A Four-Dimensional Viscoelastic Model for Deformation of the Long Valley Caldera, California, between 1995 and 2000

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Abstract

We investigate the effects of viscoelastic rheologies surrounding a vertically dipping prolate spheroid source on an active period of time-dependent deformation between 1995 and 2000 at Long Valley caldera, including a rapid inflation episode with increased seismic activity between late 1997 and early 1998. We begin with the model in Newman et al. [2001], a spherical magma chamber encased by a 1-km thick viscoelastic shell (rigidity = 5 GPa, and viscosity = 10^{16} Pa-s – similar to a 670°C rhyolite at 5% wt. water) in an elastic half-space, and extend it to include a prolate spheroid geometry more accurately representing inversion results from purely elastic studies. The volume of the deformation source is constrained to not allow the overall pressure increase to exceed the lithostatic load while maintaining agreement with seismic and borehole drilling results. This paper represents the first attempt to geodetically constrain the source volume of the volcanic deformation source at Long Valley, a critically important value to assess the internal pressures that cause deformation and possible future eruptions. The model results well explain most time-dependent deformation observed with several EDM baselines and individual components of two continuous GPS time series. Additionally, the model well explains the spatial extent of deformation observed by InSAR covering the 1997-98 inflation episode. For the time period studied, the viscoelastic model required modest pressure changes, maximum of 14.3 MPa, are far lower than the overburden pressure, ~115 MPa. The modeled source pressure grew slowly before 1997 before rapidly accelerating until Nov 22nd, 1997, the onset of major seismic moment release along the south moat. At that point, the source pressure continued to grow but at a considerably lower rate before decaying in early 1998. For a purely elastic model with the same geometry and rigidity, the pressure change necessary to describe the 1995 through 2000 inflation is near 40 MPa, thus the inclusion of a viscoelastic component significantly lowers the necessary pressures. Though the model described here is non-unique, it provides considerable advances over purely elastic models in defining the time-dependent nature and pressures necessary to create the observed deformation at Long Valley Caldera.

Keywords: Global Positioning System, Finite Element Analysis, Long Valley Caldera, Viscoelasticity, Magma Chambers.

1 Introduction

Long Valley Caldera, in east-central California is situated on the eastern edge of the Sierra Nevada range. The 17x32 km caldera, (Figure 1), was created approximately 760,000 years ago in a massive paroxysmal ignimbrite eruption which expelled more than 600 km^3 of pyroclastic material and ash that also formed the locally >1 km thick Bishop tuff [Bailey et al., 1976]. Shortly after caldera formation the central resurgent dome, which raises approximately 500 m above the caldera floor, formed from magmatic uplift and coincident lava flows.

In the last 40 k.y. eruptions have been isolated to the Inyo and Mono chain, running north-south from the western part of Long Valley Caldera to Mono Lake (figure 1), and consisting predominantly of rhyolitic-rhyodacitic flows [e.g., Miller, 1985; Fink, 1985; Vogel et al., 1989]. The most recent eruptive period was between 700-500 years ago along the length of the Inyo-Mono chain, occurring in a series of small eruptions and phreatic explosions[Miller, 1985].
developed was the first to directly explain observed volcanic deformation incorporating such a viscoelastic rheology. However, using various geodetic datasets several authors have found that for a purely elastic half-space approximation, deformation is better described by a near vertically dipping prolate spheroid than by a spherical source [Tiampo et al., 2000; Fialko et al., 2001; Battaglia et al., 2003a; Langbein, 2003]. Here we again examine this phase of unrest, now with a vertically dipping prolate spheroid, with an axis ratio constrained by previous studies and surrounded by a 0.5 to 1 km thick viscoelastic shell within a purely elastic half-space. By making the assumption that the pressure within the source cannot exceed the lithostatic load, we constrain the minimum plausible source volume. We compare these results to 6 years of data from eight single-component EDM lines and two three-component continuous GPS receivers on the resurgent dome, as well as a single Interferometric Synthetic Aperture Radar (InSAR) image that spans the most significant inflation period. The close approximation of the viscoelastic prolate source model to each of these data suggest, though do not require, that the model adequately reflects the source pressures and rheology responsible for the inflation between 1995 and 2000 at Long Valley Caldera.

