t \).

42. The line that joins (0, 0, 0) to (1, 2, -1), and the line that joins (1, 1, 1) to (2, 3, 4).

43. \( \frac{x - 1}{2} = \frac{y + 2}{3} = \frac{2z - 1}{4} \) and \( \frac{x + 2}{-1} = \frac{2 - y}{2} = \frac{z - 1}{2} \).

44. The z axis and \( r = 1 - k + (2i - 3j + k)t \).

45. A particle is traveling along the line \( \frac{x - 3}{2} = \frac{y + 1}{-2} = z - 1 \). Write the equation of its path in the form \( r = r_0 + At \). Find the distance of closest approach.
16.5 \( v = |z_1 - z_2|; \ a = 0 \)

16.6 (a) Series: 3 - 2i; parallel: 5 + i
    (b) Series: 2(1 + i\sqrt{3}); parallel: i\sqrt{3}

16.8 \( R = i(\omega CR^2 + \omega^3 L^2 C - \omega L)/(\omega CR)^2 + (\omega^2 LC - 1)^2); \) this simplifies to \( L/(iRC) \) at resonance.

16.9 (b) \( \omega = 1/\sqrt{LC} \)

17.12 \( e^{-x^2} = -5.17 \times 10^{-5} \)

17.16 \( e^{x/2} = 4.81 \)

17.28 \( 1 + (a^2 + b^2)(2ab)^{-2} \sin^2 b \)

17.30 \( e^x \cos x = \sum_{n=0}^{\infty} (2^n/n!)(2^n/n!) \cos n\pi/4 \)
    \( e^x \sin x = \sum_{n=0}^{\infty} (2^n/n!)(2^n/n!) \sin n\pi/4 \)

Chapter 3

2.4 \[
\begin{pmatrix}
1 & 0 & -\frac{1}{2} \\
0 & 1 & 0 \\
-\frac{1}{2} & 0 & 1
\end{pmatrix}, \quad x = \frac{1}{2}(z + 1), \ y = 1
\]

2.8 \[
\begin{pmatrix}
0 & 0 & 1 \\
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 1 & -1 \\
0 & 0 & 0
\end{pmatrix}, \quad x = y - 11, \ z = 7
\]

2.9 inconsistent, no solution

2.12 \[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad x = -2, \ y = 1, \ z = 1
\]

2.17 \( R = 2 \)

3.1 -11 \( \quad \quad \quad 3.5 \quad -544 \quad \quad \quad 3.12 \quad 16 \)

3.16 \( A = -(K + ik)/(K - ik), \ |A| = 1 \)

4.12 \( \arccos(-1/\sqrt{2}) = 3\pi/4 \)

4.14 (a) \( \arccos(1/3) = 70.5^\circ \)

4.14 (c) \( \arccos \sqrt{2}/3 = 35.3^\circ \)

4.15 (b) \( 8i - 4j + 8k \)

4.18 \( 2i - 3j - 3k \)

4.19 \( i + j + k \)

4.22 Law of cosines \( A^2 B^2 \)

5.1 \( r = (2i - 3j) + (4i + 3j) \) \( t \) \( \) [Note that \( 2i - 3j \) may be replaced by any point on the line; \( 4i + 3j \) may be replaced by any vector along the line. Thus, for example, \( r = 0i - (8i + 6j) \) \( t \) is just as good an answer, and similarly for all such problems]
5.11 \( \frac{x - 4}{1} = \frac{x - 3}{(-2)}, \; y = -1; \) or \( r = 4i - j + 3k + (i - 2k)t \)
5.12 \( \frac{x - 5}{5} = \frac{y + 4}{(-2)} = \frac{x - 2}{1}; \) or \( r = 5i - 4j + 2k + (5i - 2j + k)t \)
5.14 \( 36x - 3y = 22x = 23 \) \( 5.16 \) \( 5x - 2y + z = 35 \)
5.18 \( x + 6y + 7z + 5 = 0 \) \( 5.20 \) \( x - 4y - x + 5 = 0 \)
5.21 \( \cos \theta = 25/(7\sqrt{50}) = 0.652, \theta = 49.3^\circ \)
5.22 \( \cos \theta = 2/\sqrt{6}, \theta = 35.3^\circ \)
5.24 \( r = 2i + j + (1 + 2k)t, \; d = 2\sqrt{6}/5 \)
5.25 \( r = i - 2j + (4i + 9j + k)t, \; d = (3\sqrt{3})/7 \)
5.29 \( 2/\sqrt{6} \) \( 5.31 \) \( 5/7 \) \( 5.33 \) \( \sqrt{43}/15 \)
5.34 \( \sqrt{11}/10 \) \( 5.36 \) \( 3 \) \( 5.38 \) \( \arccos \sqrt{21/22} = 12.3^\circ \)
5.39 Intersect at \( (3, 2, 0) \); \( \cos \theta = 5/\sqrt{60}, \theta = 49.8^\circ \)
5.42 \( 1/\sqrt{5} \) \( 5.43 \) \( 20/\sqrt{21} \) \( 5.45 \) \( d = \sqrt{2}, \; t = -1 \)

