Connecting near and farfield earthquake triggering to dynamic strain

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Abstract: Any earthquake can trigger more earthquakes. This triggering occurs in both the classical aftershock zone, as well as the farfield. These populations of triggered earthquakes may or may not be distinct in terms of triggering mechanism. Here we look for a distinction between the populations by examining how the observed intensity of triggering scales with the amplitude of the triggering strain in each. To do so, we apply a new statistical metric based on earthquake interevent times to a large dataset and measure earthquake triggering as a function of dynamic strain amplitude, where strain is estimated from empirical ground motion regressions. This method allows us to identify triggering at dynamic strain amplitudes down to $3 \times 10^{-9}$, orders of magnitude smaller than previously reported. This threshold appears to be an observational limit, and shows that extremely small dynamic strains can trigger faults that are sufficiently near failure. Using a probabilistic model to transform measured interevent times to seismicity rate changes, we find that triggering rates in the farfield scale with peak dynamic strain. This scaling, projected into the nearfield, accounts for 15-60% of earthquakes within 6 km of magnitude 3-5.5 earthquakes. Statistical seismicity simulations validate the interevent time method, and show that the data are consistent with the number of farfield triggered earthquakes being linearly proportional to peak dynamic strain. We interpret the additional nearfield component as reflecting either static stress triggering, more effective
dynamic triggering at higher frequencies, or a concentration of aftershock nucleation sites very near mainshocks.

1. Introduction

Triggered earthquakes provide a window into the physics of earthquake nucleation because the forces initiating rupture can be inferred. Because the strain at which a fault is triggered is a measure of its strength, it may be possible to gain insight into the distribution of fault strength by studying the statistics of earthquake triggering [Brodsky and Prejean, 2005; Gomberg, 2001].

One strategy for ultimately determining the processes involved in triggering is to first identify the stresses that activate the process. The types of stresses that have been proposed as the agents by which one fault transmits a triggering signal to another include coseismic static strain changes, progressive postseismic strain changes (including afterslip and viscous creep), and dynamic strains from radiated seismic waves [Freed, 2005]. These agents may each trigger earthquakes through a number of mechanisms, including direct Coulomb frictional failure, reduction in fault strength, and pore fluid pressure changes. Different triggering agents may be expected to be relatively more or less effective at activating different failure mechanisms. Therefore, understanding the relative contribution of triggering agents may help constrain triggering mechanisms.

The proposed triggering agents have different strengths and weaknesses in explaining observed triggering. Coseismic static strain changes are the increase or decrease in strain
at one fault due to the relaxation of strain at another during an earthquake. Static strain changes are permanent and therefore can easily explain triggering for an extended period of time [King et al., 1994], but the stresses decay in amplitude quickly with distance away from a fault and are thus unlikely to trigger distant earthquakes. Multiple stress interactions, i.e. secondary triggering, or aftershocks of aftershocks, can extend the reach of static stresses to a few times the length of the primary rupture [Ziv, 2003], but not to the tens of times observed for remote triggering [Hill et al., 1993]. Postseismic static strain changes are generated by afterslip or lower crustal ductile creep and also produce prolonged stresses. Afterslip produces a quasi-static nearfield stress change that has comparable distance decay to coseismic static stress changes and thus cannot explain triggering at great distances. Viscous deformation can propagate to great distances but takes years to reach hundreds of kilometers [Freed and Lin, 1998; Pollitz et al., 1998], and thus cannot explain distant triggering that is sometimes seen within days or even seconds [West et al., 2005]. Dynamic triggering is associated with transient strains carried by radiated seismic waves [Hill and Prejean, 2007]. Dynamic strains decay less quickly with distance, and therefore do well at explaining remote triggering, but present a challenge in explaining prolonged triggering [Brodsky, 2006; Gomberg, 2001].

The above summary highlights that there is more certainty about the agent of triggering for a special subset of earthquake triggering: distant triggering over timescales of less than a few years. These remotely triggered earthquakes are believed to result entirely from dynamic triggering, both because static strains are negligible at large distances, and because distant triggering often coincides with the arrival of surface waves [Anderson et
al., 1994; Brodsky et al., 2000; Gomberg and Johnson, 2005; Hill et al., 1993]. On the other hand, short-range triggering is much more ambiguous, and has been attributed to static, postseismic, or dynamic agents by different researchers [Felzer and Brodsky, 2006; Gomberg et al., 2003; Kilb et al., 2000; Perfettini and Avouac, 2007; Pollitz and Johnston, 2006; Stein et al., 1994; Velasco et al., 2008]. Spatial correlations between calculated coseismic static stress fields and aftershock patterns appear to support coseismic static stress triggering [King et al., 1994; Stein et al., 1994]. On the other hand, correlations between temporal aftershock evolution and geodetic strain measurements support progressive postseismic strain [Freed and Lin, 1998; Peng and Zhao, 2009; Perfettini and Avouac, 2007]. Still other studies that compare nearfield aftershock locations to the static and dynamic strains for earthquakes with strong directivity conclude that dynamic strains correlate better with the ensuing seismicity [Gomberg et al., 2003; Kilb et al., 2000].

In this study, we exploit the understanding that dynamic strain is the dominant triggering agent at large distances and relatively short timescales in order to constrain the contribution of additional triggering agents in the nearfield. We first determine an empirical relationship between a measure of triggering intensity and peak dynamic strain in the farfield, based on the waiting time to early triggered earthquakes. Then we compare this farfield relationship to nearfield observations to assess the proportion of nearfield earthquakes that can be explained by the farfield proportionality. We ultimately find that dynamic triggering can account for a significant portion of nearfield aftershocks, but that there is an additional triggering component in the nearfield. Whether this reflects
additional triggering agents (e.g. static strain, afterslip) or the effect of second-order aspects of the dynamic strain (e.g. duration, frequency) we cannot resolve. In the process, we develop a measure of earthquake triggering that is significantly more sensitive to low triggering rates than previous measures and place a new bound on the threshold for farfield dynamic triggering.

The first several sections of this article concern the development of the triggering metric. First, we define a statistic based on earthquake interevent times and establish the expectation for this statistic, assuming a simple probabilistic model for earthquake occurrence times. Next, we delineate populations on the basis of local dynamic strain and describe the data selection and processing. In Section 4 we apply the method and show that the dynamic triggering relationship determined for farfield quakes can account for roughly half of nearfield aftershock triggering. We also report a new estimate on the dynamic triggering threshold in California of $3 \times 10^{-9}$ strain, several orders of magnitude lower than previous estimates. A statistical seismicity simulation is then used to interpret and validate the results. Finally, we evaluate the implications and robustness of the results.

2. Measuring earthquake rate changes using interevent times

A comparison of triggering rates in the near and farfield requires a metric that can be applied to both populations of earthquakes. This metric needs to be sensitive enough to detect the very small triggering rates associated with the very small dynamic strains common to the farfield. Previously, triggered earthquakes have been identified by
inspecting seismicity rates [Harrington and Brodsky, 2006; Hill et al., 1993; Stark and Davis, 1996] or by filtering waveforms to emphasize short-period energy within the surface wave trains of large, distant earthquakes [Brodsky et al., 2000; Hill and Prejean, 2007; Velasco et al., 2008]. Quantitative estimates of triggering usually involve calculating the likelihood of observing a number of post-trigger events given the previous seismicity rate [Anderson et al., 1994; Gomberg et al., 2001; Hough, 2005]. If the likelihood of the rate increase occurring by chance is low enough, triggering is inferred.

Any estimate that computes likelihood of triggering based on counting the number of triggered earthquakes relative to a pre-trigger count, like the $\beta$-statistic [Matthews and Reasenberg, 1988], is limited in several ways. First, the pre-trigger seismicity rate must be resolved for comparison, and this is inherently difficult. Because most earthquakes occur as clusters of aftershocks, the seismicity rate is always changing. Background seismicity level should therefore be measured at a time as close to the purported trigger as possible in as short a window as possible. Different areas will permit different length windows depending on their background level of seismicity, and thus a constant window for an entire dataset may not sufficiently capture the data. Second, an earthquake count can only resolve an integer increase in the number of earthquakes for any individual sequence. A slight advancement in the timing of subsequent earthquakes will only rarely result in an additional triggered earthquake within the counting time window, so only large levels of triggering can be resolved with statistical significance. Finally, an earthquake count also includes all secondarily triggered earthquakes, that is, aftershocks of aftershocks. These secondary earthquakes are not strictly problematic, because they
should still be produced in proportion to the number of primary triggered earthquakes when averaged over many events, but they complicate the relationship between trigger amplitude and number of triggered quakes by introducing variance into the measurements.

To detect triggering at very low dynamic strain amplitudes, our metric must use an adaptive time window to measure background rates, be sensitive to small increases in seismicity rates, and be insensitive to secondary aftershocks.

2.1 The interevent time ratio \( R \)

We meet the above requirements by developing a statistic based only on the interevent times between the last earthquake before a trigger and the first earthquake after. We define the interevent time ratio \( R \) as

\[
R = \frac{t_2}{t_1 + t_2}, \tag{1}
\]

where \( t_1 \) and \( t_2 \) are the waiting times to the first earthquake before and after the putative trigger (Figure 1). The interevent time ratio \( R \) was originally developed to study triggered quiescence [Felzer and Brodsky, 2005]; we use it here to look for a triggered rate increase.

Because \( R \) is normalized by the average seismicity rate at the time of the trigger, we can pool and compare measurements made within a highly heterogeneous earthquake catalog. The strategy in this study is to measure the distribution of \( R \) on a population of earthquakes that are subject to similar triggering conditions. For instance, the population
can be drawn from a variety of areas subject to the same dynamic strain. The metric $R$ is a random variable distributed between 0 and 1. If there is no triggering in the population, and $t_2$ is on average equal to $t_1$, then $R$ is distributed uniformly with a mean value $\bar{R} = \frac{1}{2}$. On the other hand, if triggering does occur, $t_2$ will be on average smaller than $t_1$ and $\bar{R} < \frac{1}{2}$. More triggering results in a smaller $\bar{R}$ (Figure 2). Therefore, the statistic $\bar{R}$ provides a measure of triggering intensity within a population of earthquakes.