2 Data

Because of the increased seismic activity and notable elevation changes in surveyed leveling data between 1978 and 1982, the USGS along with other agencies and universities began intensely studying the Long Valley region. Geophysical instrumentation included short period and broad band seismometers, a two-color EDM network, two continuous GPS receivers, volumetric strain meters (dilatometers) and tiltmeters. Seismic and geodetic information gathered from these instruments are useful for assessing location, intensity and type of local earthquake activity [e.g., Newman et al., 1997; Dreger et al., 2000; Prejean et al., 2003; Hill et al., 2003], developing low resolution images of subsurface rheology [e.g., Steck and Prothero, 1994; Sanders et al., 1995; Sanders and Nixon, 1995; Foulger et al., 2003, 2004], and defining the extent of surface deformation caused by both earthquake and volcanic sources [e.g., Langbein, 1989; Langbein et al., 1993, 1995; Battaglia et al., 1999; Tiampo et al., 2000; Fialko et al., 2001; Newman et al., 2001; Battaglia et al., 2003a; Langbein, 2003].

2.1 Geodetic data

In addition to the near continuously recorded GPS, daily EDM, tilt and strainmeter data, deformation at Long Valley is also measured through infrequent leveling (every 2-3 years), frequent InSAR imagery (~1 per month depending on satellite), and less frequent EDM at additional
sites. Here we utilize only the available daily EDM, continuous GPS and InSAR data as they best captured the spatial and temporal deformation for the 1997-1998 inflation episode and seismic crisis.

2.1.1 GPS

Continuous GPS data was measured at two sites, CASA and KRAK, within the caldera and atop the resurgent dome during the 1997-1998 inflation episode (figure 2). The GPS data were analyzed at the University of Miami’s Geodesy Laboratory following Dixon et al. [1997] and are further explained in Newman et al. [2001]. The data are reduced in a global reference frame daily to yield geographic north, east and vertical components of deformation. The combination of white and flicker noise in the north, east and vertical components are ~7, 11 and 21 mm and random walk error, caused by monument noise and reduces with time, is ~2 mm/yr$^2$ [Mao et al., 1999; Langbein and Johnson, 1997].

In order to get at the local deformation we removed the effects of North American Plate (NAP) Euler pole in the 2000 International Terrestrial Reference Frame ($\theta_{NAP}$, $\phi_{NAP}$, $\omega_{NAP}$ = 4.59°S, 82.91°W, 0.195 °/my) following Sella et al. [2002]. Additionally we removed Sierra Nevada block (SNB) motion relative to NAP ($\theta_{SNB}$, $\phi_{SNB}$, $\omega_{SNB}$ = 17.0°N, 137.3°W, 0.28 °/my) following Dixon et al. [2000]. There are no assumed vertical components to NAP or SNB motions. After removing these vectors, the resultant vectors are the local deformation for the Long Valley region. Shown in figure 3 are the time series for the vertical, east and north components of the residual GPS information for sites CASA and KRAK. In each of the components there still exists a considerable amount of deformation information, starting with low rates through mid-1997, before all components from both stations increase exponentially in the same direction. Because the resulting vectors (figure 2) do not appreciably change direction with the rapid increase in magnitude it is reasonable to assume no significant change in the location or depth of the deformation source. In Newman et al. [2001], we noted that the inflation source responsible for the steady inflation between 1995 through mid-1997 and the exponential change shown between mid-1997 through early 1998 did not appreciable change horizontal location.
or depth from the source responsible for most deformation between 1989 and 1992 as defined within 95% confidence by Langbein et al. [1995] and shown in figure 2.

Because GPS data provides a continuous time series of geo-centrally referenced three-component vectors it is extremely useful understanding the true nature of surface deformation. If errors are low enough, two three-component continuous GPS stations are all that are necessary to define a point source within a homogeneous half-space. However, because the true source of deformation has a volume, is likely non-spherical, includes possible deeper sources and offsets along the south moat, and because the 1997-98 episode involved rapid non-linear inflation and possible aelastic rheology, the examination of other data including EDM and InSAR are particularly helpful.

