A Geological Fingerprint

of Extremely Low Viscosity Fault Fluids

During an Earthquake

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Abstract

Physical models of earthquakes require knowledge of the stress on the fault during slip, but there are few constraints on this quantity on real faults. Here we use the geological record of an exceptional outcrop to calculate the slip rate and stress on an ancient fault in a new way. On Kodiak Island, Alaska, we found a fine-grained, fluidized fault core with a series of asymmetrical intrusions of the fine-grained fault rock into the surrounding country rock. The intrusive structures are interpretable as products of a gravitational (Rayleigh-Taylor) instability. The wavelength to thickness ratio of the intrusions is much shorter than can be explained by a low Reynolds number flow. The short wavelengths between buoyant intrusions are best explained by moderately inertial flow with fluid velocities of order of 10 cm/s. These slip velocities are typical of an earthquake or its immediate afterslip and thus require a coseismic origin. The Reynolds number of the buoyant flow requires a
very low viscosity (≤1 Pa s) and thus implies local dynamic weakening to extraordinarily low stress (<10 Pa) during an earthquake.
Introduction

Determining stresses on a fault during an earthquake is critical for building any physical model of the earthquake [Abercrombie et al., 2006; Kanamori and Brodsky, 2004; Zoback, 2000]. The problem is complicated by a plethora of processes that happen only at the high velocities of seismic rupture [Brodsky and Kanamori, 2001; Fialko and Khazan, 2005; Rice, 2006]. There are few tools to determine the slip rate of ancient faults and almost no recognized record of the seismic stresses [Cowan, 1999].

We use an exceptional site to address this problem. The key outcrop formed at 12-14 km depth in a megathrust in an accretionary prism and is now exposed on Kodiak Island, Alaska [Rowe et al., 2005; Rowe, 2007]. This type of fault generates giant earthquakes like the 2004 M_w9.2 Sumatra earthquake. The fault core is comprised of a ~3.5 cm layer of ultrafine-grained black rock that is embedded in a 13.5m-thick cataclastic subduction thrust (Figure 1). We are studying the geometry of intrusions out of this core in order to determine the rheology and emplacement speed.

In this paper, we will first provide some geological context and then present the observations of geometry from the Kodiak Island site. We will highlight the features indicative of buoyant intrusions. The role of gravitational instabilities will be further illustrated by the similarities to buoyant, ductile features in sedimentary rocks. We will then proceed to use a linear stability analysis to interpret the wavelengths of the intrusions in terms of fluid dynamics. This analysis will result in a constraint on the
Reynolds number of the flow and hence the viscosity. Ultimately, we will determine the velocity and stress at the time of intrusion and interpret these in terms of the rheologies of plausible materials.

One of the principal advantages of the analysis performed here is that it is entirely based on the geometry of the outcrop. No prior knowledge about the origin of the rock is required. As will be discussed near the end of the paper, the rheology inferred from the flow analysis is consistent with either a granular flow or a frictional melt origin as suggested by previous work at this site [Rowe et al., 2005]. The actual genesis of the rock is likely complex and its proper classification as either a pseudotachylyte (frictional melt) or previously unknown granulated rock-type is a thorny issue [Meneghini et al., 2006]. For the bulk of this paper, we will postpone a discussion of origin by referring to the fine-grained fault rock simply as the “black rock.”

**Observations**

**Geological Context**

The Kodiak accretionary complex of Alaska is an exhumed subduction zone (Figure 1) that has modern analogs offshore in the Eastern Aleutian Trench [Plafker et al., 1994]. Plate boundary thrust faults are preserved as mélanges in several accreted units. These mélanges have been interpreted by previous workers as ancient decollements formed during subduction to ~12-14 km [Byrne, 1984]. Within one mélange, four parallel, high-strain, strongly foliated cataclastic shear zones are mapped within a structural thickness of less than 1 km (Figures 1, 2). They are interpreted as episodes of localized of
deformation in the decollement evolution. Three of the cataclasites are cut and complexly intruded by thick layers of ultrafine-grained black fault rocks.

The black layers develop in the cataclasite or crop out at the sharp boundary between the shear zone and an overlying 3-10 m thick massive sandstone unit from the mélange. These layers are distinguished in the field by hardness, black color and vitreous to earthy luster. Grain size in the black rocks is on the order of 1-10 microns. The black rocks in the fault are exceptionally continuous along strike for distances of more than 2.5 km (Figure 1).

The black rocks are derived from the cataclasite. Both units have nearly identical chemical compositions and the major mineral constituents for both units are quartz, albite, chlorite and illite. Minor differences in bulk chemistry suggest very minor fractionation between the cataclasite and the black rock [Meneghini et al., 2006]. In particular, the ultrafine-grained matrix of the black rocks is slightly depleted in phyllosilicates and enriched in tabular albitic feldspar relative to the cataclasites (Table B1). However, the overall similarities indicate a close relationship between the units.

Geometry of black rock intrusions

Flame-like intrusions of black rock originate from the layer and intrude the upper cataclasite (Figures 2). Elongate swirls at the decimeter-scale indicate ductile behavior similar to soft-sediment deformation [Potter and Pettijohn, 1977; Ronnlund, 1989; Twiss and Moores, 1992; Visher and Cunningham, 1981]. The flame structures occur
exclusively where granular cataclasite overlies a horizon of black rock. The stratigraphic younging directions of the surrounding geological units indicate that the current upward direction preserves the original orientation [Rowe, 2007]. The lower boundary of the black layer is relatively flat and shows little evidence of downward mixing. The vertical asymmetry implies that the flame structures are buoyant features driven by a density contrast between the black rock and the upper cataclasite (Rayleigh-Taylor instabilities).

To verify that the density difference between the units is significant, we measured the densities of the preserved rocks (Table 1). The grain density of the black rock is presently 5% less than the surrounding cataclasites. The difference is explained by a slight depletion of sheet silicate minerals in the black rock. The preserved porosity in the black rock is less than in the cataclasite, so the net density difference is now 1.4% with a standard deviation of 1.1%. If the rocks were mobilized by frictional melting or agitation of a granular flow, the black rock may have been dilated and thus been less dense relative to the cataclasite. At the extreme end, the density of the cataclasite is 30% more than a silicate melt at the pressures, temperatures and water content inferred for the black rocks in the last section of this paper.

The intrusive structures are sheared in a direction consistent with motion on the thrust (bending leftward in Figure 2) [Rowe, 2007]. The cross-section is within 10° of parallelism to the mean transport direction in the cataclasite. The taller intrusions, such as that in the center of Figure 2, are progressively inclined with increasing distance from the source layer of black rock, varying from 15° from vertical at the base to 68° at the crest.
The shape indicates the intrusions were sheared at the same time as they grew vertically so that the points furthest from the fault surface accumulated the most shear.

The deformation is gradational throughout the intrusive zone without any sign of relative overprinting or cross-cutting so horizontal and vertical motions were simultaneous. The inclination of the intrusive structures averaged over their entire heights is $38^\circ \pm 15^\circ$ from vertical and thus the horizontal and vertical velocities are comparable. Therefore, by constraining the emplacement rate of the structures, we learn about the shear rate during their formation.

Another key feature of the outcrop is the cuspate, pointed shape of the intrusions. Such cusps in ductile rocks are normally interpreted as indicative of the relative viscosity of the two layers [Davis and Reynolds, 1996]. The lower viscosity unit typically forms sharper angles as it is more easily deformed. In this case, the elongate stringers of black rock indicate that the black rock is less viscous than the overlying cataclasite.

Even though the cusps suggest that the overlying layer is stiffer than the black rock, some flow needed to occur in the cataclasite to accommodate the growing intrusions. This deformation is recorded by the curvature of the otherwise planar fabric in the cataclasite between the intrusions (See the fabric between the lens cap and the black fault rock in Figure 2). The fabric stretches and bends so that it remains parallel to the curved black rock-cataclasite boundary between intrusions.
In summary, we infer from the geometry that the flame structures in Figure 2 are buoyant, ductile intrusion of the black rock into the overlying cataclasite. The black rock was less dense and less viscous than the cataclasite it intruded. The intrusion occurred simultaneously and at a similar rate to shear of the fault zone.

Flame Structures: A soft-sediment analog

The geometry of the black rock intrusions is similar to flame structures that are observed in soft sediment [Ronnlund, 1989; Visher and Cunningham, 1981]. The analogy is instructive and helps to substantiate the inferred buoyant origin of the intrusions. We next review the soft-sediment analog and its similarities to the rocks observed here before proceeding to an analysis of the flow in the black rocks.

Flame structures, and their complement, load casts, are a somewhat rare but distinctive structure known from soft sediment deformation [Allen, 1982]. They generally occur when sand overlies mud along a sharp contact as in turbidites [Allen, 1985]. Load casts are rounded lobes of sand which sink down into the mud layer. The flame structures are the upward injections of mud into the sand layer that separate the load casts. A detailed description is given by Allen [1985] and emphasizes that as the load casts form, laminae in the sandstone stretch and remain parallel to the sand-mud boundary.

The structures have been attributed to an instability of the sediments by many authors [Allen, 1982; Anketell et al., 1970; Brodzikowski and Haluszczak, 1987; Collinson, 1994; Kelling and Walton, 1957; Lowe, 1975; Maltman, 1994; Owen, 1987; 1996]. Most work
suggests that the instability is gravitational due to an inverse gradation in bulk density [Allen, 1982; Collinson, 1994; Kelling and Walton, 1957; Lowe, 1975; Maltman, 1994; Owen, 1996]. The gravitational model best that interprets the flames as buoyant intrusions best explains the experimental data and natural occurrence. Such gravitational (Raleigh-Taylor) instabilities are general features of layered materials whenever a lower density layer is overlain by a higher density layer and both layers behave ductilely.

Morphologically, flames show many characteristic features that require a viscous, two-fluid origin. Intrusion of mud is always perpendicular to layering, i.e., upward. The peaks of the flame structures have a characteristic wavelength and are concave-upward, pinching out at some height.

As the prevalence of planar-bedded turbidites demonstrates, the instability occurs only under special circumstances when liquefaction is triggered by an external event. As such, load cast-flame structure occurrence is sometimes attributed to vibration from the passage of turbidity currents [Allen, 1982] or shaking from earthquakes [Bhattacharya and Bandyopadhyay, 1998; Horváth et al., 2005]. It is not uncommon for flame structures to occur over only a limited area even when bedding surfaces continue over a greater extent [Dasupta, 1998].

Figure 3 shows an example from the Carmelo Formation turbidites at Pt. Lobos in central California, USA (Figure 3; for a detailed description of the sedimentological context see Clifton [1984]). In the Carmelo Formation, flame structures formed when a sand-rich
turbidity current overran saturated, high-porosity muds. The lighter, low-density mud injected upward through the higher density sands. The instability deformed the upper part of the layer, but the lower surface remained smooth. In this case (Figure 3) the bottom of the unit is wavy due to the sedimentary deformation of the layer below it, but this waviness is unrelated to the flame structure formation.

An additional key feature observed in Figure 3 is the leftward curvature of the intrusion tips. Two main mechanisms have been proposed to explain this leaning from the vertical direction: the drawing up of clay flames by current drag [Kuenen and Menard, 1952] or shear stress imprinted on a hydroplastic substrate. Potter and Pettijohn [1977] interpreted the vergence shown by the flame structures as an indicator of flow direction.

In summary, the features which characterize Raleigh-Taylor instability structures in soft sediments include:

1. Intrusions occur within essentially flat-lying layers (perpendicular to gravity) [Allen, 1985].

2. The lower density layer is overlain by a higher density layer [Anketell et al., 1970].

3. Layers are (originally) roughly planar, base of source layer remains smooth after intrusions form.

4. Intrusions have a characteristic wavelength [Allen, 1985].

5. Intrusions narrow from base to peak.
6. If intrusions are deflected, the direction of deflection is consistent [Anketell and Dzulynski, 1968; Potter and Pettijohn, 1977].

7. Intrusion shape or size is not systematically limited by rigid structure in denser layer; denser layer flows to accommodate intrusions from less-dense layer.

Comparison of fault rock intrusions and soft-sediment structures

All seven of the criteria established above for the soft sediment Raleigh-Taylor instabilities are met by the black rocks in the fault in Figure 2. The most striking difference is that the wavelengths are much shorter relative to the layer thickness in the fault rock case. We will return to this observation in our flow analysis below. The wavelengths are also much more uniform in the fault rock case. The standard deviation of wavelength is 47% of the mean wavelength in sediments and 33% in fault rocks.

In other places on the fault zone, the black rocks also occur beneath consolidated meta-sandstone in the hanging wall rather than cataclasite (Figure 2). These sandstones would have been brittle at the conditions of deformation of the paleo-decollement. The black rock injects into the sandstones but the style and scale are distinct from the flame structures case. These outcrops form a useful counterpoint that elucidates the behavior of a viscous, low-density fluid layer when the denser hanging wall is brittle rather than viscous. The sandstone injections satisfy the first three criteria above: layered structure, lower density material below, and (probably) near constant thickness in the black rock layer. However, characteristics of the sandstone-bounded intrusions violate the other four criteria. The injection orientation and spacing are consistent with activation of pre-
existing joint patterns in the sandstone hanging wall. The outcrop where black rock underlies a solid sandstone hanging wall does not at all resemble sedimentary Raleigh-Taylor instabilities, while the outcrop where black rock underlies granular fault rocks closely resembles it. Flame structure morphologies develop only where both materials behave ductilely at the strain rate of deformation.

**Flow Analysis**

Buttressed by the analogy with soft-sediment structures, we can now more confidently proceed to apply a gravitational instability model to fault rocks for the first time. We now use the insight that the driving force of the flow is a density instability to analyze the vertical motion of the black rock. The intrusion of a low density layer into an overlying high density layer is a well-studied phenomenon known as a Rayleigh-Taylor instability [Turcotte and Schubert, 2002]. When the density of the lower layer, \( \rho_1 \), is less than that of the upper layer, \( \rho_2 \), then the system is unstable and the lighter, black rock will intrude into the cataclasite (Figure 4). The buoyancy forces are balanced by viscous stress and the inertia of the fluid. The vertical velocity, \( U \), of the intrusions varies with the wavelength. Linear stability analysis shows that there is generally an optimal wavelength, \( \lambda_c \), that grows fastest and thus dominates the resulting structure [Chandrasekhar, 1961; Conrad and Molnar, 1997; Turcotte and Schubert, 2002]. The linear stability analysis prediction of the optimal wavelength is consistent with physical laboratory experiments [Berner et al., 1972; Wilcock and Whitehead, 1991]. If shear is also present, the intrusive features will be inclined relative to the vertical; this horizontal motion is independent of the buoyant growth that determines \( \lambda_c \).
We numerically solve for the growth rate as a function of wavelength for a viscous fluid of thickness $H$ overlain by an infinite layer of denser, viscous fluid (Figure 5). The boundary conditions are no-slip at the top and bottom of the system (Appendix A). Figure 5 shows the results for a fluid with negligible inertia in both dimensionless form and with units appropriate for the Kodiak Island black rock. For the cases in which the viscosity is high enough to dominate over the inertia and the viscosity is the same in both layers, a single dimensionless curve completely describes the solution for a particular geometry \[\text{Turcotte and Schubert, 2002}\]. For instance, the solid blue curve in Figure 5a is the solution for a thin viscous layer overlain by an infinite layer. As expected, there is an optimal wavelength $\lambda_c$ that grows fastest. For the case of equal viscosity in both layers, the most unstable wavelength $\lambda_c = 3.7 H$. Note that $\lambda_c$ is independent of viscosity and density as long as inertia is negligible.

We also compute the growth as a function of wavelength for a power law fluid where strain rate is proportional to shear stress to the power of $n$ ($n=1$ for a Newtonian fluid) using a modification of a standard analysis for folding and boudinage \[\text{Smith, 1977}\] (Appendix A). Magma can have $n=1.5$ and solid state rocks can have $n=2-5$. Figure 6 shows the most extreme case with $n=5$ \[\text{Sonder et al., 2006; Twiss and Moores, 1992}\]. The growth rate for the power law fluid is significantly greater than the Newtonian case, but the wavelength of the maximum growth is nearly unaffected by the rheology ($\lambda_c/H = 3.2$).
The black rock is more stretched and cuspate than the cataclasite and therefore we infer that the black rock is the less viscous of the two layers [Davis and Reynolds, 1996]. Incorporating this complication increases the values of both $\lambda_c$ and $\lambda_c/H$ as shown by the red curve in Figure 6.

For the Kodiak Island black rocks, the wavelength of the features over seven measurable intrusions (Figure 2) is $4.7\pm1.5$ cm and the layer thickness is $3.5\pm0.6$ cm where the error ranges are 1 standard deviation. Therefore, the ratio $\lambda_c/H$ is $1.4\pm0.5$ (vertical line in Figure 5 with horizontal error bars). The real system generated instabilities at much shorter wavelengths than can be explained easily by a Rayleigh-Taylor instability model without inertia. This result holds true even if we consider a finite thickness of the upper layer (dotted and dashed lines) or a viscosity contrast between the layers. Therefore, we conclude that the non-inertial model is a poor fit to the observed structures.

Inertial flows have a broader spectrum of unstable wavelengths and the most unstable value is smaller than in the purely viscous case. Using the existing formalism of Chandresekhar [1961] and Mikaelian [1996], we repeat the calculation of growth rate as a function of wavelength for parameter ranges where the inertial effects are significant (Figure 6). The Reynolds number, Re, measures the relative strength of the inertial and viscous effects. For the present problem, $Re=UH\rho_1/\eta_1$ and values greater than 1 indicate that inertia is important.
We can increase the Reynolds number by decreasing the viscosity. Reynolds numbers of 20-60 have peak growth rates at wavelengths consistent with the observations for the range of plausible densities (Figure 6). For the maximum density contrast of 30%, the viscosity that will allow sufficient inertia to match the $\lambda_c/H$ data is 0.5-1 Pas. For a smaller density contrast of 1.4%, the requisite viscosity is 0.1-0.3 Pa s. In no case can the viscosity of the black rock during emplacement be much greater than 1 Pa-s.

In Figure 6a and b, both viscosities are the same. If the upper layer (cataclasite) is appreciably stiffer than the black rock, then the critical wavelengths for a given black rock viscosity and density contrast will be higher (Figure 6c). For instance, if the cataclasite is 10 times more viscous than the black rock, a 30% density contrast requires a black rock viscosity contrast of 0.1-0.2 Pa-s.

From Figure 6b, the growth rate of the instabilities with the observed values of $\lambda_c/H$ is $1-10 \text{ s}^{-1}$. For $H=0.035$ m, the growth rate is 0.04-0.4 m/s. The horizontal shear rate and vertical growth rate are comparable, so the structures were generated at a slip rate of 10’s of cm/s, which is typical of an earthquake or immediately after. Thus, the black rock was mobilized during an earthquake. The mobilization could have occurred by a variety of processes including melting, elevated pore pressure or intergranular collision [Bagnold, 1956; Otsuki et al., 2003]. The exposure preserves the structures solidified after the generation of the fluid and thus represents the last stages of the earthquake slip. Once fluidized, it moved rapidly and buoyantly intruded the overlying cataclasite at a relatively high Reynolds number.
The shear stress in the fluid layer can be inferred by combining the velocity and viscosity constraints. The viscous shear resistance in the core is $\eta U/H$. For the maximum velocity (0.4 m/s) and viscosity (1 Pa-s) inferred above, the maximum shear stress for the 0.035 cm thick fault zone is 10 Pa.

**The Origin of the Black Rock**

Now that we have a constraint on the viscosity of the black rock, we can use it to evaluate the candidate origins of the black rock. Pseudotachylytes (frictionally-induced melts) are one of few established ways to infer rapid slip from geologic data [Biegel and Sammis, 2004; Cowan, 1999; Di Toro et al., 2005; Magloughlin, 1992; Sibson and Toy, 2006]. However, pseudotachylytes are apparently rare and their identification is difficult [Otsuki et al., 2003; Sibson and Toy, 2006; Ujiie et al., 2007]. We can calculate the required temperature for a melt of the observed composition, pressure and fluid conditions to achieve the geometrically constrained viscosity. Should the inferred temperature be plausible, then pseudotachylytes are a permissible mechanism.

The high pressure and water content of the accretionary prism permits fully saturated melts of unusually high water contents. The black rock composition is equivalent to a high silica andesite (Table B1). At 380 MPa (13 km) and 1400°C, a melt of the composition of the black rock would be fully saturated with 6.6 wt% water and have a viscosity of 1 Pa-s (Appendix B). A pseudotachylyte at these conditions is consistent with the wavelength constraint of the intrusions. Such a high temperature is also consistent
with the minimum bound of 1100°C imposed by the rarity of feldspar survivor grains in the black rock [Rowe et al., 2005]. The inferred temperature and water content is consistent with that inferred for fault slurries with local melting observed in the Nojima fault [Otsuki et al., 2003].

Another possible origin of the fault rock is as a fluidized granular flow [Meneghini et al., 2006; Rowe et al., 2005; Ujiie et al., 2007]. Unfortunately, a similar test is not possible for the possibility of a fluidized granular flow as the rheology has never been investigated at the appropriate pressure and temperature. The little data that exists suggests that mobilized slurries can have viscosities of up to ~1-10 Pa-s [Brodsky and Kanamori, 2001; Major and Pierson, 1992; Otsuki et al., 2003]. The bottom of the viscosity range may be consistent with the geometrical conditions.

We therefore conclude that either the fluidized slurry or the extremely high-temperature melt are consistent with the rheological constraints on the black rock. The geometry may not help resolve the question of the origin of the black rocks, but the consistency with plausible regimes in both cases helps bolster the Rayleigh-Taylor interpretation of the flow.

**Summary and Conclusions**

The extremely short wavelengths of the intrusions observed on the exhumed Kodiak Island megathrust indicate that the black fault core rocks represent a very low viscosity fluid emplaced during an earthquake. This conclusion is based on the geometry of the
intrusive structures and supported by measurements of the densities of the units. No a priori knowledge of the origin of the fault rock is necessary.

Although the buoyant intrusive features are only formed where the overlying cataclasite is fluid enough to accommodate ductile intrusions, the black rock persists over 2.5 km of strike-parallel exposure. At this locale the fault was extensively lubricated with a fluid that supported little shear. Elsewhere on the fault, direct contact between the wall rocks may have resulted in high shear stresses, but in the fluid layer the shear stress across the fault is at most 10 Pa.

The rock record has provided an example of extreme fault weakening that easily satisfies geophysical constraints suggesting low friction [Lachenbruch and Sass, 1980]. The picture that begins to emerge is a fault zone controlled by multiphase processes including local, extraordinarily weak zones of low-viscosity fault fluids flowing rapidly during an earthquake.

Appendix A: Linear Stability Analysis

The main text solves four closely related linear stability problems: the buoyant intrusion of a Newtonian fluid into another Newtonian fluid for both finite and infinite overlying layers, the buoyant intrusion of a power law fluid into another power law fluid and the buoyant intrusion of a Newtonian fluid into an infinite Newtonian fluid with inertia. All of the Newtonian fluid configurations have either been solved in the literature or require
only minor modifications of existing solutions. The power law fluid case requires a more significant modification.

The general solution method is outlined below. Here we primarily follow the notation of Smith [1977] who studied the closely related problem of viscous folding. For a Newtonian fluid with negligible inertia, combining the momentum and continuity equations results in the biharmonic equation

\[ \nabla^4 \psi = 0 \]  

(1)

where \( \psi \) is the stream function. The velocity field of the fluid is \( u = \frac{\partial \psi}{\partial y} \) and \( v = -\frac{\partial \psi}{\partial x} \)

where \( v \) is the flow velocity in the \( x \) direction and \( u \) is the flow velocity in the \( y \) direction. The shear measurements indicate that vertical growth rate \( u \) is comparable to horizontal shear rate \( v \), establishing a basis for control on \( v \). The linear stability of the system is investigated by assuming a separable solution to \( \psi \) of the form

\[ \psi(x, y, t) = \phi(y) \exp[iax] \exp[\gamma t] \]

where \( \phi(y) \) is an appropriate function of \( y \), \( \gamma \) is the growth rate, the wavenumber \( a = 2\pi / \lambda \) and \( \lambda \) is the wavelength. Surface tension is neglected.

Substituting \( \psi \) into eq. 1 results in

\[ \phi''' - 2a^2 \phi'' + a^4 \phi = 0 \]  

(2)

where primes are derivatives with respect to \( y \). The boundary conditions can also be posed in terms of \( \phi \). The coordinate system is chosen such that the bottom of layer 1 is \( y = 0 \). The thickness of layer 1 is \( h \) and if the top layer is finite, the thickness of the two
layers together is $h_2$. There is no slip and a continuity of traction between the layers, so the boundary conditions at $y = h$ are

$$\varphi_1 = \varphi_2$$  \hspace{1cm} (3)

$$\varphi_1' = \varphi_2'$$  \hspace{1cm} (4)

$$(\varphi_1'' + a^2 \varphi_1) = m(\varphi_2'' + a^2 \varphi_2)$$  \hspace{1cm} (5)

$$(\varphi_1''' - 3a^2 \varphi_1) - m(\varphi_2''' - 3a^2 \varphi_2) = -\Delta \rho g a^2 \varphi_1 / \eta_1$$  \hspace{1cm} (6)

where the subscripts 1 and 2 indicate the bottom and top layer, respectively, $\Delta \rho$ is the density difference between the top and bottom layer ($\rho_2 - \rho_1$), $\eta_1$ is the viscosity of the bottom layer and $m$ is the ratio of the viscosities ($m = \eta_2 / \eta_1$). Note that the subscript convention is reversed from Smith [1977]. The last boundary condition eq. 6 comes from the stress equilibrium at the interface and incorporates buoyancy [Conrad and Molnar, 1997]. This term is the most important difference between the folding problem of Smith [1977] and the gravitational instability studied here.

An appropriate form of $\varphi(y)$ that respects the boundary conditions must be chosen. For the Newtonian cases, $\varphi(y) = A \exp[ay] + B \exp[-ay] + C \exp[ay] + D \exp[-ay]$. If the top layer is infinite, $A = B = 0$ in that layer (layer 2). Otherwise, at the top of the layer $y = h_2$, no slip is achieved by

$$\varphi_2 = 0$$  \hspace{1cm} (7)

$$\varphi_2' = 0$$  \hspace{1cm} (8)
The no slip condition at the bottom of the lower boundary is achieved at \( \gamma = 0 \) with the equations

\[
\phi_1 = 0 \quad (9)
\]
\[
\phi_1' = 0 \quad (10)
\]

The solution for the growth rate \( \gamma \) as a function of wavenumber \( a \) is found by posing the boundary conditions as a matrix system \( MG = 0 \) where \( G \) is the vector of constant coefficients (\( A, B, D \) and \( E \) for each layer). For instance, for the case of an infinite top layer, the matrix \( M \) =

\[
\begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 \\
-a & 1 & a & 1 & 0 & 0 \\
-e^{-ah} & -e^{-ah}h & -e^{ah} & -e^{ah}h & e^{-ah} & e^{-ah}h \\
-ae^{-ah} & e^{-ah}(ah-1) & -ae^{ah} & -e^{ah}(1+ah) & -ae^{-ah} & e^{-ah}(1-ah) \\
2a^2 e^{-ah} & 2a^2 e^{-ah} & 2a^2 e^{ah} & 2a^2 e^{ah} & -2a^2 e^{-ah} & -2ae^{-ah}(ah-1) \\
2a^3 e^{-ah} & 2a^3 e^{-ah} & -2a^3 e^{ah} & -2a^3 e^{ah} & \frac{a^2 e^{-ah}(\Delta \rho g - 2a\gamma \eta)}{\gamma \eta} & \frac{a^2 e^{-ah} h(\Delta \rho g - 2a\gamma \eta)}{\gamma \eta}
\end{bmatrix}
\]

Solving the equation \( Det(M) = 0 \) provides a solution for the growth rate \( \gamma \) as a function of wavenumber \( a \). We solve this equation numerically for every case although analytical solutions are possible for the simplest configurations studied here.

For power law fluids the rheology is \( \sigma^n \propto \dot{\epsilon} \) where \( \sigma \) and \( \dot{\epsilon} \) are the stress and strain rate tensors and \( n \) is a material constant. Smith [1977] performs a perturbation on the rheology as well as the deformation field for a basic flow of horizontal compression or
extension. He shows that the linear stability equations are similar to the Newtonian fluid case. We extend the analysis for a basic flow of simple shear. In this case, the equations are identical to the Newtonian fluid case except eq. 6 becomes

\[
(\phi_1'' - (2W_1 + 1)a^2 \phi_1') - m(\phi_2'' - (2W_2 + 1)a^2 \phi_2') = -n \Delta \rho g a^2 \phi_i / \gamma \eta_i
\]  

where \( W = 2n - 1 \) and the subscripts of 1 and 2 denote the bottom and top layers, respectively. In this case, the function \( \phi(y) = \Sigma_{i=1,4}A_i \exp[l_i y] \) where \( l_i \) are the four values of \( \pm a \sqrt{W \pm \sqrt{W^2 - 1}} \) for each layer [Smith, 1977].

For the high Reynolds number case, the inertial term makes the right-hand side of eq. 1 non-zero and a streamfunction is no longer an appropriate tool for solving the problem. Chandrasekhar [1961] uses a very similar method based on a perturbation expansion of the flow velocity rather than the streamfunction. He explores the case with negligible advection, but significant inertia from the local acceleration of the fluid. The eigenfunctions in this case include an inertial term [Chandrasekhar, 1961, Sect. 94]. This method is used here with the modifications for a finite layer on the bottom introduced by Mikaelian [1996].

**Appendix B: Melt Viscosity and Density Calculation**

We calculated the viscosity of a silicate melt with the bulk composition measured from the black rock samples (Table B1) using the program Conflow [Mastin and Ghioroso, 2000]. We renormalize the composition to 100% without the LOI and find that the composition is equivalent to a high silica andesite. Conflow implements the MELTS algorithm to minimize the energy of the silicate melt-water mixture using laboratory
values for the thermodynamic properties [Ghiorso, 1995]. The energy minimization yields
the water solubility at a given pressure, temperature and composition. The result is then
used to calculate the viscosity for the appropriate composition in accordance with Shaw
[1972]. MELTS also calculates the melt density at the prescribed temperature, pressure
and water content by again combining empirical laboratory data with thermodynamic
constraints.

The volume percentage of survivor grains from the pre-melted structure was measured in
thin sections. The resulting percentage of clasts during melt mobilization is << 10%,
therefore, the melt viscosity is a good indicator of the mixture viscosity. At the
emplacement conditions (380 MPa, water-saturated), the temperature must be 1450°C to
achieve a viscosity of 1 Pa-s with 10% solid fragments.
<table>
<thead>
<tr>
<th>Unit</th>
<th>Mean (kg/m$^3$)</th>
<th>Standard deviation (kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black rock - bulk</td>
<td>2685</td>
<td>19</td>
</tr>
<tr>
<td>Cataclasite - bulk</td>
<td>2722</td>
<td>23</td>
</tr>
<tr>
<td>Sandstone - bulk</td>
<td>2683</td>
<td>7</td>
</tr>
<tr>
<td>Black rock - grain</td>
<td>2726</td>
<td>45</td>
</tr>
<tr>
<td>Cataclasite - grain</td>
<td>2849</td>
<td>19</td>
</tr>
</tbody>
</table>

**Table 1. Density Measurements.** Reported values are the dry bulk density of eight cataclasite, seven black rock, and two sandstone samples, and grain density on three powdered samples. We measured the volume of one inch core or sawed rectangular solid samples, and the volume of powders with a Quantachrome gas comparison pycnometer. We determined mass with a scale sensitive to $10^{-6}$ kg. Up to four repeat measurements were performed on most samples with standard deviations of 0.5%.
<table>
<thead>
<tr>
<th>Oxides</th>
<th>Black Rock Weight %</th>
<th>Cataclasite Weight %</th>
</tr>
</thead>
<tbody>
<tr>
<td>SiO₂</td>
<td>58.93</td>
<td>60.38</td>
</tr>
<tr>
<td>TiO₂</td>
<td>0.90</td>
<td>0.80</td>
</tr>
<tr>
<td>Al₂O₃</td>
<td>17.55</td>
<td>16.28</td>
</tr>
<tr>
<td>FeO</td>
<td>6.66</td>
<td>6.21</td>
</tr>
<tr>
<td>Fe₂O₃</td>
<td>0.33</td>
<td>0.52</td>
</tr>
<tr>
<td>MnO</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>MgO</td>
<td>2.71</td>
<td>2.85</td>
</tr>
<tr>
<td>CaO</td>
<td>1.24</td>
<td>1.74</td>
</tr>
<tr>
<td>Na₂O</td>
<td>4.13</td>
<td>2.34</td>
</tr>
<tr>
<td>K₂O</td>
<td>1.52</td>
<td>2.37</td>
</tr>
<tr>
<td>P₂O₅</td>
<td>0.25</td>
<td>0.23</td>
</tr>
<tr>
<td>Lost on Ignition</td>
<td>4.93</td>
<td>5.43</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>99.28</strong></td>
<td><strong>99.29</strong></td>
</tr>
</tbody>
</table>

Table B1. Median composition Kodiak Island black rock and foliated cataclasites.
Figure 1. Geologic Map of Pasagshak Peninsula showing location of ultrafine-grained fault rock, i.e., the “black rock” that is the subject of this paper. A. Regional location map. B. Map of Pasagshak Peninsula. Thickness of black rock is exaggerated to better show its location and its lateral extent of 2.5 km. C. Cross section. Note that black rock occurs in three layers; only the structurally lowest black rock layer is included in the associated cataclasite; the others occur at the boundary with sandstone beds. Surface datum for cross section is sea level and three separate sub-sections connect through equivalent structural positions along strike. The position of the photo in Figure 2 is marked on the cross-section in inset C.
Figure 2. Photograph and explanatory sketch of the outcrop of black rock in Kodiak Island, AK. Lens cap diameter on bottom pictures is 6 cm for scale. Upper left sketch shows general outcrop scale appearance of black rock layer cutting at a low angle the cataclasite fabric (Thin discontinuous lines mark structural fabric). Close up view in lower photograph and sketch show ductile intrusions of black material in hanging wall cataclasite. Cataclasite fabric is locally disrupted around intrusions (see fabric around lens cap and compare it with sub-planar fabric of lower cataclasite layer), suggesting that cataclasites were able to ductilely deform at time of intrusion and did not respond brittly to upward flow. Deflection of the intrusive structures due to shear consistent with thrust motion is also visible, especially in taller intrusions. Upper right inset illustrates only the intrusion geometry, showing the wavelengths between intrusions as measured on the outcrop. These are the wavelengths used in calculations.
Figure 3: Sedimentary flame structures, Carmelo Formation, Pt. Lobos, California. **Pencil diameter is 8 mm.** Note systematic wavelength and direction of deflection, and intrusion perpendicularity to basal bedding contact.

Figure 4. **Geometry of the Rayleigh-Taylor problem.** The densities $\rho_1$ and $\rho_2$ and viscosities $\eta_1$ and $\eta_2$ are of the lower and upper layers, respectively. The intrusions rise with a vertical velocity $U$. 
Figure 5. Growth rate of the instability as a function of wavelength for viscous (low Reynolds number) flows. (a) Dimensionless and (b) dimensional results shown for each case. Calculations are done with \( H = 0.035 \, \text{m} \), \( \rho_1 = 2685 \, \text{kg/m}^3 \), \( \rho_2 = 1.014 \rho_1 \) and \( \eta_1 = 10^4 \, \text{Pa s} \). As discussed in the text, the blue solid line in (a) is non-dimensionalized to be independent of these values as long as inertia is negligible. The dotted and dashed lines are for two layers of finite thickness and equal viscosity. The top layer is the same thickness as the bottom for the dotted line (0.035 m) and is 0.3 m (the observed thickness) for the dashed line. The gravitational acceleration \( g \) is 9.8 m/s\(^2\).
Figure 6. Growth rate of the instability as a function of wavelength for a variety of Reynolds numbers. (a) Dimensionless and (b) dimensional results for identical viscosity in both layers. (c) Dimensionless results for a more viscous upper layer. Plus symbols are a density contrast of 1.4% and solid lines are 30% for each case. As in Fig. 5, the thickness of the bottom layer is 0.035 m, the top layer is infinite and $\rho_1 = 2685 \text{ kg/m}^3$. In (a) and (c), the Reynolds number Re are given for the most unstable wavelength of each curve.
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