The interevent time ratio $R$ naturally solves the three problems identified with earthquake counting methods by defining an appropriate time window for each event based on the interevent times, utilizing the statistics of large populations, and focusing on the first recorded earthquake rather than the entire triggered sequence.

One of the unusual features of the interevent time method is that there is no time limit for the inclusion of triggered events. Both instantaneous and delayed triggering are included in the measurements. This comprehensiveness is desirable because of issues of catalog completeness, as well as the physical implications of delayed triggering. Instantaneous triggering ($t_2 \sim 0$) should be reflected in the $R$ distribution as a strong spike at $R \sim 0$ with a uniformly depressed probability density at higher values. Delayed triggered ($0 < t_2 < t_1$), should cause the spike to be spread out to larger $R$-values (e.g. Figure 2). In principle, the distribution of $R$ should therefore reflect the time-decay of the triggered rate change. For the purposes of this study, however, we restrict ourselves to calculations involving the sample mean $\bar{R}$ and do not consider the precise shape of the distribution. We will return to the issue of delayed triggering at the end of the paper after the metric has been
implemented.

2.2. Interpreting $R$ as seismicity rate change

The purpose of this study is to compare dynamic triggering levels in farfield and nearfield populations of triggers. However, the range of dynamic strain amplitudes observable in each population is different, at least in the California study region focused on here. This is because even the largest farfield earthquakes generate only low amplitude waves at great distance – so small that equivalent nearfield triggers are too low in magnitude to be detected in regional catalogs. Consequently, we measure the scaling of $R$ with dynamic strain in each population and compare the respective trends. Because $R$ must asymptote to 0.5 for zero rate change and to 0 for extreme rate increases, $R$ must scale nonlinearly with rate change. This nonlinearity complicates the comparison of trends between the two dynamic strain ranges. In order to compare triggered rate changes in the two populations, we first establish the expected behavior of $R$ as a function of rate change.

To derive the expectation of $R$ as a function of seismicity rate change, we use a model that relates interevent times to earthquake rate. If earthquake occurrence were perfectly periodic and uniform, the expectation of $R$ would follow directly from Equation 1 with $t_1$ and $t_2$ equal to the inverse of the earthquake rate. However, earthquakes are for the most part not periodic. Detailed studies of earthquake catalogs find that the distribution of interevent times is best characterized by a combination of Poisson distributed background earthquakes and triggered aftershocks that decay in time according to Omori’s law [Corral, 2004; Gardner and Knopoff, 1974; Hainzl et al., 2006; Molchan, 2006; Saichev
The time decaying Omori’s law component can be considered a nonhomogeneous Poisson process, that is, a Poisson process where the intensity is a function of time. This is a standard model for aftershock forecasting and is frequently used to analyze the significance of earthquake triggering metrics [Kagan and Jackson, 2000; Marsan, 2003; Matthews and Reasenberg, 1988; Ogata, 1999; Reasenberg and Jones, 1989; Wiemer, 2000]. We use it here to determine the expectation of $R$ as a function of triggered rate change.

Calculating the expectation of $R$ for this nonhomogeneous Poisson process requires an estimate of the background rate and the parameters in Omori’s law (Appendix A). These parameters may be heterogeneous in time and space and are impossible to determine for the single earthquakes that go into measurements of $R$. However, since we deal only with single earthquakes before and after the trigger, the details of the underlying interevent time distribution are relatively unimportant. Rather, we expect $R$ to be primarily sensitive to average rates. This restricted sensitivity allows us to approximate the nonhomogeneous process as a simple stepwise homogeneous process, characterized by an average rate $\lambda_1$ before the trigger and a new average rate $\lambda_2$ afterward. For the stepwise homogeneous Poisson process, the expectation of $R$ is given by (Appendix A)

$$
\langle R \rangle = \frac{1}{n^2} \left[ (n + 1) \ln(n + 1) - n \right],
$$

(2)

which is a function only of the normalized difference in average rate, or fractional rate change,

$$
n = \frac{\lambda_2 - \lambda_1}{\lambda_1}.
$$

(3)
As anticipated, the expectation of $R$ scales nonlinearly with rate change for the three models mentioned above: periodic, nonhomogeneous Poisson, and step-wise homogeneous (Figure 3). To emphasize the scaling for the small fractional rate changes, Figure 3 shows both the value of $\langle R \rangle$ and $\langle \Delta R \rangle = 0.5 - \langle R \rangle$. The stepwise homogeneous model is found to be an excellent approximation of the more complex nonhomogeneous model, especially for the small rate changes we expect to deal with in this study. This demonstrates that the metric $R$ is primarily sensitive to average seismicity rate change, rather than the details of the interevent time model.

As stated above, the nonlinearity in the scaling of $\bar{R}$ complicates the assessment of continuity between discontinuous farfield and nearfield populations. We can address this complication by transforming the observations, using Equation 2 to map the measured $\bar{R}$ into a modeled fractional rate change $n$. That is, we can interpret the measured $\bar{R}$ in terms of the number of triggered earthquakes required to produce that $\bar{R}$. Although we had no a-priori expectation for the scaling of $\bar{R}$ with dynamic strain, we do have an expectation for the scaling of $n$ with dynamic strain. Studies of traditional aftershock zones have found that the number of aftershocks scales with mainshock magnitude as $\log N \propto M$ [Felzer et al., 2004; Gasperini and Lolli, 2006; Helmstetter et al., 2005; Ogata, 1992; Yamanaka and Shimazaki, 1990]. Since peak ground velocity (a proxy for dynamic strain) also scales with magnitude as $\log PGV \propto M$, the aftershock scaling with magnitude is consistent with a power-law scaling of aftershock rate with dynamic strain. Studies comparing aftershock rates directly to peak ground velocities suggest that
aftershock rates may even be linearly proportional to dynamic strain, i.e. with a power-law exponent of ~1 [Gomberg and Felzer, 2008]. If our probabilistic model (Equation 2) is adequate, and assuming that at least the more conservative power-law scaling hypothesis holds, then the transformed statistic $n$ should scale linearly on a log-log scale. In Section 4 we will show that this hypothesis is confirmed by the data. This gives us the tool we need to compare relative triggering rates in near and farfield populations.

In what follows, we refer to $n$ as triggering intensity, to emphasize that it is a transformed statistic and not a direct measurement of fractional rate change. Nevertheless, Equation 2 represents a smoothly continuous transformation, and therefore the transformed statistic $n$ should not scale continuously with dynamic strain amplitude unless the primary statistic $\overline{R}$ does so as well.

3. Defining Populations

The statistic $\overline{R}$ (or the transformed statistic $n$) measures triggering intensity in a population of earthquakes. Therefore, the first step in applying the interevent time method to a real dataset is to define reasonable populations so that we can evaluate the different rate changes in each one.

Because long-range triggering is clearly associated with dynamic strain, we start by constructing sets of earthquakes with common dynamic strain amplitude. Other aspects of the seismic waves may be important in terms of triggering earthquakes, such as duration or frequency of the seismic waves. For instance, some studies suggest that
higher frequency waves are more effective triggers [Gomberg and Davis, 1996], while some attribute more triggering power to lower frequencies [Brodsky and Prejean, 2005]. Other studies find that frequency or number of oscillation cycles may modulate triggering power slightly, but these aspects are of secondary importance to peak amplitude [Savage and Marone, 2008]. Comparison between radial aftershock decay and ground motion attenuation at intermediate distances supports a scaling with peak or average ground velocities, independent of duration [Gomberg and Felzer, 2008]. Given the uncertainties regarding other aspects of the seismic waves, we choose to define populations based only on peak amplitude. Observed differences between near and farfield triggering intensities may therefore be attributed to second-order aspects of the wave field, and not only to other triggering agents such as static and postseismic strain.

Peak dynamic strain is roughly proportional to the amplitude of seismic waves

$$\varepsilon \approx \frac{A}{\Lambda} \approx \frac{V}{C_s},$$  \hspace{1cm} (4)

where $A$ is displacement amplitude, $\Lambda$ is wavelength, $V$ is particle velocity and $C_s$ is seismic wave velocity [Love, 1927]. In principle, dynamic strain can therefore be calculated wherever there is a seismogram. In this study, we use empirical ground motion regressions to approximate seismic wave amplitude, allowing us to extend strain estimates to any point on the map. Ground motions are converted to dynamic strain estimates by dividing by wavelength or wave speed, depending on whether the regression is for displacement or velocity, respectively.

In the nearfield, S-waves should carry the largest dynamic strains, with a dominant period
on the order of 1 second [Boatwright et al., 2001]. S-wave velocity at seismogenic depths is roughly 3.5 km/s. In the farfield, surface waves with periods near 20 seconds dominate ground motion, as smaller period waves tend to be scattered and attenuated [Lay and Wallace, 1995]. Both Love and Rayleigh waves can trigger earthquakes [Hill, 2008; Velasco et al., 2008], but the 20 second Rayleigh wave, with a velocity of ~3.5 km/s, is usually associated with the surface wave magnitude equation we use to estimate peak ground motions [Lay and Wallace, 1995]. We therefore treat this phase as representative of farfield dynamic strains. We note that if triggering is in fact dominated by Love waves, with a velocity of ~4.3 km/s, Equation 4 may overestimate strain by ~20% for equivalent amplitude waves.

Previous work has investigated the accuracy of using ground motions as a proxy for dynamic strain at depth by comparing strain estimated from seismometer data to strain measured by strainmeters [Gomberg and Agnew, 1996]. This study found that although there was considerable deviation in observed strains from those expected for a simple layered earth model, the dynamic strain amplitudes calculated from ground motions generally agreed within ±20% of the strainmeter measurements for periods between ~10-25 seconds, and within ±50% outside this band. These uncertainties are smaller than the factor of 2 uncertainties related to the empirical regressions used to estimate peak ground motions (see below). Since we are interested in the first-order scaling of triggering intensity with dynamic strain amplitude averaged over many earthquakes, we consider peak ground motions to be an adequate order-of-magnitude proxy for peak dynamic strains.
3.1. Estimating dynamic strain with empirical ground motion regressions

In order to take full advantage of the large number of earthquakes in the catalog, we must calculate dynamic strain at any point on the map for any trigger in the catalog, not only where we have seismic stations and archived waveforms. We therefore use empirical ground motion regressions to estimate strain as a function of distance from the potential trigger earthquakes. Peak ground motions are well studied at near and intermediate distances for estimating seismic hazard, and at regional and teleseismic distances for calibrating magnitude scales [Abrahamson and Silva, 2008; Boatwright et al., 2003; Campbell and Bozorgnia, 2007; Joyner and Boore, 1981; Lay and Wallace, 1995; Richter, 1935]. However, published regressions rarely focus on the very nearfield distances and small magnitudes of interest in this study, and are insufficient for our purposes. Previous researchers comparing aftershock distribution to peak ground motions have performed their own regressions with an emphasis on the nearfield [Gomberg and Felzer, 2008]. We also perform our own small-magnitude, nearfield PGV regression, using California Shakemap data [Wald et al., 1999]. Boatwright [2003] also used Shakemap data to make an empirical ground motion regression, but used only a small subset of the data available today and did not focus on the small distances of interest in this study.

For the nearfield regression, we follow the Next Generation Attenuation study of Campbell and Bozorgnia [2007] and use an equation of the form

$$\log_{10} \text{PGV} = c_1 + c_2 M - c_3 \log_{10} \sqrt{r^2 + c_4^2},$$

(5)
where PGV is peak ground velocity in cm/s, $M$ is earthquake magnitude, $r$ is hypocentral distance in km, and $c_i$ are fit parameters. The fit parameters are found by regression analysis using ~2000 PGV measurements from the Shakemap archives (Table 1). The details of the regression are given in Appendix B.

For farfield dynamic strain we use the surface wave magnitude relation [Lay and Wallace, 1995],

$$\log_{10} A_{20} = M_s - 1.66 \log_{10} \Delta - 2,$$

(6)

where $A_{20}$ is in $\mu$m and $\Delta$ is in degrees. This equation is commonly used to assign a catalog magnitude based on measured amplitude at some distance. We turn the procedure around and use the catalog magnitude to calculate amplitude. This approach uses the long-period waves ($T = 20$ s) as indicators of the peak dynamic strain, implicitly assuming that the short-period body waves are attenuated at large distances. The displacement $A$ is converted to velocity for the 20 second waves by the approximation $V \approx 2\pi A_{20}/T$ [Aki and Richards, 2002]. Equation 6 was historically calibrated using a similar catalog of global earthquakes to the one we use for potential triggers, and so provides a good measure of average amplitude despite being imperfect for any individual earthquake. The standard magnitude error for this regression, reflecting both inter-event variations (e.g. source depth) and intra-event variations (e.g. radiation pattern), has been tabulated for a selection of seismic stations [Rezapour and Pearce, 1998], and we compute a pooled standard error of 0.28 units. Through Equation 6, this corresponds to a factor of ~2 uncertainty in the estimated strain amplitude. The surface wave magnitude equation is designed for distances on the order of at least 800 km [Lay and Wallace,
1995] and this sets the minimum distance for the population of long-range triggers in this study.

3.2. Defining populations over space

A large farfield quake may trigger numerous earthquakes distributed throughout the study area. We therefore split the study region into spatial bins and calculate $R$ for each of these bins (Figure 4). This generates a number of $R$-values for each trigger and ensures that the measurements are not dominated by any single region with particularly high activity. Using a spatial bin that is much smaller than the wavelength of the long-range trigger also ensures that measured triggering intensity reflects the dynamic strain at that site. These $R$-values are then pooled according to peak dynamic strain as calculated by Equation 6, rather than according to their particular trigger. In this way, each strain bin incorporates numerous triggers at various combinations of distance and magnitude.

The higher the number of bins, the higher the number of bracketing pairs for each trigger quake, down to a lower size limit where single earthquakes begin to be isolated. A bin size of $0.1^\circ \times 0.1^\circ$ gives the maximum number of bracketing pairs for the whole catalog of potential triggers, but we perform the analysis using several different bin sizes to ensure robustness of the results with respect to parameter choices (Section 6.3).

To measure the time ratio $R$ for nearfield triggering, we search for the first earthquake before and after a potential trigger in a disk centered on the trigger earthquake epicenter. The radius of the disk is set to give the same area as in the farfield bins, e.g. for a farfield
bin of 0.1° × 0.1° we define the nearfield radius as ~6.2 km. The time ratios $R$ are then associated with the mean peak dynamic strain over the area of the disk. Assuming radial symmetry (i.e. approximating the trigger as a point source) and using Equations 4 and 5, the average peak dynamic strain in the nearfield disk is

$$\bar{\varepsilon} = \frac{PGV}{C_S} = \frac{1}{C_S \pi D^2} \int_0^D PGV(r) 2\pi r \, dr,$$

where $C_S$ is shear wave velocity, $PGV(r)$ is given by Equation 5, and $D$ is the radius of the disk.

We estimate the uncertainty in nearfield peak dynamic strain by bootstrap resampling the Shakemap data directly, refitting the regression formula, and recalculating the mean peak strain according to Equation 7. This is preferable to using the uncertainties in the regression constants themselves, because it accounts for strong correlations between the regression parameters. We resample 1000 times and report 95% confidence levels.

### 3.3 Earthquake catalogs

Using the interevent time method on earthquake populations requires large catalogs for both potential trigger earthquakes and for local seismicity. The trigger catalog is drawn from the global ANSS catalog from 1984 through April 2008. We choose the ANSS catalog because it includes both global earthquakes and local California earthquakes in a self-consistent catalog. More specialized regional or global catalogs may contain more carefully determined moment magnitudes and refined earthquake locations, but given the order of magnitude nature of our strain estimates, the small potential increase in accuracy is not worth the sacrifice in consistency.
For the global catalog, a depth cutoff is imposed at 100 km, because deep earthquakes do not generate significant surface waves. Only earthquakes with surface wave amplitude greater than ten micrometers displacement are treated as potential triggers. This minimum corresponds to a M$\text{L}$4.5 earthquake at 800 km. We find that this cutoff is sufficiently small to resolve an observational threshold for long-range triggering.

Potential nearfield triggers are drawn from the ANSS catalog for the California and Japan study regions. Other regional catalogs have considerably smaller location errors than the ANSS catalog, but contain considerably fewer earthquakes. However, location error should not be a significant source of error for this study, because the required spatial precision is on the order of the spatial bin size. We therefore choose the catalog with the largest number of earthquakes. The interevent time method should not be sensitive to regional variations in completeness magnitude, because the incompleteness should affect the pre and post-trigger catalogs in a consistent way. However, we impose a magnitude threshold of 2.1, based on the roll-off in the Gutenberg Richter distribution for the catalog as a whole, to protect against large swings in completeness level with time. An upper magnitude cutoff is imposed to prevent the rupture length from exceeding the radius of aftershock collection. For a spatial bin size of 0.1° × 0.1°, this corresponds to roughly magnitude 5.5. The study area extends from 114° to 124° west, and from 32° to 42° north.

We also look at the scaling of triggering intensity with dynamic strain in Japan. Here we
use the JMA catalog from 1997 through March 2006. For consistency with California, we limit the catalog of local events to shallower than 15 km within the land area of the four main islands of Japan. The magnitude of completeness for the JMA catalog of shallow crustal earthquakes may be below 2.1, but we impose this larger magnitude cutoff for consistent comparison with California.

3.4. Practicalities of implementation

In order to evaluate the significance of $\bar{R}$ as an indicator of triggering, we require confidence bounds on $\bar{R}$. We use the bootstrap method to generate confidence bounds by randomly resampling the $R$ distribution for a given population to generate a suite of estimates of $\bar{R}$ [Casella and Berger, 2002]. The confidence bounds on triggering intensity $n$ are then calculated by applying the transformation (Equation 2) to the bounds computed for $\bar{R}$.

We also take into consideration two potential sources of undesirable bias for realistic sets of earthquakes: (1) the superposition of Omori’s law on measurements made in aftershock sequences and (2) the finiteness of the earthquake catalog. The reader should note that understanding these practicalities is important for implementing the interevent time method, but not crucial for understanding the results presented in subsequent sections.

The superposition of Omori’s Law on the measurements is a significant issue for local triggering, but not farfield, and thus could affect the two populations differently.
Consider the three earthquakes shown in Figure 1. The metric $R$ is designed to measure whether or not earthquake B affects the timing of earthquake C. However, for the nearfield case, it is possible that earthquakes B and/or C are aftershocks of earthquake A. In this case, the times $t_1$ and $t_2$ are not uncorrelated. An Omori rate decay ($\sim t^{-1}$) is instead superimposed on the timing of both the trigger quake (quake B) and the subsequent quake. Measuring $R$ in an aftershock sequence, where seismicity rates are decreasing, can spuriously associate the trigger with a seismicity rate decrease. On the other hand, the farfield population is not subject to this effect, as it is very unlikely that a distant earthquake B is an aftershock of a local earthquake A.

Since the Omori’s Law bias only affects the nearfield population of quakes, it could interfere with the comparison of triggering intensity between the populations. This bias is suppressed by requiring that any event treated as a trigger (earthquake B) must be larger than the preceding local event (earthquake A). The condition ensures that the rate increase due to the trigger is much larger than any Omori-law rate decrease associated with the prior quake. A higher magnitude difference better ensures against bias, at the cost of reducing the number of eligible trigger quakes. We find that $\bar{R}$ is stable for a magnitude difference of at least one unit. One magnitude unit corresponds to a roughly tenfold increase in total triggering power [Reasenberg and Jones, 1989], and Omori’s law ensures that the difference is in general much greater than this, because the influence of the previous earthquake decays rapidly with time.

The second potential source of bias is related to the finiteness of the catalog. This is
especially problematic in regions where seismicity rates are low. To understand this effect, consider an ideal catalog with no triggering and uniformly distributed earthquake interevent times. Now consider a distant earthquake with a time near the beginning of the catalog as a potential trigger. Since there is no triggering in this hypothetical catalog, the times to the first earthquake before and after \((t_1 \text{ and } t_2)\) should be distributed identically, taking values between 0 and infinity, and \(\bar{R}\) should be 0.5. However, for a finite catalog, larger values of \(t_1\) and \(t_2\) are missing, because they fall outside the bounds of the catalog. For our hypothetical trigger near the beginning of the catalog, we can only measure \(R\) when \(t_1\) happens to be small enough to appear in the catalog, while \(t_2\) can take much larger values and still make it into the catalog. This sampling bias causes \(\bar{R}\) to differ from 0.5. The bias is unique for each spatial bin, as it depends on the particular combination of trigger times and the average seismicity rate in the bin.

We compute this bias stochastically by determining \(\bar{R}\) for 1000 simulated catalogs in which local earthquake occurrence times are replaced by uniformly distributed random times. The calculated bias is subtracted from the values of \(\bar{R}\) measured for the actual catalog. This means that, in effect, \(\bar{R}\) is reported relative to a simulated control case with zero triggering. For simplicity, the bias-corrected mean is referred to below as \(\bar{R}\).

Correcting for the finite catalog bias somewhat reduces the apparent triggering for the farfield triggers, but does not significantly alter the nearfield data, presumably because nearfield triggers tend to be located in regions with very high seismicity rates. The selection criteria for avoiding the Omori’s law bias is only applied to the nearfield
triggers. We demonstrate that these bias corrections are adequate by applying the interevent time method to a simulated earthquake catalog in Section 5. First, however, we show the results for the real seismicity catalogs.

4. Observed triggering intensity as a function of dynamic strain

4.1 Proof of concept: Denali

We begin attacking real data by measuring the interevent times for a well-known case of pervasive triggering in order to establish that $R$ and $n$ behave as designed. The 2002 magnitude 7.9 Denali earthquake generated peak dynamic strains on the order of $2-3 \times 10^{-7}$ for the California study area, according to the empirical regressions, and is known to have triggered significant seismicity [Gomberg et al., 2004; Prejean et al., 2004]. For this initial case study, we disregard dynamic strain amplitude variations and define a population consisting of the full gridded study area. The resulting $R$ distribution reflects significant triggering (Figure 5). The sample mean $\overline{R}$ (with 95% confidence limits) is $0.475 (0.461-0.488)$, corresponding to a fractional rate change $n$ of $0.16 (0.08-0.26)$ according to Equation 2. A simple earthquake count in the 24 hours before and after the Denali earthquake indicates a 22% seismicity rate increase in the following 24 hours. These estimates agree within error. We conclude that $R$ is capable of capturing triggering in a case with known seismicity rate increases.

4.2 Triggering intensity in the full catalog

The real utility of the method becomes apparent when it is applied to the full ANSS dataset with over 3000 potential farfield triggers, and 12,000 nearfield triggers meeting
our criteria. Figure 6 shows measured $R$ distributions for different dynamic strain amplitudes, corresponding to various combinations of magnitude and distance for the long-range dataset, and various magnitudes at constant distance for the short-range data. The distributions show evidence of both immediate triggering ($t_2 \sim 0$), in the form of large spikes at $R \approx 0$, and protracted or delayed triggering ($0 < t_2 < t_1$), in the form of a gradual decrease in probability density with increasing $R$. The distributions show larger proportions of small $R$ for higher dynamic strain amplitudes, as expected.

To assess whether the intensity of dynamic triggering in the farfield can account for the observed intensity of triggering in the nearfield, we plot the sample mean $\bar{R}$ and the transformed triggering intensity $n$, calculated from Equation 2, as a function of peak dynamic strain for both nearfield and farfield populations (Figure 7a,b). To emphasize the scaling of $\bar{R}$ at small strains, Figure 7a shows the value $\Delta \bar{R} = 0.5 - \bar{R}$, as in Figure 3. Triggering intensity in the farfield scales with the amplitude of peak dynamic strain. Comparing the triggering intensity for the nearfield population to a trendline fit to the farfield data, we find that the farfield (dynamic) triggering relationship can account for roughly half of the nearfield triggering intensity (Figure 7d). Although triggering intensity is best interpreted as a qualitative measure of actual rate change, the relative values should be fairly robust. The uncertainty is large, but the hypothesis that both populations of triggered quakes are produced in simple proportion to peak dynamic strain can be rejected at the 95% confidence level. Nevertheless, the farfield triggering relationship accounts for a significant portion of the nearfield aftershocks, accounting for about 60% of earthquakes within ~6 km of a M3.1 nearfield trigger and about 15% within
the same distance of a M5.5 trigger.

We can define a dynamic triggering threshold in the farfield as the smallest dynamic strain for which the 95% confidence limits on $\bar{R}$ fall above 0.5. By this definition, the dynamic triggering threshold in California is approximately $3 \times 10^{-9}$ strain. Note that this threshold is not dependent on the transformation from $\bar{R}$ to triggering intensity $n$, because the significance is evaluated through bootstrap resampling of $\bar{R}$ itself. For a crustal shear modulus of 30 GPa, this corresponds to a dynamic stress of 0.1 kPa. This estimate is several orders of magnitude smaller than previously reported for dynamic triggering [Brodsky and Prejean, 2005; Gomberg and Davis, 1996; Gomberg and Johnson, 2005; Stark and Davis, 1996]. We attribute this improvement in sensitivity to the interevent time method, which can detect small rate changes by using large populations. The threshold is discussed in more detail below.

We also apply the interevent time method to Japan, assuming that the nearfield regression determined in California can be applied to shallow crustal earthquakes in Japan at these small distances. In Japan, the dynamic strains from the largest farfield earthquakes are large enough to overlap in amplitude with the smallest of the nearfield triggers. Where the two populations overlap, we again find a small additional nearfield component, though not at the 95% confidence level (Figure 7c). Unfortunately, the uncertainty ranges are too large to assess the relative contributions with any confidence.

Triggering intensity in shallow crustal Japan (Figure 7c) is reduced relative to California
in both the near and farfield populations, with a higher dynamic triggering threshold of $10^{-6}$ strain. The relative paucity of long-range triggering in Japan has been documented before [Harrington and Brodsky, 2006], but this study shows that the reduced triggering susceptibility extends to the nearfield, as well. This difference in triggerability may reflect the difference in tectonic style (compressive vs. transpressive) between the two study areas.

4.2 Dynamic strain threshold

The interevent time method resolves triggering at dynamic strains as low as $3 \times 10^{-9}$. Many faults are regularly exposed to such small dynamic strain amplitudes without being triggered [Spudich et al., 1995]. The scaling of $\bar{R}$ with dynamic strain may therefore best reflect the distribution of fault strengths. The very low triggering intensity at the threshold would then reflect the scarcity of faults so very near failure.

The dynamic strain threshold is also smaller than tidal strain fluctuations [Cochran et al., 2004; Scholz, 2003]. This is somewhat puzzling, because tidal strains might be expected to activate all available nucleation sites on a daily basis and set a lower limit for dynamic triggering. Strain tensors associated with crustal earthquakes are likely oriented with more variety than those due to the tides, however, and may access faults that tides are incapable of triggering. In addition, the forcing at the relatively long periods of the tides may be intrinsically different from the dynamic strains imposed at the short periods of seismic waves [Beeler and Lockner, 2003; Gomberg et al., 1997; Savage and Marone, 2008].
Evaluating the probability of triggering as a function of dynamic strain (Figure 8) aids in the interpretation of the observed triggering threshold. Probability is calculated from the triggering intensity $n$ using the same Poissonian statistical model as before (Appendix C). This is not a new development, as the Poisson model is commonly used to transform an estimate of the number of triggered events into a probability of earthquake occurrence in a given time period [Reasenberg and Jones, 1989]. We discuss the probability of triggering in order to address 1) whether we would expect to resolve triggering below the threshold we have identified, and 2) why previous studies may have failed to identify triggering at the very low levels identified here.

The probability of triggering an earthquake within its own recurrence interval, as a function of $n$, is given by

$$P(N_{Eq} \geq 1) = 1 - \exp \left\{ -(n+1) \right\}. \quad (8)$$

The baseline probability of having an earthquake in the absence of any rate change ($n = 0$) is ~63%. A positive $n$ produces a positive probability gain. Figure 8 shows that the probability gain decreases smoothly to zero as $n$ decreases. For the ~$3 \times 10^{-9}$ strain bin in California, there are ~$10^5$ interevent time measurements (~$10^3$ triggers $\times$ ~$10^2$ bins containing local earthquakes) and this number of events is sufficient to find $\bar{R}$ less than than 0.5 at the 95% confidence level. We estimate (based on an assumed $\sqrt{n}$ scaling of the confidence bounds) that an order of magnitude more observations would be needed to push the observable threshold an order of magnitude lower. This exceeds the size of the earthquake catalog, and we infer that the absence of detected triggering at dynamic
strains of less than $3 \times 10^{-9}$ reflects an observational limit and not necessarily a physical threshold. If triggering occurs at lower dynamic strain amplitudes, we would not expect to resolve it.

Understanding the probability of triggering also helps explain why previous studies have not identified dynamic triggering at the threshold reported here. Previous estimates of the dynamic triggering threshold have been based on waveform inspection, counting statistics, and/or likelihood methods. If triggering intensity $n>1$, the change in seismicity rate is comparable to the background rate, and triggering is easily observable by inspection. Perhaps the conventional division between aftershocks and the more recently discovered farfield triggered populations results from the ease of observing large seismicity increases ($n > 1$) compared to the more subtle farfield triggering ($n < 1$). For a Poisson process, the variance is equal to the average rate, so $n > 1$ also roughly corresponds to the threshold for statistical significance using an earthquake count. Therefore, only the seismicity rate increases corresponding to $n > 1$, i.e. dynamic strains of nearly $10^{-5}$, are easily observable by these methods. Likelihood-based methods, in which triggered earthquakes are identified by determining whether the modeled likelihood of their occurrence is otherwise small, cannot resolve triggering where the triggering probability itself is very small. Figure 8 shows that the probability of observing a triggered earthquake does not exceed 5% below $10^{-6}$ dynamic strain. If triggering results in an additional earthquake fewer than 5% of the time, we cannot be 95% confident that the resulting earthquake count did not occur by chance. These considerations may explain why previous studies have not identified dynamic triggering
at the level of $3 \times 10^{-9}$ strain.

5. Validation and calibration through statistical seismicity simulations

5.1 The effect of triggering cascades on the measured scaling of triggering intensity with strain.

We have shown that triggering intensity scales with dynamic strain in both near and farfield populations, with a moderate additional component in the nearfield. However, there is a problem interpreting the quantitative slope of this trend. Previous work using earthquake counting and carefully declustered seismicity catalogs has shown that the number of local aftershocks following a mainshock of magnitude $M$ goes as $10^{\alpha M}$, with $\alpha \approx 1$ [Felzer et al., 2004; Helmstetter et al., 2005]. Since dynamic strain scales with magnitude as $\varepsilon \propto 10^M$ (Equations 5 and 6), this implies a linear scaling of aftershock rate with peak dynamic strain. Studies comparing aftershock spatial decay directly to peak ground velocities also find aftershock rates to be consistent with a linear scaling with strain [Felzer and Brodsky, 2006; Gomberg and Felzer, 2008]. In contrast, this study suggests that triggering intensity varies with dynamic strain roughly as $n \propto \varepsilon^{0.5}$ in both populations (Figure 7b). We will now show that this discrepancy arises from the application of a probability model derived for isolated earthquake sequences to a catalog containing superimposed triggering cascades. In essence, the transformation fails to take into account that the interevent times are effectively sampled from two underlying distributions: the distribution of interevent times for triggered earthquakes, and the distribution of times for background or uncorrelated quakes. The observed distribution of $R$ is therefore a superposition of the two distributions illustrated in Figure 2. Fortunately,
this effect can be quantified and calibrated using statistical simulations.

As described in Section 2.2, the sample mean $\bar{R}$ is transformed to triggering intensity $n$ via a probabilistic model for earthquake occurrence times. This transformation is adequate for recovering the qualitative scaling of triggering rate change with dynamic strain amplitude. However, in order to recover absolute rates, we must employ a more sophisticated model that considers the effect of earthquake cascades. The simple transformation from $\bar{R}$ to $n$ implicitly assumed that we correctly associate earthquakes with their respective triggers, but a real catalog contains numerous superimposed triggering cascades. The first earthquake before and after a trigger may or may not then be causally related. If they belong to a different earthquake sequence, they will introduce $R$-values sampled from a uniform distribution, and the resulting distribution will be some combination of the two curves illustrated in Figure 2. This has the effect of dampening the observed triggering signal.

### 5.2 Modeling earthquake cascades: ETAS

To evaluate whether the presence of superimposed earthquake cascades can explain the discrepancy between our recovered slope in Figure 7b and previous work based directly on earthquake counts, we generate an artificial earthquake catalog that follows the usually observed statistics of magnitude, timing, and triggering distributions (Appendix D). Because causality is known in the simulation, we can investigate how the use of the first earthquake before and after the trigger affects the recovered scaling relationship.
A well-established method for generating such a catalog is the epidemic triggered aftershock sequence (ETAS) [Ogata, 1992]. ETAS uses well-known empirical statistical seismicity laws as probability distributions to generate stochastic seismicity catalogs. Numerous researchers have used ETAS models to study the complex statistical repercussions of simple earthquake cascades [Felzer et al., 2002; Felzer et al., 2004; Hardebeck et al., 2008; Helmstetter and Sornette, 2003; Holliday et al., 2008]. Here we use ETAS to study the effects of superimposed earthquake triggering sequences on our transformation of $R$ to an estimate of fractional rate change.

We first apply the interevent time method to a zero-dimensional ETAS catalog. The zero dimensional model simulates earthquakes in time only, disregarding spatial distribution, in order to isolate the effect of the earthquake cascade. The triggering law in the simulation corresponds to the case of the nonhomogeneous Poisson process with an Omori decay, and the number of triggered earthquakes scales with $\alpha = 1$. We measure $\bar{R}$ for this simulated catalog using the first earthquake before and after each trigger (unknown causality) and also along individual branches of the triggering cascade (known causality). Both the stepwise homogeneous Poisson and the nonhomogeneous (Omori) model are then used to transform $\bar{R}$ to a triggered rate change.

If interevent times are measured with respect to known branches of the cascade, that is, if the causal relationships are known, the transformation from $\bar{R}$ to $n$ recovers the scaling law that was put into the model (Figure 9). If we instead use the first earthquake before and after a putative trigger, a scaling with $\alpha = 0.5$ is recovered, similar to that recovered
for the real catalog. This demonstrates that the discrepancy between our scaling and that found in other studies is due to the inclusion of some non-causally related interevent times in the $R$ distribution. The simulation therefore suggests that our interevent time observations are consistent with the number of triggered earthquakes being directly proportional to dynamic strain, as found in previous studies. Consequently, the triggering intensity calculated by our transformation represents a lower bound on the real fractional rate change.

We have claimed that triggering intensity $n$ serves as a good qualitative measure of the scaling with dynamic strain amplitude as long as it scales in a consistent manner with $\bar{R}$. The zero-dimensional simulation shows that $n$ indeed scales consistently, but the zero-dimensional simulation only considered nearfield aftershock triggering, not triggering from distant earthquakes unconnected to the local earthquake cascade. It is possible that the effect of unknown parentage will be different in the near and farfield. Since our conclusions about the relative contribution of dynamic strains in triggering nearfield earthquakes is based on a projection of the farfield relationship into the nearfield, we need to confirm that the two populations are affected identically, despite the imperfect transformation to modeled rate change (i.e. Equation 2).

5.3 Space-time ETAS simulation

To verify the robustness of the comparison between populations, we apply the interevent time method to a simulated earthquake catalog in which earthquakes are produced in simple proportion to peak dynamic strain at all distances. If the interevent time method
recovers a continuous trend between the simulated near and farfield populations, we can then interpret the observed offset between the real populations as reflecting an additional nearfield triggering component.

To appropriately represent the two populations defined in this study, we use a full space-time simulated ETAS catalog (Appendix D). We make a key modification to the model, introducing farfield triggering in direct proportion to dynamic strain, as calculated from the empirical ground motion regression in Equation 6. We also modify the nearfield triggering rules to reflect the empirical PGV constants in Equation 5. This requires very minor adjustments of published ETAS parameters. In order to match the known scaling of aftershock productivity with mainshock magnitude, we set the regression constant \( c_2 = 1 \). This produces a negligibly larger misfit than the unconstrained PGV regression, and does not significantly change the spatial decay (Table 1). This allows us to generate both nearfield aftershock triggering and long-range triggering from distant sources with a consistent triggering rule.

Applying the interevent time method, we recover a continuous trend for a representative set of ETAS parameter taken from the literature (Table 2)(Figure 10). This verifies that the method is capable of qualitatively measuring triggering in an ideal catalog with triggering proportional to strain. We are therefore justified in interpreting the offset in nearfield and farfield triggering as reflective of an additional nearfield triggering process.

In the previous section, we found that the transformation from \( \bar{R} \) to triggering intensity \( n \)
underestimates the actual productivity scaling $\alpha$, due to the superposition of triggering cascades. In the space-time ETAS simulations we find that the slope of the trend is also reduced relative to the input value, and varies slightly for different simulation runs, perhaps correlating with the fraction of triggered vs. background earthquakes in a particular realization. The precise relationship between the absolute value of $n$ and the other statistics of the catalog is beyond the scope of this study.

6. Discussion

6.1 Implications for dynamic triggering

Triggering intensity scales with peak dynamic strain in the farfield. Triggering in the farfield (further than 800 km) can be confidently attributed to a dynamic agent, because the triggered earthquakes are well beyond the several source dimensions affected by static and postseismic stresses, and the waiting times to the first triggered earthquakes are much less than the ~10 years required to propagate stresses viscously to these distances, as discussed in the introduction. The empirical proportionality between dynamic strain and triggering intensity can account for a significant portion of triggering in the nearfield, but not all. Additional nearfield triggering may reflect any or all of the following factors: 1) the effect of prolonged static or postseismic strain near the mainshock, 2) a dependence on frequency content, where higher frequency dynamic waves are more effective triggers, or 3) the concentration of potential nucleation sites (e.g. secondary fault strands, damage zones) in the regions very near to mainshocks. If the nearfield triggering component reflects static stress triggering, then this observation suggests that dynamic and static strains produce roughly equivalent numbers of earthquakes in the
nearfield of intermediate magnitude earthquakes. Therefore, both effects must be taken into account to explain aftershock numbers and spatial distributions.

It is important to note that the interevent time measurements do not simply reflect unusual behavior at the beginning of aftershock sequences. In fact, for small nearfield triggers near the magnitude of completeness, the first triggered event is often the only aftershock in the sequence. Furthermore, stacked sequences consisting of only the first aftershocks of small mainshocks follow the same Omori’s law decay as single aftershock sequences of large mainshocks [Felzer et al., 2004; Felzer and Brodsky, 2006]. This implies that there is no physical distinction between the first cataloged aftershock and subsequent aftershocks in a sequence.

How do dynamic strains, which produce no permanent load change, nonetheless account for a significant portion of nearfield triggering? The low threshold for dynamic triggering suggests that arbitrarily small dynamic strains can trigger earthquakes on nucleation sites that are sufficiently near failure. Without a physical threshold for dynamic triggering, the question is one of a balance of timescales – the timescale over which a nucleation site is loaded to failure quasi-statically vs. the time between dynamic strain events large enough to push the fault the rest of the way. If the dynamic trigger recurrence time is smaller than the quasi-static time to failure, the fault will be triggered dynamically. A fault far from failure is unlikely to be triggered by any but the largest dynamic strain events. However, as the fault nears failure, not only are smaller and smaller dynamic strains required for triggering, but the availability of sufficient triggers
increases due to the greater abundance of small earthquakes.

In fact, a simple scaling argument shows that a fault is just as likely to be triggered by a small dynamic strain event as by a large one. The ETAS models show that the data are consistent with the number of dynamically triggered earthquakes being linearly proportional to dynamic strain. The number of earthquakes triggered by an earthquake of magnitude $M$ therefore goes roughly as $\sim 10^M$. The Gutenberg-Richter distribution gives that the number of earthquakes with magnitude $M$ goes as $\sim 10^{-M}$. Therefore, the total triggering power for each magnitude bin as a whole is constant:

$$N = N_{\text{Triggers}} \times N_{\text{Triggered}} = 10^{-M} \cdot 10^M = \text{constant}.$$  

Small amplitude triggers and earthquakes with magnitudes below the level of catalog completeness are therefore very important in triggering subsequent earthquakes. Similar arguments for the importance of small earthquakes have been made previously based on statistical considerations [Felzer et al., 2004; Felzer and Brodsky, 2006; Helmstetter et al., 2005; Sornette and Werner, 2005a; b].

The low observational threshold and the linear relationship between triggering intensity and dynamic strain amplitude place constraints on the mechanics of triggering. For example, these observations are not consistent with the exponential dependence between stress/strain and the number of triggered earthquakes predicted by classical rate and state friction [Brodsky, 2006; Dieterich, 1994; Gomberg, 2001]. The simplest way to reproduce the farfield observations may be to invoke a population of nucleation sites with a uniform distribution of dynamic triggering thresholds and a Coulomb-type nucleation
criterion.

6.2 How delayed earthquakes can be triggered earthquakes

Studies frequently identify only those earthquakes occurring during the passage of seismic waves as dynamically triggered. In applying the interevent time technique, we include arbitrarily delayed ‘first’ earthquakes after the trigger. This inclusion is based on several considerations.

First, the designation of an earthquake as ‘first’ is a threshold-dependent, observational distinction. For a lower catalog detection threshold, we should always be able to find an earlier quake. This consideration is especially important for nearfield triggering, where the detection threshold can be temporarily elevated, and a tremendous number of early aftershocks are usually missing from earthquake catalogs [Kagan, 2004; Peng et al., 2007]. Operationally, it is impossible to distinguish between primary, secondary, and delayed triggered earthquakes, as they share identical space, time, and magnitude statistics [Brodsky, 2006; Felzer et al., 2004; Kagan, 2004]. Regardless, the rates of all of these classifications of earthquakes should be correlated, and secondary or delayed triggered earthquakes should reflect the intensity of the primary triggering process.

Second, we wish to allow for the possibility that dynamic strains can trigger earthquakes by inducing a semi-permanent change in the properties of the fault patch, rather than only through transiently exceeding the fault strength. Several studies have identified or posited long-lasting changes in the mechanical properties or effective stresses within fault
zones related to the passage of high-amplitude seismic waves [Brodsky et al., 2003; Elkhoury et al., 2006; Johnson and Jia, 2005; Parsons, 2005; Taira et al., 2009]. Delayed triggering may then simply reflect the prolonged nature of the triggering process.

Regardless of whether delayed triggering reflects an incompletely observed earthquake cascade or a prolonged physical perturbation of the fault conditions, the inadvertent inclusion of uncorrelated (non-triggered) events will not invalidate the interevent time method, because these interevent times are drawn from a uniform distribution of $R$ values and will not impart a false positive bias.

6.3. Robustness of the observations with regard to parameters

The binning of the data and the separation of triggered quakes into farfield and nearfield populations required the introduction of arbitrary parameters. We want to be certain that our conclusions are robust with respect to these parameter choices. The success of the method in recovering a continuous trend for a control case using a simulated catalog with a continuous triggering law is a good confirmation, but we also check the robustness of the observations with respect to the data selection parameters.

The first arbitrary parameter is the spatial bin size. The results shown in Figure 7-8 use a bin size of 0.1°, because this maximizes the number of $R$ values we can calculate. Figure 11 confirms that the measurements are not sensitive to the spatial bin size, as long as the number of data points remains high. We show results for bin sizes between ~8 km² and
123 km² (0.025º - 0.1º on a side). For larger bins, either the reduced quantity of data or the masking of triggered activity by unrelated local aftershocks causes confidence limits to exceed the mean triggering signal.

The distance cutoff for farfield triggers also does not influence the results. Trials using minimum far-field cutoff distances of 800 km through 3200 km all recover essentially the same farfield scaling (Figure 12).

Finally, to make sure the long-range triggering signal is not generated entirely by isolated geothermal areas, we plot the contribution of each spatial bin to the total measured triggering intensity, combining all the farfield amplitude bins (Figure 13). Geothermal regions (particularly Long Valley and the Salton Trough) contribute strongly, but virtually all regions of active seismicity in California contribute to the long-range triggering signal.

6.4 Relation to previous work

Dividing earthquakes into populations with common strain is a novel way of looking at the scaling of triggering intensity with dynamic strain. Previous work has shown the relevance of dynamic triggering in the nearfield by comparing the falloff in aftershock density away from a mainshock to the falloff of seismic waves at near and intermediate distances [Felzer and Brodsky, 2006; Gomberg and Felzer, 2008]. These correlations cannot be trivially mapped to a particular function of dynamic strain, however, because the decay in triggering intensity is superimposed on the decay of available nucleation
sites away from the mainshock. It is therefore necessary to carefully analyze the statistics of non-triggered (background) seismicity in order to extract the triggering function. The method defined here does not suffer from this ambiguity with respect to a background distribution, because each value of $R$ reflects the seismicity rate change at a single site. We therefore do not need to be concerned about the geometry of the local fault network.

7. Conclusion

The observations presented here have the following implications: Farfield triggering scales with peak dynamic strain. This scaling, projected into the nearfield, accounts for 15-60\% of earthquakes within 6 km of magnitude 3-5.5 earthquakes. The additional nearfield triggering component may reflect static stress triggering, frequency dependence for dynamic triggering, or concentration of nucleation sites very near mainshocks. Extremely small dynamic strains can trigger faults if they are sufficiently near failure, down to the observed level of $3 \times 10^{-9}$ dynamic strain. ETAS simulations in which earthquakes are produced in direct proportion to dynamic strain reproduce the observed scaling of triggering intensity vs. strain, suggesting that dynamic triggering intensity is linearly proportional to peak dynamic strain amplitude. This places a useful constraint on models for earthquake triggering mechanisms.

Appendix A: Expectation of $R$ for a Poisson process with a step change in intensity

To find the expectation of the interevent time ratio $R$ as a function of rate change, we first derive the distribution of $R$ using an assumed distribution of interevent times $t_1$ and $t_2$, based on a probabilistic model. As discussed in the text, the most widely accepted model
for the interevent time distribution is the nonhomogeneous Poisson process. We approximate this model with a stepwise homogeneous process. In a homogeneous Poisson process events occur randomly in time with some average rate $\lambda$, known as the intensity. If the intensity $\lambda$ is independent of time, the interevent times follow an exponential distribution.

$$ f(t) = \lambda \exp \left\{ -\lambda t \right\}. \quad (A1) $$

In a non-homogeneous Poisson process, the intensity $\lambda$ is a function of time, and the interevent times are distributed as

$$ f(t) = \lambda(t) \exp \left\{ -\int_0^t \lambda(\tau) d\tau \right\} \quad (A2) $$

where $\lambda(t)$ is the intensity at time $t$. The term within the exponential in both cases is the expected number of events at time $t$. We define the expected number of events $N(t)$ as

$$ N(t) = \int_0^t \lambda(\tau) d\tau \quad (A3) $$

for use in subsequent equations.

The joint distribution of two independent interevent times is the product of two exponential distributions.

$$ f(t_1, t_2) = \lambda_1(t_1) \lambda_2(t_2) \exp \left\{ -N_1(t_1) - N_2(t_2) \right\}. \quad (A4) $$

where the subscripts 1 and 2 refer to intensities and times before and after the trigger, respectively.

We derive the joint distribution of $R$ (Equation 1) and a dummy variable $T = t_2$, by substituting these variables into Equation A4 and multiplying by the absolute value of the
Jacobian of the variable transformation [Casella and Berger, 2002; Walpole and Myers, 1989]. Expressed in terms of $R$ and $T$,

\[
t_1 = \frac{T}{R} - T, \\
t_2 = T.
\] (A5)

The Jacobian of the transformation can be thought of as describing how areas under the distribution are expanded or contracted through the transformation. It is given by

\[
J = \det\begin{pmatrix}
\frac{\partial t_1}{\partial R} & \frac{\partial t_1}{\partial T} \\
\frac{\partial t_2}{\partial R} & \frac{\partial t_2}{\partial T}
\end{pmatrix} = \det\begin{pmatrix}
\frac{-T}{R^2} & \frac{1}{R} - 1 \\
0 & 1
\end{pmatrix} = -\frac{T}{R^2}.
\] (A6)

The joint distribution for $R$ and $T$ is then

\[
f_{R,T}(R,T) = \lambda_1 \left( \frac{T}{R} - T \right) \lambda_2(T) \exp\left\{ -N_1\left( \frac{T}{R} - T \right) - N_2(T) \right\} \left| \frac{T}{R} \right|.
\] (A7)

The marginal distribution of $R$ is obtained by integrating Equation A7 with respect to $T$.

\[
f(R) = \int_0^\infty \lambda_1 \left( \frac{T}{R} - T \right) \lambda_2(T) \exp\left\{ -N_1\left( \frac{T}{R} - T \right) - N_2(T) \right\} \frac{T}{R^2} dT.
\] (A8)

The expectation of $R$ is defined as

\[
\langle R \rangle = \int_0^1 R f(R) dR.
\] (A9)

Substituting Equation A8 into A9 gives

\[
\langle R \rangle = \int_0^1 \int_0^\infty \lambda_1 \left( \frac{T}{R} - T \right) \lambda_2(T) \exp\left\{ -N_1\left( \frac{T}{R} - T \right) - N_2(T) \right\} \frac{T}{R} dT dR.
\] (A10)

For an otherwise homogenous Poisson process with a step change in intensity $\lambda$, the solution to Equation A10 is
Let us now define the fractional rate change as the number of triggered earthquakes in some post-trigger time interval normalized by the number that would be expected for the pre-trigger rate:

\[
n \equiv \frac{N_2(t) - N_1(t)}{N_1(t)}.
\]  

(A12)

For the step-wise homogeneous Poisson process, where \( \lambda_1 \) and \( \lambda_2 \) are each independent of time, this becomes

\[
n = \frac{\lambda_2 - \lambda_1}{\lambda_1}.
\]  

(A13)

The expectation of \( R \) for the stepwise homogeneous process (Equation A11) can then be rewritten solely as a function of fractional rate change \( n \)

\[
\langle R \rangle = \frac{1}{n^2} \left[ (n + 1) \ln(n + 1) - n \right].
\]  

(A14)

Equation A14 is Equation 2 in the main text with the parameter \( n \) identified as triggering intensity. The sample mean \( \bar{R} \) is transformed to triggering intensity \( n \) by equating \( \bar{R} \) with the expectation \( \langle R \rangle \) and solving numerically for \( n \).

We can also calculate the expectation of \( R \) for the non-homogeneous (Omori-decaying) Poisson process, given an estimate of the parameters in Omori’s law. In this case, the post trigger rate is given by
\[ \lambda_2(t) = \lambda_1 + \frac{k}{(t+c)^p}. \]  

(A15)

The number of expected events as a function of time is then

\[ N_2(t) = \lambda_1 t + \frac{k}{1-p} \left[ (t + c)^{-p} - c^{-p} \right], \quad p \neq 1. \]  

(A16)

\[ N_2(t) = \lambda_1 t + k \ln \left( \frac{t}{c} + 1 \right), \quad p = 1. \]

Substituting these definitions of \( \lambda_2 \) and \( N_2 \) into Equation A10, we integrate numerically to find the expectation of \( R \).

The transformation from observed \( \bar{R} \) to fractional rate change \( n \) with the nonhomogeneous Poisson process is carried out by iteratively solving for the parameter \( k \) through Equation A10, and then calculating the fractional rate change \( n \) (Equation A12) using Equation A16 for \( N_2(t) \). A natural timescale for calculating \( n \) is \( t = \lambda_1^{-1} \), the expected time to the first event given the pre-trigger rate \( \lambda_1 \). The definition of fractional rate change \( n \) for the nonhomogeneous model then reduces to

\[ n = \frac{k}{1-p} \left[ (\lambda_1^{-1} + c)^{-p} - c^{-p} \right], \quad p \neq 1. \]  

(A17)

\[ n = k \ln \left( \frac{1}{c\lambda_1} + 1 \right), \quad p = 1. \]

In this case, we need to estimate a representative background rate, as well as the Omori’s law parameters. The fractional rate change recovered using Equation A17 is very similar to that recovered by the stepwise homogeneous transformation, as reflected by Figure 3.
Appendix B: Shakemap Peak Ground Velocity regression

We begin with an equation modified from the Next Generation Attenuation study of Campbell and Bozorgnia [2007],

\[ \log_{10} \text{PGV} = c_1 + c_2 M - c_3 \log_{10} \sqrt{r^2 + c_4^2}, \]  

(B1)

where PGV is peak ground velocity in cm/s, \( M \) is earthquake magnitude, \( r \) is hypocentral distance in km, and \( c_i \) are fit parameters. This is Equation 5 in the main text. This equation differs from Campbell and Bozorgnia [2007] only in the lack of magnitude dependence for the attenuation with distance. We also try a functional form that includes an exponential attenuation term, but find that attenuation is negligible at the small distances studied.

The regression dataset consists of peak velocities from seismic stations within 15 km hypocentral distance from all California earthquakes having an archived Shakemap (http://earthquake.usgs.gov/earthquakes/shakemap). As of Nov. 2009, this constitutes just over 2000 PGV measurements, with 140 within the ~6.2 km range defined as the nearfield in this study. PGV is corrected to hard rock values following the Shakemap methodology, which uses NEHRP site classifications based on shallow shear wave velocity [Wald et al., 2006, Section 2.4.3]. Data flagged as outliers during the generation of the Shakemaps are excluded. Since Shakemaps are generated very rapidly after a quake, earthquake magnitudes and locations listed in the Shakemap archive are often preliminary. Earthquake magnitudes and locations are therefore taken from the ANSS catalog for California, which combines locations from both Northern and Southern California seismic networks and is a more authoritative source. To reduce the potential
impact of errors in earthquake locations, only data from earthquakes with catalog depths of at least 2 km are considered.

A MATLAB nonlinear optimization algorithm (FMINCON) is used to solve for the constants in Equation B1, with the constraint that constants $c_2$, $c_3$, and $c_4$ be positive. To make the regression as representative of nearfield measurements as possible, the data are weighted by the space between data points. This is roughly equivalent to fitting the regression curve to binned data. The best-fit regression parameters are given in Table 1, row 1. The 95% confidence levels for each parameter are computed from 1000 bootstrap resamplings of the Shakemap data. The regression always finds $c_4 = 0$ km, and therefore we do not report error bounds on this parameter. However, due to the lack of data at very small distances, we must consider this parameter somewhat ill-constrained. The regression and regression misfit are plotted against distance and magnitude in Figure B1, along with several other published regressions for comparison. This regression is the most appropriate proxy for dynamic strain in our magnitude and distance range, as it is conditioned entirely on data within this range.

For use in the ETAS simulation, we also perform a regression in which the magnitude scaling constant $c_2$ is constrained to equal 1 (Table 1, row 2). This is for consistency with known aftershock scaling with magnitude. This constraint does not significantly alter the spatial component of the regression.

Appendix C: Calculation of Earthquake Probability from $\bar{R}$
Appendix A shows how to transform the distribution of $R$ into fractional earthquake rate change $n$, assuming Poisson distributed interevent times. The same statistical model can be used to calculate the probability of triggering an earthquake given the estimated rate change. For a homogeneous Poisson process, the probability of observing exactly $m$ events in time period $t$, given the average event rate $\lambda$, is given by

$$P(N_{EQ} = m \mid \lambda t) = \frac{(\lambda t)^m e^{-\lambda t}}{m!}. \quad (C1)$$

The probability of observing one or more events is equal to one minus the probability of observing zero events.

$$P(N_{EQ} > 0 \mid \lambda t) = 1 - P(N_{EQ} = 0 \mid \lambda t) = 1 - \exp \{ -\lambda t \}. \quad (C2)$$

Over one recurrence interval (time $t = \lambda_1^{-1}$), the probability of observing at least one event given fractional rate change $n$ is

$$P(N_{EQ} \geq 1 \mid n) = 1 - \exp \{ -(n + 1) \}. \quad (C3)$$

That is, if the average seismicity rate were one earthquake per day, the probability of seeing an earthquake on any given day, in the absence of triggering, is about 63%.

Equation C3 then gives the adjusted probability of seeing that “daily” earthquake given the increase in seismicity rate measured by $n$. Equation C3 is Equation 8 in the main text.

Appendix D: Modified ETAS simulation

The Epidemic-Type Aftershock Sequence (ETAS) model uses empirical probability distributions to stochastically generate realistically clustered earthquake catalogs [Ogata, 1998]. We briefly summarize the governing equations here and direct the interested reader to the studies cited in the main text for more information.
(1) Earthquake magnitudes are assigned from a Gutenberg-Richter probability distribution,

\[ N(M) = 10^{a-bM}, \]  

(D1)

where \( N \) is the number of earthquakes with magnitude greater than or equal to \( M \), and \( a \) and \( b \) are constants, with \( b \) typically around 1.

(2) The temporal decay of aftershock sequences is governed by the Modified Omori’s law, which states that aftershock rate decreases approximately as 1 over the time since the mainshock.

\[ R(t) \propto \left(c + t\right)^{-p}, \]  

(D2)

where \( R \) is the instantaneous aftershock rate, \( c \) is a constant that effectively keeps the rate finite at zero time, and \( p \) is the decay exponent.

(3) An aftershock productivity law is necessary to close the equations in time and magnitude. Previous work shows that number of aftershocks scales exponentially with mainshock magnitude [Felzer et al., 2004; Helmstetter et al., 2005].

\[ N_{AS} \propto 10^{\alpha M}, \]  

(D3)

where \( \alpha \) is a constant near 1 and \( M \) is mainshock magnitude. Equations D2 and D3 are related, in that the integral of \( R(t) \) over the duration of the aftershock sequence equals \( N_{AS} \), and we define a productivity constant \( A \) such that

\[ N_{AS} = \int_0^{\infty} A \cdot 10^{\alpha(M-M_{min})} \left(c + t\right)^{-p} \, dt, \]  

(D4)
where $M_{\text{min}}$ is the minimum magnitude in the simulation. For $p > 1$, $N_{\text{AS}}$ is finite. For $p \leq 1$, $N_{\text{AS}}$ must be calculated over a finite time period. Equation D4 is calibrated to reproduce Båth’s Law (with $\alpha = 1$), which states that the largest aftershock of a sequence is on average $\sim 1$ magnitude unit below the mainshock magnitude.

(4) A full space-time simulation also requires a law describing the spatial clustering of aftershocks. For example, Felzer and Brodsky [Felzer and Brodsky, 2006] give

$$\rho(r) \propto r^{-\gamma}, \quad (D5)$$

where $\rho$ is linear aftershock density at distance $r$ from the mainshock, and $\gamma$ is a constant.

We replace rules (3) and (4) with an equivalent rule that also reproduces Bath’s Law and a power-law decrease in linear aftershock density, but is based on dynamic strain rather than magnitude. The equivalent rule specifies that the number of aftershocks per unit area scales linearly with peak dynamic strain.

$$N_{\text{AS}} = \kappa \varepsilon_{\text{dyn}}, \quad (D6)$$

Accordingly, we set $\gamma = c_3$ from the constrained PGV regression (Table 1, row 2).

The constant of proportionality $\kappa$ is found by dividing the number of aftershocks predicted by Equation D4 by the peak dynamic strain integrated over the aftershock zone (Equation 7). This gives

$$\kappa = \frac{C_S A \cdot 10^{-c_1 - M_{\text{min}}} c_1^{-p} \left(1 - \gamma\right)}{2\pi \left(p - 1\right) \left(D_{\text{max}}^{-\gamma} - D_{\text{min}}^{-\gamma}\right)}, \quad (D7)$$

where $C_S$ is the shear wave speed, $c_1$ is the first PGV regression parameter (Table 1, row
2), $A$, $c$, $p$, are the ETAS constants, and $D_{\text{max}}$ and $D_{\text{min}}$ are the maximum and minimum bounds of the local aftershock zone, imposed to make the simulation numerically tractable. For the ETAS simulation, we refit the PGV regression constraining $c_2 = 1$, consistent with $\alpha = 1$ in Equation D3. The fit parameters for the constrained regression are reported in Table 1, row 2.

Remotely triggered earthquakes are generated in proportion to $\kappa$ times their dynamic strain amplitude. In this way, triggering associated with both local earthquakes and the surface waves of distant earthquakes is simulated simultaneously in a self-consistent manner.

We use a version of Felzer and Felzer’s Matlab code, modified to include a separate catalog of global triggers (http://pasadena.wr.usgs.gov/office/kfelzer/AftSimulator.html) [Felzer et al., 2002]. This code was also used by Hardebeck et al., [2008]. An estimate of the spatially varying California background seismicity rate is included with the code (Figure 13b), as well as estimates of ETAS parameters representative of California (Table 2). The dimensions of the aftershock zone, specified for computational efficiency and to keep the number of aftershocks finite, are left at the default values of $D_{\text{min}} = 0.001$ km and $D_{\text{max}} = 500$ km, respectively.

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**Figure Captions**

**Table 1.** Best-fit regression constants for Equation 5. Values in parentheses are 95% confidence limits from 1000 bootstrap resamplings of the underlying peak ground velocity dataset. The regression always prefers $c_4 = 0$. The second row of constants is used in the ETAS simulation, in which case $c_2$ is constrained to equal 1 to match observed aftershock scaling.

**Table 2.** Parameters used in the ETAS simulation, representative of California. $A$ is the productivity constant that controls the number of aftershocks per mainshock, $c$ is the time offset in the Modified Omori’s law, $p$ is the time decay of aftershock rate in Omori’s law, and $\kappa$ is the productivity constant as a function of dynamic strain, calculated according to Equation D7.

**Figure 1.** Cartoon timeline illustrating the variables contributing to the interevent time ratio $R$ (Equation 1). The trigger earthquake is labeled (B), and the first earthquake
before and after the trigger are labeled (A) and (C), respectively. The time $t_1$ is the time to the first earthquake before the trigger, and $t_2$ is the waiting time to the first earthquake after the trigger.

**Figure 2.** Schematic cartoon illustrating the distinction between the distribution of $R$ in a case with no triggering (dashed) and a simulated case of strong triggering (solid). The integral of the probability density is 1 in both cases. The mean value of $R$ is 0.50 in the non-triggered case and 0.46 in the triggered example.

**Figure 3.** Expectation of $R$ (Equation A10), i.e. predicted $\bar{R}$, as a function of fractional rate change for three probabilistic models for earthquake occurrence, as described in the text. ‘Omori’ refers to the nonhomogeneous Poisson model with Omori decay in triggered rate change. Inset shows the expectation of $R$ on a linear-log scale, while the larger figure shows the value $\langle \Delta R \rangle = 0.5 - \langle R \rangle$ on a log-log scale to emphasize the scaling at small fractional rate changes, like those sought in this study. Fractional rate change is the normalized triggered earthquake rate defined in Equation 3. The curves are all very similar, especially similar for small rate change.

**Figure 4.** Cartoon illustrating the construction of earthquake populations for analysis with the interevent time method. For the long-range case (red), various combinations of magnitude and distance are combined to create populations of earthquakes bracketing potential triggers of common dynamic strain amplitude at the site of the triggered quakes. The study area is gridded, and the interevent time ratio $R$ is calculated in each bin for
each trigger. Zones of common dynamic strain form arcs within the study zone. For the
short-range case (blue), populations are constructed by combining all earthquakes within
some small radius of potential triggers of common magnitude. Spatial grid is not to scale.

**Figure 5.** Distribution of $R$ for the 2002 M7.9 Denali earthquake. Individual $R$ values
are computed in $0.1^\circ \times 0.1^\circ$ spatial bins. The high proportion of small $R$ values
demonstrates triggering in a large proportion of the bins. The mean of $R$ is 0.475. Data
are smoothed by a 0.09 unit cosine filter for clarity.

**Figure 6.** Empirical probability densities (distributions) for $R$. (a) Long-range California
data (trigger distance > 800 km) for four dynamic strain increments. (b) Short-range data,
for magnitudes 3.1 to 5.1 in five increments. Curves have been smoothed for clarity
using a cosine weighted running average with a window width of 0.09 units. The
oscillations at this wavelength are therefore a spurious effect of the smoothing. Curves
are truncated at the limits to avoid plotting edge effects of the smoothing.

**Figure 7.** (a) The mean interevent time ratio $\bar{R}$, in terms of the deviation from the value
in the absence of triggering ($\Delta \bar{R} = 0.5 - \bar{R}$). Compare to Figure 3. Long-range (>800
km) triggers are in red and short-range triggers (<6 km) are in blue. Vertical and
horizontal error bars are 95% confidence limits. The green point corresponds to the
Denali earthquake. The red horizontal bar shows the $2\sigma$ uncertainty associated with the
farfield peak ground motion estimates. (b) **Triggering** intensity $n$ ($\bar{R}$ transformed via
Equation 2) as a function of peak dynamic strain in California and (c) Japan. The black dashed line in panels (a) and (b) shows the weighted least squares fit to the California farfield data, along with 95% confidence levels. The best-fit curve for California is also shown in (c) for comparison with Japan. (d) The fraction of nearfield triggered quakes accounted for by the farfield scaling relationship. First and second error bars represent 64% and 95% confidence limits, respectively.

**Figure 8.** Probability of having an earthquake within the pre-trigger recurrence interval (Equation 8) as a function of peak dynamic strain, in California. Colors and error bars are as in Figure 7. The horizontal dotted line shows the baseline probability of having an earthquake within its own recurrence interval in the absence of triggering (~63%). The black dashed line is the best-fit line from Figure 7a,b transformed using Equation 8, and shows that the rapid increase in probability is consistent with a smooth increase in triggering intensity. The threshold for dynamic triggering (black arrow) is seen to be an observational threshold, with the probability of observed triggering going smoothly to zero with decreasing dynamic strain.

**Figure 9.** Testing the effect of the earthquake cascade on the transformation of $\bar{R}$ to triggering intensity $n$. A zero-dimensional (time only) ETAS model is used to generate a simulated seismicity catalog in which the triggering law and causal relationships are known. The nonhomogeneous transformation, using Omori’s law, (thick lines) and the stepwise homogeneous transformation (thin lines) recover similar triggering intensities. If the interevent time ratio $R$ is calculated using the first causally related earthquake
before and after the trigger, the transformation recovers the imposed triggering law (solid curves). If the \( R \) calculation is not restricted to known causally related earthquakes (as occurs for real catalogs), the method recovers a reduced scaling exponent (dashed curves). Error bars are 95% confidence limits. Reference lines show a slope of 1 (the input relationship) and 0.5.

**Figure 10.** Triggering intensity \( n \), determined by the interevent time method, plotted as a function of peak dynamic strain for simulated seismicity catalogs. Triggering is simulated as an identical function of peak dynamic strain in both near (blue hues) and farfield (red hues) populations. Different point brightnesses correspond to different simulation realizations. The recovery of a continuous trend between near and far-field populations validates the method, and demonstrates that the offset in the observed trend (Figure 7b) can be confidently interpreted as reflecting an additional nearfield triggering component. Lines of slope = 1 (the input relationship) and slope = 0.5 are plotted to show that the recovered scaling is again reduced relative to the input.

**Figure 11.** Triggering intensity as a function of dynamic strain for different bin sizes. Curves on the left are farfield data; curves on the right are nearfield. The legend gives the farfield spatial bin dimension in degrees for each curve. The nearfield aftershock radius is scaled to cover the same area as the farfield bins. For clarity of presentation, data are plotted against the peak dynamic strain integrated over the bin, rather than the averaged dynamic strain, and are therefore offset. The slopes of the curves and the absolute values of the triggering intensity change slightly with bin size, but the offset
between trends is robust.

**Figure 12.** Sensitivity to long-range trigger cutoff distance for the California dataset. The legend gives the minimum distance used for potential farfield trigger earthquakes. Results are not sensitive to distance cutoffs above 800 km, although uncertainties grow larger because of the reduced catalog size for larger cutoffs. We do not investigate distance cutoffs below 400 km, because the surface wave magnitude relation (Equation 6) is not appropriate for such small distances.

**Figure 13.** The geographical distribution of triggering susceptibility in California partially reflects the background activity rate. **a)** Triggering intensity in 0.1° spatial bins for dynamic strains above $10^{-9}$. **b)** Background seismicity rate [Hardebeck et al., 2008], expressed in terms of the number of magnitude 4 and greater earthquakes per year in each ~0.5° bin. The plots are qualitatively similar, implying that all regions of active seismicity are triggerable, and no single region dominates the triggering signal. Both maps are smoothed by a 0.3° Gaussian kernel.

**Figure B1.** Peak Ground Velocity regression based on California Shakemap data. **(a)** PGV as a function of distance for a M4 reference earthquake, using several published regressions (blue [Atkinson and Boore, 1997], magenta [Boatwright et al., 2003], green [Abrahamson and Silva, 2008], cyan [Campbell and Bozorgnia, 2007]), as well as the regression found in this study (red curve). Grey dots are individual PGV measurements, rescaled to the reference magnitude, using the regression from this study. The vertical
dashed line at ~6.2 km marks the boundary of the nearfield region defined in this study.

(b) $\log_{10}$ misfit (the predicted PGV divided by the observed PGV) for each of the regressions in panel (a) as a function of distance. Misfits are averaged over magnitudes 3.5-5.5. Vertical dashed line as in panel (a). (c) PGV as a function of magnitude at a reference distance of 5 km. (d) $\log_{10}$ misfit as a function of magnitude for the various regressions, averaged over distances from 0 to 15 km.
### Table 1. PGV regression parameters

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<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$ (km)</th>
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<td>1.29 (1.58, 1.00)</td>
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### Table 2. ETAS Parameters

<table>
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<th>$A$</th>
<th>$c$ (days)</th>
<th>$p$</th>
<th>(events km$^{-2}$ strain$^{-1}$)</th>
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<td>0.095</td>
<td>1.34</td>
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Nearfield trigger

Farfield trigger

Zones of common peak dynamic strain
A graph showing the relationship between triggering intensity and peak dynamic strain. The graph includes data points for different distances: 800 km (red), 1600 km (green), and 3200 km (blue). The intensity is plotted on a logarithmic scale on the y-axis, ranging from $10^{-1}$ to $10^{0}$, while the peak dynamic strain is plotted on a logarithmic scale on the x-axis, ranging from $10^{-10}$ to $10^{-6}$. The data points are accompanied by error bars to indicate variability.