2.1.2 EDM

EDM uses laser ranging to measure a single component length change between a base laser and a series of reflectors. At Long Valley, a 2-color (dual frequency) laser is used to minimize errors in travel time, converted to length, due to atmospherice pressure and moisture. Per km of baseline, the white noise errors are between 0.1-0.2 mm an random walk errors, which reduce with length of recording, are ~ 0.3 mm/yr^2 [Slater and Huggett, 1977; Langbein and Johnson, 1997]. At Long Valley Caldera, frequent, near daily EDM measurements are made from a base site at Casa Diablo (CASA) immediately next to the continuous GPS station by the same name, toward eight reflectors (KRAK, SAW, KNOLLS, HOT, SHERWIN, SHARK, MINER and TILLA) (figure 4). Additional, less frequent baselines were measured from CASA to nearly 30 additional sites between 4 and 15 times per year and extremely infrequent, annual to subannual measurements were taken from an additional five base stations within the caldera and along the Inyo-Mono chain.

For this study we only examine data from the near daily measurements from site CASA that were between 1998 and early 2000. The EDM data are useful because they augment the available continuous GPS data with more available continuous data, however showing only one component of relative motion between reflector sites and the base station. The time series of these data (figure 3) have lower errors and fewer outages than GPS data. The four baselines across the resurgent dome (KRAK, SAW, KNOLLS and HOT) all clearly capture the long steady inflation between 1995 and 1997 before increasing exponentially during the crisis. The baselines which near or cross the south moat (SHERWIN, SHARK, MINER and TILLA) are less clear. Two of these sites (SHERWIN and TILLA), show a clear step in late 1997 while the other two (SHARK and MINER) show a slow gradual increase or no clear length-change. This step in late 1997 is most likely due to active faulting across the south moat [Langbein, 2003].

Though the GPS and EDM data give near continuous deformation data, without extreme cost, they are unable to give dense spatial coverage necessary to fully define...
2.1.3 InSAR

Being that InSAR data come primarily from existing satellites with near global coverage, individual images are relatively inexpensive to obtain thus making it incredibly useful for finding previously unknown or new sources of deformation or for spatially augmenting existing spatially limited geodetic data. InSAR works by revealing phas differences between reflected radar signals off the surface of the earth. The data are inherently affected by topography thus it is necessary to remove known elevation using a digital elevation model (DEM) for the region. The resulting phase differenced information gives relative length changes between individual points on the earth’s surface in the line-of-sight (LOS) direction of the satellite.

The InSAR data we examine for Long Valley come from the European Space Agency C-band satellite ERS 2, which has wavelength of 56.6 mm and a repeat orbit every 35 days. Though it is fruitful to stack multiple images to obtain a robust image of deformation with minimal errors and regions of decorrelation, such as done by Fialko et al. [2001], in order to capture the magnitude and location of time-dependent deformation it is necessary to examine individual pairs. Thus, we examine an individual InSAR pair that spans the major inflationary 1997-98 episode. Shown in figure 4, is an InSAR image of Long Valley caldera showing the wrapped 1/2 wavelength phase change in LOS direction between August 12, 1997 and May 19, 1998 (period of image also shown in figure 3 for clarity). LOS for this image, a descending orbit, is 76° CCW from north and 23° from vertical, hence the data is mostly vertically with a small 14 degrees south of east horizontal component. Each color “fringe” represents 28.3 mm of motion, showing a total of approximately 100 mm of LOS inflation at the central “bull’s eye”. The data is reasonably radially symmetric with a slight elongation in the east-west direction (same as caldera rim).

Repeat InSAR images from the same orbit give only 1-dimensional information, and are spatially continuous (excluding areas of decorrelation, such as in the western portion of figure 3). However, the combination of these data with GPS and EDM all help to give a full four-dimensional, three spatial dimensions plus time, image of deformation activity at Long Valley during the 1997-1998 inflation episode.