6.2 \( \mathbf{A} = \begin{pmatrix} -2 & 1 \\ -2 & 2 \end{pmatrix}, \; \mathbf{B} = \begin{pmatrix} -6 & 17 \\ -2 & 6 \end{pmatrix} \), \( \mathbf{A} + \mathbf{B} = \begin{pmatrix} 1 & -1 \\ -1 & 5 \end{pmatrix} \)
\( \mathbf{A} - \mathbf{B} = \begin{pmatrix} 3 & -9 \\ -1 & 1 \end{pmatrix} \), \( \mathbf{A}^2 = \begin{pmatrix} 9 & -5 \\ -5 & 14 \end{pmatrix} \), \( \mathbf{B}^2 = \begin{pmatrix} 1 & 4 \\ 4 & 0 \end{pmatrix} \), \( 5\mathbf{A} = \begin{pmatrix} 10 & -25 \\ -5 & 15 \end{pmatrix} \), \( 3\mathbf{B} = \begin{pmatrix} -3 & 12 \\ 0 & 6 \end{pmatrix} \), \( \text{det}(3\mathbf{A}) = 5^2 \text{det} \mathbf{A} \) for a \( 2 \times 2 \) matrix

6.4 You should have found \( \mathbf{BA}, \; \mathbf{C}^2, \; \mathbf{CB}, \; \mathbf{C}^3, \; \mathbf{C}^2 \mathbf{B}, \) and \( \mathbf{CBA} \); all others are meaningless. \( \mathbf{C}^2 \mathbf{B} = \begin{pmatrix} 53 & 7 \\ -13 & -9 \end{pmatrix} \), \( \mathbf{CBA} = \begin{pmatrix} 36 & 46 & 14 & -36 \\ -8 & 2 & 1 & -29 \end{pmatrix} \)

6.13 \( \begin{pmatrix} 5/3 & -3 \\ -1 & 2 \end{pmatrix} \) \( 6.15 \) \( \frac{1}{2} \) \( \begin{pmatrix} 5 & 8 \\ -2 & -2 \\ 2 & 3 \end{pmatrix} \)

6.19 \( \mathbf{A}^{-1} = \frac{1}{7} \begin{pmatrix} 1 & 2 \\ -3 & 1 \end{pmatrix} \), \( (x, y) = (5, 0) \)

6.22 \( \mathbf{A}^{-1} = \frac{1}{12} \begin{pmatrix} 4 & 4 \\ -7 & -3 \end{pmatrix} \), \( (x, y, z) = (1, -1, 2) \)

6.30 \( \sin h \mathbf{A} = \mathbf{A} \sin h = \begin{pmatrix} 0 & \sin h \\ \sin h & 0 \end{pmatrix}, \; \cos h \mathbf{A} = \mathbf{I} \cos h = \begin{pmatrix} \cos k & 0 \\ 0 & \cos k \end{pmatrix} \)
\( e^{kh} = \begin{pmatrix} \cosh k & \sinh k \\ \sinh k & \cosh k \end{pmatrix}, \; e^{ik} = \begin{pmatrix} \cos k & i \sin k \\ i \sin k & \cos k \end{pmatrix} \)

7.1 Not linear \( 7.4 \) Linear \( 7.6 \) Not linear
7.8 Not linear \( 7.11 \) Not linear \( 7.12 \) Linear
7.14 Not linear \( 7.15 \) Linear
7.22 \( D = 1 \), rotation \( \theta = -45^\circ \)
7.24 \( D = -1 \), reflection line \( x + y = 0 \)
7.26 \( D = -1 \), reflection line \( x = 2y \)
7.30 \( R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \), \( S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \)