Slip-predictability and dynamically fluctuating rupture speeds on a glacier-fault, Whillans Ice Plain, West Antarctica

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Abstract

Bi-daily, tidally modulated stick-slip speed-ups of the Whillans Ice Plain (WIP) provide insight into glacier dynamics and failure at a naturally repeating fault asperity. We deployed a network of continuously-operating GPS receivers in 2007 and operated on-ice broadband seismometers during the austral summer of 2008 on Whillans Ice Stream (WIS), West Antarctica and recorded 26 glacier speed-up events. Previous work during the 2004 field season suggested that these speed-ups initiate as failure of an asperity on or near “Ice Raft A” that triggers rupture across the entire WIP. Our results for 2008 locate the slip initiation farther to the south of this feature, closer to the grounding line and the southernmost extent of the Ross Ice Shelf. A strong correlation between the amplitude of seismic waves generated at the rupture front and the total slip achieved over the duration of the slip event (~ 30 min) suggests slip-predictable behavior, or the ability to forecast the eventual slip based on the first minute of seismic radiation. Successive slip events propagate with different rupture speeds (100-300 m/s) that strongly correlate ($R^2 = 0.73$) with the recurrence interval. In addition, the amount of slip achieved during each event appears to be correlated with the rupture speed. We calculate one component of the basal shear stress, which we call frictional stress and show the transmission of seismic radiation to the far-field is dependent upon the degree of basal friction. Our work suggests that the far-field transmission of seismic waves from glacier action is not dependent upon bulk ice movement, but rapid basal stress changes. We constrain basal shear stress using two approaches a.) considering the stick-slip cycle, including slip event unstable sliding and inter-event stable sliding and b.) assessing conditions for basal freezing. The combined constraints yield a range of 0.2 – 4 kPa for the basal shear stress. Our observations yield important information regarding mechanics and dynamics of ice stream beds at the scale of 10s to 100s of km. Subglacial processes are
notoriously difficult to constrain on these large scales, which are relevant to the understanding of regional and continental ice motion.

1. Introduction

The recently discovered episodic slip events on the Whillans Ice Stream in Antarctica [Bindschadler et al., 2003] indicate that locally, the glacier suddenly speeds up to a slip rate of 10s of cm per hour for 20-30 minute periods, twice a day. Wiens et al. [2008] have shown that during these slip events slip velocity of the bulk ice becomes fast enough to generate seismic waves, observable at far-field stations near the South Pole (QSPA) and the Dry Valleys, Antarctica (VNDA).

Twice daily stick-slips are intrinsically interesting as a direct indication of the mode of glacier motion (and failure). The episodic slip also provides a window into the physical conditions at the base of the glacier. Periodic episodic movement in a system driven by far-field, steady motion is a hallmark of stick-slip as most commonly evidenced by earthquakes on tectonic faults [Brace and Byerlee, 1966]. Winberry et al. [2009] suggest that this basic stick-slip model is applicable to the ice sheet. The base of the glacier can be locked by friction between events. As stress accumulates elastically in the glacier, the base is loaded. Eventually the local frictional stress is overcome and the glacier jerks forward, releasing the elastic stress and beginning the cycle again. In this scenario, the timing, size, and location of the slip events provides information about the basal friction and its control on overall glacial movement.

In this paper, we use field data to closely study these episodic slip events in order to identify observational constraints on the conditions at the base of the glacier. We follow previous modeling with our own data collection efforts, and show that the transmission of far-field
seismic signals from slip initiation is dependent upon the bed friction. Also, we show that the seismic radiated energy at the rupture front scales with slip displacement. The rupture speed of the events is strongly modulated by the stress accumulation during inter-event times and scales with slip displacement, suggesting that a pervasive healing process occurs across the entire WIP.

2. Fieldwork

During December of 2007, we established a network of continuously-sampling GPS stations on WIS/WIP at strategic locations to capture the hydrology and dynamics of an active subglacial lake network [Fricker et al., 2007]. GPS receivers record a position every 15 seconds and have operated continuously since installation, with plans to operate through 2014. The operation of the GPS network will make the WIS/WIP the longest-continually monitored glacier system at sub-minute temporal resolution.

Following the establishment of the GPS network, in the November/December 2008 field season, we operated broadband seismometers on WIS. The broadband seismometers operated at 100 Hz and all but one (A702) were co-located with continuous GPS stations. Figure 1 shows the positions of GPS stations and seismometers. In addition to on-ice broadband seismometers, we used data from the Global Seismic Network (GSN), including stations QSPA and VNDA, which are about 650 and 990 km away, respectively, from WIP. Their location relative to our field site appears on Figure 1.

3. Observations

3.1 Slip Event Description

3.1.1 Slip events in the near-field
We recorded a series of slip events and Figure 2 displays a typical slip event at an on-ice station, which included a co-located continuous GPS receiver and a broadband seismometer. The top pane of Figure 2 includes the GPS record, and the gray box encapsulates the duration and total slip of the event. Slip events typically last 25-30 min and slip 0.2-0.5 m. The middle pane of Figure 2 shows the east component of ground velocity recorded by the broadband seismometer, high-pass filtered at ~3,000 seconds (0.0003 Hz). The small amplitude, medium period, portion of the wave that is contained within the red box arrives at the initiation of GPS-observed slip. Throughout the remainder of the paper, we will refer to this portion of the wave as the initiation phase. The last pane of Figure 2 depicts the integrated seismic record, in order to obtain ground displacement. The remainder of the seismic record is the displacement of the station as it moves along with the slipping ice (permanent offset), corresponding with the GPS record. Our on-ice broadband seismic recordings are the first reported recordings of a stick-slip event from on-ice at WIS.

For every slip event, a seismic initiation phase arrives synchronous with the start of GPS slip. The initiation phase is a <100 s period wave, recorded by the broadband seismometers and shown in the red box on Figure 2. An example of this initiation phase at all the on-ice broadband stations is shown on Figure 3. The seismic records have been bandpassed (0.012-0.04 Hz) in order to highlight the energy contained within the initiation phase. The rupture front spreads from its origin and the amplitude of the initiation phase is proportional to the elastic energy radiated as the rupture front propagates along the ice/till interface, a relationship we will show in a later section.

In addition to recording the initiation of the slip events as the rupture front passes beneath each station, some of the on-ice stations also record evidence of the rupture front stopping at a
discrete location ~100 km from its origin. Figure 3 depicts this stopping phase as observed in the near-field records, which we call Phase B. Far-field seismic records show at least two stopping phases, and we describe these in more detail in the following section.

3.1.2 Slip events in the far-field: QSPA and VNDA

We obtained broadband seismic data from far-field stations VNDA (~990 km from WIP) and QSPA (~650 km from WIP) for the period of time our on-ice stations were operating. Examples of waveforms from these stations appear on the lower half of Figure 3. Figure 3 contains an example, for station VNDA, of a triple-phased long period waveform first described by Wiens et al. [2008]. For this paper, we refer to the three phases on the far-field record (as shown on Figure 3) as Phase A, Phase B, and Phase C. Of the 26 slip events observed during our 2008 field season, only 5 had all three phases (A, B, and C) visible on VNDA records, with the majority lacking the first phase (Record sections for all of the slip events are including as Supplementary Material). Phase B is visible on VNDA records for 22 of 26 events.

Wiens et al. [2008] suggested that the triple-phased long-period waveforms visible at stations VNDA and QSPA are Rayleigh waves representing a starting and two stopping phases of the WIP slip. They located the initiation of the slips at “Ice Raft A”, whose outline appears on Figure 1. They interpreted the Phase B and C arrivals to be stopping phases and located both using a surface wave grid-search technique. They located Phase B ~50 km south of “Ice Raft A”, near the region we suggest the slips start (see Figure 1 and below for location methods), and Phase C ~120 km north of “Ice Raft A”.

3.2 Location of Slip Start

Slip-start locations for stick-slip events recorded during our 2008 field season appear on
Figure 1. By assuming that the rupture speed of each event represents an average rupture speed across the entire ice plain and compiling a record of arrival times for the initiation phases at each near-field station, we implement a 2-D (x, y, t) linear least squares inversion to locate the positions where slip events start (See Appendix A for a detailed description). We also calculate error ellipses, with two standard deviations representing the 95% confidence interval; these are shown on Figure 1. The zone encompassed by our locations and error ellipses is south of our seismic network and adjacent to the grounding line. The zone does not intersect “Ice Raft A.”

Events 19 and 20 are particularly diagnostic (Supplementary Material), as the arrivals occur within ~10 seconds at M6 and Whigh. If the initiation was at “Ice Raft A,” the arrivals would be at a minimum 100 seconds apart.

In a previous field season, during 2007, ice penetrating radar was operated in a transect crossing the suture zone where WIS and Mercer Ice Stream (MIS) converge and continue to flow to the Ross Ice Shelf. The transect crosses the GPS stations M6-M8 and continues along this general line southwest. The cross-section is presented on Figure 4, with ice flow direction oriented directly at the reader. The slip events locate adjacent to this suture zone, in the area corresponding to low reflectivity.

3.3 Slip events are magnitude predictable

In order to understand the rupture energy, we compare the amplitude of the initiation phase with the total slip of the event. The total slip displacement over the 20-30 min duration of each slip event is well-constrained by the GPS record. The five panels on Figure 5 show the east component amplitude of the initiation phase (east axis is parallel to ice flow direction) versus GPS slip displacement at one particular GPS station, M8. There is a strong linear correlation
between amplitude and GPS slip for all stations. The correlations (see Figure 5 for $R^2$, p-value, and slope of each fit) imply that the events are slip-predictable, with the probability of random occurrence (p-value) being less than 1% at stations M6, Cookie, and Crevasse. For example, one could predict the total slip occurring over a 20-30 minute time period, merely by measuring the amplitude of the initiation pulse.

In addition to the scaling of the amplitude of the initiation phase with slip achieved during the event, the slip events appear to stop at a rate that scales with slip. In general, the amplitude of seismic waves is proportional to the moment rate (Lay and Wallace, 1995), which is proportional to the slip rate; thus, seismic waves are generated during acceleration or deceleration of slip. Figure 6 shows the relationships between slip and Phase B amplitude.

3.4 Fluctuating rupture speed

The difference in time between the arrival of stopping Phase B at station VNDA and the start of a slip event detected at on-ice stations varies greatly, ~11–24 min and the approximate travel-time for a Rayleigh wave originating at WIS to travel ~1000 km is 5.5 minutes. This suggests that the time for the slip to rupture across the WIP varies from ~5 – 18 min. Due to the clustered nature of the GPS network, we have limited constraints upon the actual area slipped during each slip event. We will assume the length of the rupture does not change for subsequent events and investigate the timing variability by inferring that the rupture velocity fluctuates for each slip event (The alternative, varying the rupture length, is also investigated below). Previous work [Wiens et al., 2008] showed that the rupture speed varies from 0.1-0.2 km/s for each event and fluctuations correlate with the tide height.

We begin our rupture velocity calculation by assuming that the far-field arrivals
propagate as a Rayleigh wave, and use an approximate Rayleigh wave speed of $\sim 3$ km/s. For the purposes of our rupture speed determination, we assume that a generalized area ruptures (length 100 km x width 100 km). We use the relation below to calculate rupture speed using our observed moveouts:

$$
V_{\text{rupture}} = \frac{L_{\text{rupture}}}{t_{\text{VNDA}} - \frac{d_{\text{far-field}}}{V_{\text{Rayleigh}}}}
$$

(1)

Figure 7 and the accompanying caption provide a schematic and description of our rupture speed inference. We use an average value of 3 km/s Rayleigh wave speed ($V_{\text{Rayleigh}}$), distance far-field is 990 km ($d_{\text{far-field}}$), and length of rupture is 100 km ($L_{\text{rupture}}$). The $t_{\text{VNDA}}$ is the difference in time between on-ice initiation and far-field arrival of the Phase B long period wave. We primarily use VNDA arrivals for rupture speed analysis, as they are detected more frequently. Typical calculated values for rupture speed vary between 0.1-0.3 km/s (See Supplementary Table 2).

Alternatively, we can allow the rupture length, $L_{\text{rupture}}$, to vary, keeping a fixed rupture speed for all of the events. A fixed rupture speed of 0.1 and 0.3 km/s correspond to average rupture lengths of 90 and 270 km, respectively. The length of 270 km exceeds any dimension of the WIP and 90 km is smaller than the distance between many of the stations and the furthest slip start location. The rupture speed fluctuations therefore seem to be the more plausible cause of the large range (~!11-24 min) of arrival time variations at the far-field station.

A closer look at the Phase B and C separation also provides important constraints upon the varying nature of the rupture speeds. In order to assess phase and/or amplitude variations between subsequent events, we plot various waveforms for VNDA with respect to each rupture velocity as inferred from Equation 1 on Figure 8a. The figure shows that the lag between Phase
B and Phase C arrivals gradually increases with decreasing rupture velocity. This suggests that
Phase B and C, as stopping phases, have fixed locations, a feature also identified by Wiens et al.
[2008]. Multiplying the time difference between the Phase B and Phase C arrivals by the
maximum and minimum rupture speeds, read on Figure 8a, yields a consistent value of ~ 60 km,
which is a minimum difference in rupture distance perpendicular to the wavefront from the
initiation point, for the two stopping locations.

Wiens et al. [2008] published a catalogue of slip event origin times for their field season
in the austral summer of 2004. We obtained VNDA data for 2004 and with use of the Wiens et
al. [2008] catalog, we repeated the steps described above and obtained rupture velocity estimates
consistent with our observations in 2008. Figure 8b shows records for 2004, plotted similarly to
Figure 8a. The 2004 data contain more Phase A arrivals than are present for slip events in 2008.
Slip events with lower rupture velocities seem to preferentially generate Phase A arrivals, in both

Another distinguishing feature of these ice slip events is that the total slip achieved over
the slip events scales with the rupture velocity. Figure 9 indicates a strong correlation ($R^2$ value
of 0.79 and p-value of 0.0005) between the GPS-recorded slip and inferred rupture velocity
calculated using the above equation. Wiens et al. [2008] qualitatively observe a correlation
between Ross Sea tidal amplitude and rupture velocity. Winberry et al. [2009] model the stress
balance on the WIP from the tides and showed that the high tide increases shear stress,
contributing to the observation that high tide correlates with slip. Therefore, the correlation
between rupture velocity and slip was implicit in the earlier work, but was not specifically
addressed.
3.5 Inter-Event Stable Sliding

The WIP does not remain locked during inter-event periods. Inter-event surface displacement account for ~50% of the total daily motion for most stations within our network. Due to the short recurrence intervals, such motion is elastically accommodated and basal sliding occurs during inter-event periods. We calculate the average stable sliding velocity by differencing positions after a slip event and prior to the subsequent event and dividing by the recurrence interval. We observe that the average stable sliding velocity (creep velocity) varies for subsequent inter-event periods, as shown on Figure 10. Figure 10 shows that for low recurrence interval events, the creep velocity is relatively high. Also, after the semi-diurnal period, ~12 hours (0.5 day), the creep velocity averages out to a value of ~0.5 m/day for stations M8, M9, and Cookie. Station Whig has a relatively low creep velocity, however, as it slips nearly twice as much as M8 and M9 during slip events (See Supplementary Table 2). Therefore, while the nature of the stick-slip episodes may vary for two stations on-ice, surface velocities are similar when averaged over a longer time period.

4. Interpretation

4.1 Slip Start Location

Our locations (see Figure 1) and error ellipses, delineating the 95% confidence interval, show that the slip starts in a region adjacent to the southern section of the WIP grounding line. Due to the overlapping error ellipses, we cannot assess whether the events nucleate in a critical slipping region or if there is a separate origin for each event. Our interpretation of the far-field seismic record is that the Phase A arrival is the start of the slip event, Phase B is a stopping phase up-ice, and Phase C is another stopping phase. The
location of the slip event origins is different than a previous study [Wiens et al., 2008], which
located the slip starts at “Ice Raft A” and suggested that “Ice Raft A” acts as an asperity. Wiens
et al. [2008] implemented a grid-search inverting for the origin using the GPS arrival times and a
surface wave grid-search to determine the source of Phase B and C arrivals. Their interpretation
of these locations was that Phase A represents the slip start at the location of “Ice Raft A” and
the two trailing phases are stopping phases of the slip located near the grounding line.

The difference between the locations determined by these studies has three possible
explanations. First, WIS is undergoing a century scale slowdown [Bougamont et al., 2003].
This slowdown likely causes a redistribution of the basal stresses and it is plausible that the slip
start zone may migrate over the period of 4 years, the time separating the two experiments. The
WIS also contains a dynamic subglacial lake network at its base [Fricker and Scambos, 2009],
which could cause stress re-distributions at relatively short timescales. Different measurements
techniques for the two experiments may also be a source of the discrepancy; Wiens et al. [2008]
locations are based on inversions of GPS data compared to our seismic data.

The locations also seem to suggest that friction plays an important role in the
accumulation of inter-event strain, as the suture zone, shown on Figure 4, extends down-ice to
the zone where the two slip start locations have the smallest error ellipses (Figure 1). As shown
on the lower panel of Figure 4, the bed is more reflective below WIS-sourced ice, compared to
MIS-sourced ice, and the height of the overlying ice increases to the southwest, towards more
clustering of our origin locations. Ice-penetrating radar reflectivity is influenced by dielectric
properties, and a strong bed reflection suggests a subglacial zone with abundant water and/or
water that is highly conductive [e.g. Raymond et al., 2006]. Weaker bed reflection indicates
transition to subglacial materials that contain less water and/or water that is less conductive. A
plausible interpretation of the observed ‘dimmer’ bed south of the suture zone is that ice there is overriding bedrock, or a thinner till layer than in the area to the north of the suture zone. This suggests that the slip origin region is frictionally stronger than the bed beneath the main part of the WIP.

4.2 Implication of Initiation Phase: Predicting Slip

Figure 5 suggests that the eventual slip of the ice stream scales with the amplitude of the pulse emitted at the rupture front. Although the amplitude of the initiation phase varies from station to station for any given event, the relative size of the initiation phase at a single station correlates with the final GPS slip of the event. This observation implies that the magnitude of the slip events is deterministic based on the conditions during the first 30-100 s of slip. This ice slip event behavior is in contrast with tectonic earthquakes where the connection between the initiation and the final size of an earthquake is subtle enough to remain controversial [Rydelek and Horiuchi, 2006].

This difference between ice and tectonic slip events is echoed by another, closely related observation. The ratio of displacement to rupture length varies by a factor of two for the ice events and the ultimate magnitude of the event is controlled by the variation in displacement, not rupture length. This again is in contrast to most earthquakes where rupture length varies in proportion to displacement [Kanamori and Anderson, 1976] with only rare exceptions in places with well-separated stick-slip patches [Harrington and Brodsky, 2009].

Both of these observations are consistent with an isolated stick-slip patch with relatively simple driving conditions. An isolated patch is expected to have regular, deterministic motion as opposed to the chaotic behavior of strongly coupled patches [Burridge and Halliday, 1971]. The
stick-slip section of the WIP appears to be poised in a transitional basal regime surrounded by a combination of steady sliding and fixed boundaries.

Examination of Figure 3 reveals that there are differences in amplitude at different stations across the Ice Plain. These differences may represent variabilities in bed properties at different scales along the ice. However, generally, the linear trend with GPS slip supports the notion that a healing process during inter-event periods occurs across the entire Ice Plain and is highly localized to the basal boundary layer. Geometrically, the stick-slip region is confined, as suggested by the scaling of the stopping phase with total slip, as shown on Figure 6. This supports the notion that the slip initiation phase runs into fixed boundaries. Thus, the momentum is transferred to the ground surface and releases elastic energy, with amplitude scaled by the momentum in the stopping ice.

4.3 Stress Drop and Basal Shear Stress

In order to better constrain evolution of basal shear stress, we will first assess the stress drop for each of the stick-slip events. Stress drop is a measure of the amount of elastic energy released during a slip event. On a circular fault plane, in an elastic half-space, \( \Delta \sigma \approx \frac{7\pi}{16} \mu \frac{d}{L} \), where the stress drop, \( \Delta \sigma \), is approximately the shear modulus, \( \mu \), times \( d \) the amount of slip, divided by \( L \), the length of rupture. However, because of the planar slab of ice with a high length-to-thickness aspect ratio, we need to perform a series of stress transformations. First, we consider the stress equilibrium, \( \nabla \cdot \tau = 0 \), and assume that the derivatives in the y-direction are negligible,

\[
\frac{\partial \tau_{xz}}{\partial z} = \frac{\partial \tau_{zx}}{\partial x} \quad \text{or} \quad \Delta \tau_{xz} = \frac{H}{L} \Delta \tau_{xx}, \tag{2}
\]
where $H$ is the thickness of the ice stream (~600 m), $L$ is length (~100 km), $x$ is the slip direction, and $z$ is depth. Next, we use Hooke’s law for the stress-strain relationship,

$$\Delta \tau_{xx} \sim \frac{d}{L} E,$$  \hspace{1cm} (3)

where $E$ is Young’s modulus (~10 GPa for ice). Combining Equations 2 and 3, the shear stress drop, $\Delta \sigma$, is

$$\Delta \sigma \sim \Delta \tau_{xz} \sim \frac{dH}{L^2} E.$$  \hspace{1cm} (4)

Equation 4 is similar to one proposed and used in modeling efforts [Bindschadler et al., 2003; Winberry et al., 2009] to approximate driving stresses up-ice in between stick-slip events. The constant of proportionality for Equation 4 is typically of order one and depends on geometry [Lay and Wallace, 1995]. Since the shape of the slipping patch is largely unconstrained, we use Eq. 4 in its simplest form, as equality, i.e., a proportionality constant of 1, to measure the stress change due to each slip event. The resultant stresses range from 50-300 kPa and appear on Figure 11 plotted against rupture speed.

As could be anticipated based on the raw observations in Figure 9, large rupture velocities occur for events with large stress drops. The correlation between displacement and initiation phase amplitude in Figure 5 implies that the initial process also becomes more vigorous with increasing stress drop. The picture that is beginning to emerge is that the stress during rupture initiation controls the resultant initiation phase, rupture velocity, far-field radiation and ultimately, total slip.

For tectonic earthquakes, it is generally accepted that earthquakes of all sizes have stress drops independent of size [Kanamori and Anderson, 1975], between 0.1-10 MPa. The consistent range of stress drops holds for even the largest M9 earthquakes. The observation in Figure 11
combined with Figure 5 is in marked contrast to tectonic earthquakes as stress drop is varying
with total slip. However, the observation is consistent with isolated asperity cases, like Parkfield,
where occasionally stress drop is seen to covary with earthquake size [Harrington and Brodsky,
2009].

To some extent, the apparent difference between glacial and tectonic behavior is an effect
of the constant value of rupture length use in eq. 4 based on the relative weak constraints on
rupture length reviewed above (constant to within a factor of 3). However, given the co-variation
of other observables such as rupture velocity and phase A coupling with the slip discussed
below, a variation of actual stress with even size appears to be reasonable for the glacial slip
events. This interpretation bears further investigation with future deployments of more dense
instrumental arrays that can capture directly both $L$ and $d$.

To consider stress drop in a broader context, we define the effective basal shear stress, $\tau_b$,
as resisting the driving stress, and neglect margin stresses,

$$\tau_b = \sigma_{\text{friction}} + \sigma_{\text{residual}}.$$  \hspace{1cm} (5)

The effective basal shear stress must be positive and so we introduce the residual stress, $\sigma_{\text{residual}}$,
as a minimum basal shear stress, needed to balance Equation 5 (as will be shown later).

Physically, this contribution beyond friction may be due to unrecoverable deformation of the till.

Previous studies have modeled the force (stress) balance at opposing ends of the WIP
[Bindschadler et al., 2003; Winberry et al., 2009], where the tidal stress resists an up-ice stress
on the WIP. We follow from the previous work and introduce

$$\sigma_{\text{driving}} = \sigma_{\text{up-ice}} - \sigma_{\text{tide}} + \sigma_{\text{residual}}.$$  \hspace{1cm} (6)

If the system is in stress equilibrium, the driving stress will balance with stress drop plus basal
shear stress, such that
In Equation 7, the residual stress cancels and the stress drop, within a stress balance equation for glacier stick-slips is

\[ \sigma_{\text{driving}} = \Delta \sigma + \tau_b. \]  

We calculate driving stress (as in Equation 6) using the aspect-ratio stress relation in Equation 4, but instead, substitute the creep displacement, \( d_{\text{creep}} \), for \( d \), while assuming the residual stress is zero (We will return to this assumption shortly). We use a Young’s modulus \( (E) \) of 10 GPa, length \( (L) \) of 100 km, thickness \( (H) \) of 600 m. For tidal stress, we use the equation from Winberry et al. (2009),

\[ \sigma_{\text{tide}} = \rho_{\text{ice}} g z_{\text{tide}} \frac{H}{L}. \]

We obtained tide height, \( z_{\text{tide}} \), from an inverse Ross Sea tidal model [Padman et al., 2003] and use 915 kg/m\(^3\) for density of glacier ice. Previous analysis [Winberry et al., 2009], assumed that the driving stress is best represented by an annually average velocity of the WIS at upstream of the WIP. However, the actual observed creep displacement, \( d_{\text{creep}} \), is a more appropriate choice for our analysis, because it introduces friction into the force balance.

We can solve for frictional stress from Equation 8. The motion of the ice stream during inter-event stable sliding is highly modulated by the tide (e.g. Figure 10) and so the frictional stress varies with time. Figure 12 shows the frictional stress plotted against recurrence interval for various slip events. The figure suggests that the frictional stress is highly sensitive to tidal periodicities. Negative values should not interpreted as negative stress, but rather reflect a minimum value for residual basal stress, \( \sigma_{\text{residual}} \), that is not consumed during the previous slip event. Based on this logic and Figure 12, the minimum residual stress is 200 Pa.

The residual stress is any stress within the till/basal ice interface not relieved during a slip.
event, in Equation 5. To assess the remnant elastic till shear strain associated with such a small
residual stress, we consider that the shear wave speed in a material, is \( V_s = \sqrt{\mu/\rho} \), where \( \mu \) is
shear modulus of till and \( \rho \) is till density. Blankenship et al. [1987] report a shear wave speed of
170 m/s for Ice Stream B till. Assuming a till density of 2,000 kg/m\(^3\), we calculate a till shear
modulus of 58 MPa. Equation 3 provides the stress-strain relationship, however, we substitute
shear modulus for Young’s modulus, and the strain associated with a 200 Pa shear stress is
3.5x10\(^{-6}\). Given this strain, if till deformation were concentrated within a 5 m zone,
displacement of the till would be 17 microns. Joughin et al. [2002] reported values for basal
shear stress of the WIP of 1.1 ± 0.9 kPa. The lower range of the error estimate falls within our
minimum residual stress. The added frictional stress increases the total basal shear stress, until a
till failure threshold is reached. A 17 micron deformation would represent less than 0.01% of a
~0.5 m slip event. Only small strains are fully recoverable elastically [e.g. Lay and Wallace,
1995]. Therefore, we infer that the residual stress is till deformation, not fully recoverable when
considering the large deformation occurring at the ice/till interface.

Of particular note on Figure 12, is that the low recurrence interval slip events occur
during a rapidly falling tide, where the observed driving stresses (\( d_{\text{creep}} \) is relatively high) are at a
maximum and stress drop is low due to the low recurrence interval (less accumulated strain).
During 2008, we observe Phase A arrivals in the far-field during just 5 of the 26 events. To
begin with, based on the location of the slip origins on Figure 1 and our interpretation of Figure
4, the slip start origins locate in an area of higher friction. In addition, when we examine the
frictional conditions under which Phase A is propagated at a sufficient amplitude above the noise
level, we see that Phase A events qualitatively correspond to high frictional stress (Figure 12, red
squares are superimposed upon M8 frictional stress and red diamonds upon Cookie frictional
stress). This suggests that the moment rate increases for these local accumulations of stress.

Under such conditions, the ice would be relatively well-coupled to the bed. Another contributing factor to the absence of far-field signals is that, for all other events, much of the energy in the slip initiation may not be transmitted to the bedrock. Poor coupling, due to rising tides, would reduce the amplitude, making Phase A undetectable.

4.4 Inter-Event Basal Freezing

The variation in rupture speed with slip (Figure 9) indicates a strong strength dependence upon the basal boundary layer. For typical earthquakes, the shear wave speed limits the rupture velocity, so that typically earthquakes rupture at a speed approximately 0.9 times the shear wave speed [Kanamori and Brodsky, 2004]. Blankenship et al. [1987] reported a shear wave speed for in-situ till of approximately 170 m/s at a location approximately 300 km away from our field area. We report rupture velocities for the WIP in the range 100-300 m/s for our highly idealized length-scale. In most fracture mechanics relations, the shear wave speed is not solely representative of only the till layer, but is the surrounding elastic medium, inclusive of basal ice, till, and marine sediments. Wiens et al. [2008] report rupture velocity variations within a similar range (0.1-0.2 km/s) that we observe in 2008.

The WIP undergoes stick-slip events usually twice daily, which is evidence for a relatively fast healing process. The healing is time-dependent, as supported by the correlation between rupture speed and recurrence interval on Figure 13. The healing process must also be pervasive, across the length and width of the WIP/WIS (100 km x 100 km), in order to achieve elastic slip during the subsequent event. In order to explain the correlation between increased tidal loading and recurrence interval, Winberry et al. [2009] suggested a time-dependent healing
process. They suggested basal freezing as the likely inter-event healing process. In order to evaluate the likelihood of such a process, we modify an equation from Patterson [1994] for the dependence of basal melting/freezing on a basal heat budget so that freezing is positive,

\[ f_r = \frac{-G - \tau_b u_b + k_i \Theta_b}{L_i \rho_i} \]  

(10)

where \( f_r \) is freeze rate, \( G \) is the geothermal gradient, \( \tau_b \) and \( u_b \) are basal shear stress and velocity, \( k_i \) is thermal conductivity of ice, \( \Theta_b \) is the basal temperature gradient, \( L_i \) is latent heat of fusion, and \( \rho_i \) is the density of ice. In assigning values for the above constants, we follow previous assumptions for the area [Tulaczyk et al., 2000; Joughin et al., 2002], and use \( G = 0.07 \) W/m\(^2\), \( k_i = 2.1 \) W/m°C, \( \Theta_b = 0.046 \) °C/m, \( L_i = 333 \) kJ/kg, and \( \rho_i = 917 \) kg/m\(^3\). We use our observed values of inter-event stable sliding at station M8 for values of \( u_b \). For basal shear stress, we use a constant value of 1 kPa, as previous work suggests a value of 1.1 ± 0.9 kPa [Joughin et al., 2002]. We calculate melt rate, \( f_r \), and multiply by the recurrence interval to obtain an estimate of the freeze, where positive values represent freezing. Figure 14 shows the linear relationship between inter-event basal freezing and rupture speed.

The freeze rate calculation (Equation 10) is highly sensitive to the product of \( \tau_b u_b \). For the basal shear stress, \( \tau_b \), we used the nominal value of 1 kPa, based on Joughin et al. [2002]. In order to assess the validity of this value and place a maximum constraint on basal shear stress, we varied the basal shear stress and calculated the associated freezing. Figure 15 shows the associated freezing expected based on Equation 10, when the basal shear stress is allowed to vary between 0 – 5 kPa. The basal interface transitions into melting near the value of 4 kPa for the range of velocities we observe at our stations. Joughin et al. [2004] shows that the annually averaged velocities for the Whillans Ice Stream have been decreasing over the last few decades and suggested basal freezing as the primary mechanism for such a slow-down. Numerical
models suggest a similar mechanism for shutdown [Bougamont et al., 2003]. During the unstable sliding (slip event), the basal velocity is at least an order of magnitude faster and promotes melting. Based on our observations of the range of inter-event sliding velocities, in order to obtain net freezing over long time periods in Equation 10, the basal shear stress is constrained to be below the 4 kPa value.

The basal slip velocity and basal shear stresses are sufficiently low to promote freezing. In addition, we showed that at its current surface velocity, the WIP promotes basal freezing, at sub-daily timescales, up to a limit of 4 kPa, where it transitions to promoting melting. Our results suggest inter-event freezing at maximum of 7 microns. Direct evidence does not exist of such small magnitude basal freezing, nor would such a small change be easily directly detected between subsequent slip events. However, such a mechanism is proposed here as the cause of the rupture speed variability.

5. Conclusions

Based on the work described in this paper, we come to the following conclusions:

1. The slip locates in a region shown on Figure 1, near a zone where friction is high, and the rupture front propagates across our network with amplitude of the initiation phase proportional to the eventual slip.

2. The higher the inferred friction, the higher the resultant rupture velocity and amplitude of the far-field initiation phase (Phase A). The frictional stress in the slip start location must reach a threshold in order for certain waves to be transmitted to far-field distances (Phase A).

3. The minimum value of residual stress after a slip event is shown to be ~200 Pa. We
suggest that the residual stress is small, non-elastic deformation of the till.

4. The rupture speed varies for successive slip events and scales with both slip and recurrence interval. This suggests a pervasive healing process and we have shown that during inter-event sliding, the shear heating equation suggests that the ice stream base undergoes freezing, which may strengthen the basal zone. The maximum basal shear stress is 4 kPa.

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Appendix A: Slip-start location inversion

We implement a 2-D (x, y, t) linear least squares inversion to locate the slip event start location. For the typical inversion for earthquake source [Stein and Wysession, 2003: 7.2], we first assume a velocity, in our case we use that the rupture speed unique to each particular slip event (whose determination is discussed in Section 3.4). Given an initial guess of the matrix \( m \),
whose components are the origin location and time \((x, y, t)\), the distance \(d\) can be obtained from \(m\), with the observations of the arrival times at each station times, \(A\), so that \(d_i = A(m_i)\). A guessed model \(m\) is improved by determining how our arrival times differ from the model-predicted arrival times. We define the matrix \(G\) as \(G_{ij} = \frac{\partial d_i}{\partial m_j}\), or simplified in matrix notation as \(\Delta d = G\Delta m\). Here, \(\Delta m\) can simply be determined by taking the inverse of \(G\), however, a better solution, for cases when the matrix \(G\) is non-square would be the generalized inverse, \(\Delta m = (G^TG)^{-1}G^T\Delta d\). We run multiple iterations of the inversion, adding successive values of \(\Delta m\) to the previous iteration \(m\), to obtain an origin location and time. In our case unique 2-D case, \(m\) is a 3x1 matrix, \(G\) is a \(n\times3\) matrix, and \(d\) a \(n\times1\) matrix, where \(n\) is the number of stations for that particular observation.

For the model regression described above, error ellipses can be determined by calculating the errors associated with the model parameters, so that \(\sigma_m^2 = \sigma^2(G^TG)^{-1}\) [Stein and Wysession, 2003: Section 7.2, Equation 34]. Two standard deviations represent the 95% confidence interval and are shown as ellipses on Figure 1.

**Supplementary Material**

Supplementary material includes record sections of all on-ice and far-field stations used for analysis. Records have been bandpassed 0.012-0.04 Hz to highlight energy associated with the on-ice initiation phase. Some stations experienced various issues with data quality deeming it unusable, however, that data is shown unedited and unaltered. Station A702 experienced power-related issues throughout the field season and data was unusable for Slip Events 8, 9, 11, 14, 15, 18, and 23. Far-field data at QSPA and VNDA experienced either electronic issues or teleseismic earthquakes that made arrivals undetectable for Slip Events 3, 6, 13, 21, and 22. No
data or poor data was collected for all stations (on-ice and far-field) for Slip Events 10, 12, 24, 25, and 26. The arrival times in Supplementary Table 2 may be used to cross-reference which data was used for each slip event.
Figure 1: Station location map depicting continuous GPS network and broadband station names for the 2008 field season. Subglacial lake geometry is shown as IceSAT tracks, adapted from Fricker et al. [2007]. Grounding line is shown, adapted from Horgan and Anandakrishna [2006]. The green shaded circles 95% confidence level error ellipses encircling slip-start locations, shown as green squares. Yellow squares indicate slip-start locations with only three station observations. The suture zone is drawn in reference to the presence of buried crevasses detected by airborne radar sounding [Shabtaie et al., 1987]. The suture zone is the boundary between Mercer- and Whillans-sourced ice.
Figure 2: GPS displacement (top), seismic ground velocity (middle) and displacement (bottom) at a co-located site. Red box indicates what is referred to as medium-period slip initiation pulse and the grey box shows magnitude and duration of the slip event.
Figure 3: Seismic record section of a single event observed at both on-ice and far-field stations. At the on-ice stations, the initial pulse (solid gray line through color stations) indicates the rupture front propagating across the ice plane and its modeled arrival at far-field stations (most events in 2008 do not transmit sufficient Phase A energy to be detectable above the noise). The dashed and dashed-dotted lines are the seismic waves associated with Phase B and Phase C. Note that the speed of the initial pulse is approximately an order of magnitude slower than the speed of the secondary pulse.
Figure 4: Ice-penetrating radar line collected across a transect from stations M6-M8, continuing southwest. Ice flow direction is directly towards the reader. Brighter reflections typically indicate the presence of conductive pore water [Raymond et al., 2006].
Figure 5: Slip displacement at a single GPS station versus amplitudes of the seismic initiation for various stations in our network. The p-value reflects the probability of the null hypothesis being valid of non-correlated values being valid. All stations show a strong correlation.
Figure 6: Far-field Phase B amplitude measured at broadband station VNDA and GPS slip.
Figure 7: Schematic of method for far-field inference of rupture speed. (1.) Rupture initiates at a location adjacent to the grounding line, (2.) propagates across the ice at an average value determined by Equation 1, (3.) the rupture stops spreading at one stopping point which transmits Phase B of the far-field wave, (4.) the rupture stops at another stopping point and transmits Phase C of the far-field wave. Note in the figure that relative locations of Phase B and C are given for illustrative purposes. Also, no Phase A is present on the shown seismogram for this particular event, but its approximate position on this schematic is shown for clarity.
Figure 8: Waveforms of a suite of slip events observed at station VNDA in a.) 2008 and b.) 2004, plotted as a function of rupture speed. The time difference between Phase B and Phase C arrivals varies linearly with rupture speed, such that an estimated distance of 60 km separates the physical locations of Phase B and Phase C decelerating points.
Figure 9: Rupture velocity as inferred from the farfield records and locally measured GPS slip.
Figure 10: Measured inter-event stable sliding (creep) velocity based on GPS and recurrence interval.
Figure 11: Stress drop plotted as inferred from the GPS slip of Figure 9 and rupture speed. Two different sites (M8 and Cookie) are used due to lack of available GPS data spanning the entire field season (See Supplementary Table 2).
Figure 12: Frictional stress ($s_{\text{friction}}$), where two different sites (M8 and Cookie) are used due to lack of available GPS data spanning the entire field season (See Supplementary Table 2).
Figure 13: Rupture velocity as inferred from the far-field records and recurrence interval.
Figure 14: Basal freeze-on calculated from Equation 10 and the GPS data from Figure 10 and rupture velocity for the resultant slip event, inferred from farfield stations. Basal freeze-on is calculated over the duration of inter-event time leading up to the slip event.
Figure 15: Freezing for a range of interevent basal shear stress $t_b$ in Equation 10. This calculation is used to constrain the range of basal shear stresses that are realistic, and as freezing must be positive over the long-term, $t_b$ must be less than 4 kPa based on the range of velocities we observe.