2.2 Seismic data

Seismic data available from the Northern California Earthquake Data Center (quake.geo.berkeley.edu) are shown for the early 1995 through mid-1997 slow inflation and peak late-1997 inflation, color coded in figure 2. The earthquakes are spatially offset, mostly occurring in the south moat, from the inflation center, but are well correlated in time, with the maximum seismic moment release following the exponential increase in inflation [Newman et al., 2001; Hill et al., 2003]. Additional, high precision double-differenced relative earthquake locations (figure 5) are available through early September (recorded as part of two temporary arrays installed by the USGS and Duke University) [Prejean et al., 2003]. Unfortunately, because these data are only available through early September, they miss the major moment release in late November.

These data are useful for assessing the location of the brittle-ductile transition because earthquakes are impossible in ductile material (generally greater than 300°C). Additionally, by defining the south moat fault and movements across it [e.g., Dreger et al., 2000], seismicity helps to explain non-radially symmetric components of observed 1997-1998 deformation.

3 Model

Deformation due to volcanic inflation at Long Valley has been assessed by many authors, using either spherical, prolate-spheroid, ellipsoidal or penny-shaped crack sources using either GPS, EDM, leveling and/or InSAR data to corroborate models [e.g., Langbein et al.,
With the exception of [Newman et al., 2001], which used a spherical source within a viscoelastic shell in an elastic half-space, all models have assumed all deformation occurred within a homogeneous purely elastic medium. Given that Long Valley caldera has an extensive eruptive history, and is currently undergoing unrest, it is logical to assume that near the source of inflation, the surrounding rock is considerably heated and weakened beyond the brittle-ductile transition, becoming viscoelastic. Here we explore a finite element (FE) model that incorporates a plausible viscoelastic region near the volcanic deformation pressure source. Additionally, we assess a realistic source volume (necessary for assessing source pressures). Because of the numerous variable parameters, the results are non-unique, however they illustrate the impact viscoelasticity has on time-dependent deformation given a plausible pressure history. Because it is non-trivial and time-intensive to geometrically parameterize and mesh appropriate FE models, we explore only one geometry that is built upon source properties determined from prior, published analytical inversions of geodetic data.

3.1 Assessing Source Volume

With geodetic data we can normally calculate volumetric change, $\Delta V$, assuming deformation occurs within a homogeneous purely elastic half-space and a given source location and depth. Using the equation by Mogi [1958], McTigue [1987] showed that for a spherical source model $\Delta V$ could be found for either horizontal, $U_x$, or vertical deformation $U_z$, by:

$$\Delta V = \frac{4\pi}{3} U_x \frac{(d^2 + r^2)^{3/2}}{r} = \frac{4\pi}{3} U_z \frac{(d^2 + r^2)^{3/2}}{d},$$

where $r$ and $d$ are the surface length between displacement measurement and source center and the depth of source. For a prolate spheroid shaped source, an ellipsoid whose two minor axes are of equal length (football shaped), the analytic equation relating $\Delta V$ and $U$ becomes quite complex, but is still controlled only by the source location and geometry, given a homogeneous purely elastic half-space [Yang et al., 1988; Davis, 1986].

In order to get bounds on the total volume involved in expansion, a few assumptions need to be made about the rheology of the media between the deformation source and the surface. Additionally, it is necessary to assume, that since no eruption had occurred due to the source inflation, the overall pressure change, $\Delta P$, will need to stay below the overlying lithostatic load, $P_{\text{litho}}$. Though the tensile strength of the overburden rock may be as high as 10 MPa, it is permeated by fractures and have no effective tensile strength.

Though aspects of published models for vertically, or near vertically, dipping prolate spheroids at Long Valley vary (table 1), rough estimates of individual parameters are sufficient [Langbein et al., 1995; Fialko et al., 2001; Langbein, 2003; Battaglia et al., 2003a]. For the 1997-98 episode, depth varies between 5.9 and 7.2 km, however Langbein et al. [1995] found only 5.5 km depth for the 1989 episode. A depth of 6 km for our model is reasonable because it is unlikely that the source depth changed significantly over this time. Between various models, the axis ratio of the semi-minor, $b$, and semi-major, $a$, $b/a = 0.43$ to 0.85, depending on the data used. Here we will choose $b/a = 0.5$, which tends slightly to the more spherical source than the average of the more pipe-like models of 0.45, but much more elliptical than the nearly spherical source, 0.85, found by Langbein [2003] for the 1997-98 source. Additionally, since volume change is cumulative, we must assess the total growth between 1978 and 200 which is approximately 0.1 km$^3$.

The geometric equation relating $\Delta V$ and $\Delta P$ for a prolate spheroid ($a = b$ for a spherical source) is defined by Tiampo et al. [2000] as:

$$\Delta P = \frac{\mu \Delta V}{\pi ab^2}.$$  

The lithostatic load, $P_{\text{litho}}$, at the top of vertically dipping prolate spheroid pressure source is defined as:

$$P_{\text{litho}} = (d - a)\rho g,$$

where $\rho$, and $g$ are the rock density and the local gravitational acceleration. Assuming the maximum allowable pressure within the chamber, $\Delta P_{\text{max}} = P_{\text{litho}}$, otherwise an eruption is expected, which at Long Valley did not occur between 1978 and 1998, and using an axes ratio $b/a = 0.5$, we obtain the following relation:

$$\Delta P_{\text{max}} = \frac{4\Delta V \mu}{\pi a_{\text{min}}^3} = (d - a_{\text{min}})\rho g.$$  

Now using $\Delta V = 0.1$ km, $d = 6$ km, and approximate values for density, $\rho = 2800$ kg/m$^3$, gravitational acceleration, $g = 10$ m/s$^2$, and rigidity suitable for hot volcanic region, $\mu = 5$ GPa [e.g., Bonafede et al., 1986], we obtain a minimum size for the prolate spheroid, where $a_{\text{min}}$ and $b_{\text{min}} = 1740$ and 870 m. The maximum allowable size mathematically, goes almost to the surface.

The maximum size of the vertically dipping magmatic pressure source at 6 km depth is realistically limited by seismicity and thermal information from the Long Valley Exploration Well (LVEW) [Fischer et al., 2003]. The seismicity in the immediate vicinity of the magmatic source region occurs along the south moat and is mostly above 7 km depth and more than 4 km from the source (figures 2 and 5). Since there is virtually no seismicity directly above the focus of the magmatic center, the source could conceivably go almost to the surface, however, no such body is detectable with local seismic tomography shallower than 3 km depth, before loosing resolution deeper [Foulger et al., 2003]. Additionally, the LVEW site, was
drilled to 3 km depth approximately 1 km from the magmatic source focus and is only ~100°C between 2500 and 3000 m depth. However, it is evident that the rocks at 2600 m depth were nearly 200°C hotter, and similar to surrounding bore-hole temperatures [Sorey et al., 1991], before changes in the hydrothermal system [Fischer et al., 2003]. Because of these maximum limitations on the size of the spheroid, we chose a model near the small end with a and b = 1800 and 900 m.

### 3.2 Viscoelasticity

Volcanoes are hot, hence material surrounding a magmatic source, if long-lived should also be heated significantly, and beyond the brittle-ductile transition. Beyond this point, rock no longer behaves in a purely elastic manner, but maintains a component of permanent deformation because the fluid strength, or viscosity, \( \eta \), is significantly lowered. However, unless the material contains a significant percent melt, it maintains a partially recoverable elastic response. When rocks behave in this manner, with both elastic and viscous responses to stress they are considered to be a Maxwell viscoelastic material. In such a material an imposed stress will have both a instantaneous and recoverable strain and a time-dependent permanent strain whose rate is controlled by its Maxwell or characteristic time, \( \tau \), which is generally described as:

\[
\tau \approx \frac{\eta}{\mu},
\]

Dragoni and Magnanensi [1989] describe a viscoelastic shell model for volcano deformation, with a spherical pressure source surrounded by a concentric, spherical, Maxwell viscoelastic shell within a purely elastic half-space. In the analytical equation’s simplest form, that of an instantaneous pressure increase, \( \Delta P \), radial displacement as a function of radial distance, \( r \), and time, \( t \), is described as:

\[
u_r(r,t) = \frac{1}{4} \frac{\Delta P}{\mu} \frac{R_2^3}{r^3} \left[ 1 - \left( \frac{R_1}{R_2} \right)^3 e^{-t/\tau} \right],
\]

where the shear modulus, \( \mu \), for the outer and inner layers are equal, \( R_1 \) and \( R_2 \) are the radii from the source center to the source wall and viscoelastic shell wall, and whose time-dependent deformation is controlled by the characteristic time, \( \tau \). The characteristic time, is controlled by:

\[
\tau = \frac{9}{5} \frac{\eta}{\mu} \left( \frac{R_2}{R_1} \right)^3.
\]

Thus, the deformation is controlled by an exponential with time, the geometry and the ratio of rigidity and viscosity.

In Newman et al. [2001], we showed that for hot wet rhyolites (containing 5% wt. \( \text{H}_2\text{O} \), similar to what is found in Long Valley) ~670°C and near the solidus temperature, have a viscosity, \( \eta \approx 10^{16} \) Pa s. Additionally, for hot quartz-bearing country rock, around 350°C, \( \eta \) is between 10^{17} and 10^{19} Pa s [Luang and Patterson, 1992; Ivins, 2000]. For our model, we use a variable thickness viscoelastic shell that is 1 km thick at top and bottom and 0.5 km thick on the sides with a viscosity of 10^{16} Pa s (figure 5). This is done to maintain \( b/a = 0.5 \) for the purely elastic half-space outside viscoelastic shell. In fact, Davis [1986] suggested that the ellipsoidal source can be a region of low viscosity rock. Determining the maximum allowable viscosity is dependent on the time period of deformation analyzed. Because previous elastic models use long time constants of deformation, ~3 yr, they are not sensitive to low viscosity rheology. Using an order of magnitude prolate ellipsoid approximation of equation 7, we find that depending on the period of the time averaged data, \( T \), the minimum resolvable viscosity, \( \eta_{\text{min}} \), can be describe as:

\[
\eta_{\text{min}} \approx T \mu \left( \frac{a}{a'} \right) \left( \frac{b}{b'} \right)^2.
\]

where \( a' \) and \( b' \) are the radii from the source center to the outer edge of the viscoelastic shell in the semi-major and semi-minor directions. Thus, the prolate spheroid in most long valley models, with \( T \approx 3 \) yr, contains material with viscosities less than ~10^{17} Pa s. Alternatively, because our model is time-dependent and is comparing data measured on a daily basis, our prolate spheroid is sensitive to viscosities less than ~10^{15} Pa s. Thus, for modeling daily sampled time-dependent deformation it is justifiable to consider a smaller region with a lower viscosity region than can be determined from longer period, \( T \approx 3 \) years, averaged models.

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**Table 1:** Published prolate spheroid source parameters from inversions of geodetic data. *Includes a second deeper source between 10-20 km depth. † Converted from stress ratios using Davis [1986]. ‡ Volume change is calculated from \( \Delta P = 8 \) MPa and \( \mu = 5 \) GPa using equation 2. § Volume here is assumed for the entire inflationary period between 1978 and 2000 since the total \( \Delta V \).

<table>
<thead>
<tr>
<th>Paper</th>
<th>Depth ( d [\text{km}] )</th>
<th>Volume change ( \Delta V [\text{km}^3] )</th>
<th>Axis ratio ( b/a )</th>
<th>Episode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Langbein [2003]*</td>
<td>6.0</td>
<td>0.03</td>
<td>0.45†</td>
<td>1989</td>
</tr>
<tr>
<td>Fialko et al. [2001]</td>
<td>7.2</td>
<td>0.07‡</td>
<td>0.43</td>
<td>1997-98</td>
</tr>
<tr>
<td>Langbein [2003]*</td>
<td>6.0-7.0</td>
<td>0.03</td>
<td>0.85‡</td>
<td>1997-98</td>
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<tr>
<td>Battaglia et al. [2003a]</td>
<td>5.9</td>
<td>0.09</td>
<td>0.48</td>
<td>1997-98</td>
</tr>
<tr>
<td><em>This study</em></td>
<td>6.0</td>
<td>0.1§</td>
<td>0.5</td>
<td>1997-98</td>
</tr>
</tbody>
</table>
3.3 Finite Element Model

Because of the geometric and temporal complexities involved in this model, a prolata spheroid within a viscoelastic shell and a varying pressure source, an analytic solution would be exceedingly difficult to derive mathematically, therefore, we utilized the finite element method and analyzed this code with the commercial software package ABAQUS [ABAQUS, Inc., 2003]. Because our model assumes radial symmetry about the source center, we use a two dimensional axisymmetric model that is far more efficient than an equivalent full three-dimensional model. Though this model’s rheology is more complex than others proposed for Long Valley deformation, it still does not account for the full spatial complexity of deformation sources and rheology as shown in figure 5; a comparison of our model and a more realistic geologic cross section, modified from Hill et al. [1998]. The source location chosen for this model is 37.687°N, 118.915°W, the source location for the 1989 source of Langbein et al. [1995], and is comparable to other inversion result locations [e.g., Fialko et al., 2001; Battaglia et al., 2003a; Langbein, 2003].

To approximate a half-space, the axisymmetric FE model, shown in figure 6, contains ~6100 quadrilateral elements and extends 50 km horizontally from the source center and 50 km down from the surface. It is bound by rollers (no off-lateral motion) on either side nor bottom. Near the source center are two elliptical shells within a larger outer spherical shell, which are all useful for evaluating multiple layers of varying viscosity material.

In Newman et al. [2001], we found good agreement between a similar FE spherical viscoelastic shell model surrounding a spherical pressure source within a full-space, with the analytic equivalent described by Dragoni and Magnanensi [1989]. However, it is beneficial to test the validity of the FE approximation of our modeled prolata spheroid as well. Thus, we compare the FE model, holding \( \mu = 5 \text{ GPa} \), and Poisson’s ratio, \( \nu = 0.25 \), for all regions outside the source to the Yang et al. [1988] analytic model for the same geometry \((\theta, \phi, P, z_0 = 90^\circ, 0^\circ, -12.825 \text{ MPa}, 6 \text{ km})\). The model geometry is the same as in figure 6 and the \( \Delta P \) was chosen to be near the maximum explored in our model. The two models are similar, however the FE model slightly under estimates the Yang model by between 5-10% at distances between 10 and 25 km. This is acceptable because most of the data being compared is within 10 km of the deformation source epicenter. The smaller model, has considerably fewer integrations and runs in 70% faster than the larger model.
in section 3.2. Using a starting point pressure history, the final model L-2 from Neuman et al. [2001], we performed a series of forward models to best fit the single EDM Baseline between CASA and KRAK (the baseline crossing the central resurgent dome and showing the most deformation). The only parameters that were variable in fitting the model were the time and magnitudes of $\Delta P$. Because the model includes a viscoelastic layer causing a significant time-dependent component to deformation, it was necessary to begin with the earliest portion of the pressure history matching the first data and maintaining the an approximate fit to the near future data, before continuing forward in time until all data from the CASA-KRAK EDM baseline were adequately fit.

Figure 8[Bottom] shows the resultant pressure history obtained by fitting the CASA-KRAK EDM baseline data while maintaining a minimal number of segments (table 3). The pressure history consists of seven piece-wise linear and continuous segments, with a maximum $\Delta P$ of only 14.3 MPa, almost an order of magnitude lower than the lithostatic load near the source top ($\sim$ 115 MPa). The model has a slow and steady increase in pressure between 1995 and early 1997 before increasing rapidly through late 1997. On November 22nd, 1997 (Day 1056 in table 3) the pressure increase decelerates rapidly before decaying in early 1998. The onset of this deceleration is intentionally coincident with the onset of major moment release across the south moat. Because this change in the pressure history is justified by changes in the GPS and EDM time series, it is likely that the sudden deceleration in pressure increase within the pressure source is directly related to the maximum moment release along the south moat. A similar result was found in Neuman et al. [2001].

### 3.4 Effect of Rheology on Observed Deformation

In order to obtain an understanding of the effects of rheology, we run additional models using the best fit pressure history from model VE1 and vary rigidity and viscosity for individual model components with a range of plausible values. Table 2 shows the values for each.

In model VE2, the viscoelastic layer was first broken into two layers with the outer layer being 100x more viscous, analogous to 350°C granodiorites (country rock) [Luan and Patterson, 1992; Ivins, 2000]. Because the model more closely approximates an elastic half-space over VE1, with less of a time-dependent character, the modeled vertical deformation at site CASA is 40% smaller and has sharper transitions (figure 8).

The next model comparison has the same viscoelastic shell as VE1, but includes a large circular shell of the higher viscosity country rock. Because this model is similar to VE1, incorporating a large outer annulus of higher viscosity material, the modeled vertical deformations for site CASA are similar; VE3 does predict about 20% more inflation after the rapid pressure increase in late 1997 due to the larger region with higher viscosity.

In model VE4, the viscous properties for all materials are the same as in VE1, however the rigidity is raised to normal crustal values of 30 GPa [e.g., Masters and Shearer, 1995], rather than 5 GPa, which has been suggested appropriate for hot volcanic regions [e.g., Bonafede et al., 1986]. As expected, because deformation scales inversely with rigidity, the 6 fold increase in rigidity results in about 1/6th the deformation.

Lastly, we test a model, LE1, assuming all materials to be purely elastic, $\mu = 5$ GPa, hence any change in pressure is translated into a completely recoverable instantaneous deformation. Here the deformation time series is a scaled image of the pressure history because there is no longer a time-dependent character. It is interesting to note that the predicted deformation is approximately 30% that of VE1 even though the elastic character of each model is the same. Thus, for the given geometry, a purely elastic model requires at least a 3 fold increase in source pressures to explain the same magnitude of deformation as does VE1.
Once the optimal pressure history was determined for model VE1, modeled deformation results were compared to additional EDM and GPS time-dependent data. Both CASA and KRAK had comparable vertical uplift (between 100-120 mm). This causes the EDM baseline to essentially become a summation of the horizontal components and is therefore not sensitive to vertical deformation. Though the model was developed to fit the CASA-KRAK EDM 1-component baseline length change, it also well describes the three-component time-dependent deformation signals from the CASA and KRAK continuous GPS; rotated in radial and transverse components relative to the inflation source (figure 9[left]). If all deformation occurred due to a single radially symmetric source at 37.68°N, 118.91°W and there remains no residual component of NAP or SNB motion, then the transverse component, $U_T$, on each GPS receiver should show zero motion. In fact, $U_T$ from both GPS sites are nearly flat, suggesting little off-axis deformation. KRAK $U_T$ does however, have a slight CCW trend and an offset that is coincident with the major moment release along the south moat. This is an interesting result because site CASA is clearly closer to the south moat but does not appear to have a similar offset. Though this model cannot resolve it, it is likely that direction of south moat motion at site CASA was similar to the direction of the radial component and had little effect on the transverse component.

Shown in figure 9[Right], is the VE1 modeled time-dependent deformation along with data for the 8 frequently observed EDM reflectors from CASA (baseline map in figure 4). The fit in the CASA-KRAK baseline is explicit since this the data used to fit the model. The baselines for the three reflectors north of CASA and transverse the resurgent dome all do reasonably well, with the best fit for the CASA-HOT baseline (~80° CW from KRAK), however the model over-predicts deformation for both the SAW and KNOLLS baselines. For each of the four baselines that cross the south, and either near or cross the seismically active south moat, poorly match the data. In the case of TILLA, the data go the opposite direction, during the peak in south moat seismicity. This is most certainly due to to fault related deformation not considered in this model.

To understand the applicability of model VE1 to regions across the caldera, model results are then compared to an InSAR image spanning most of the 1997-98 inflation episode. Figure 10 [Top-left] shows the same InSAR image from figure 4, from August 12, 1997 through May 19, 1998, now unwrapped and converted to velocity. Also shown are the VE1 modeled results for that time and the spatial residual. Note that along with the data, the modeled peak inflation is offset from the source axis, because LOS is about 20° off-vertical from the east. Overall, the modeled results closely approximate the InSAR data, with a mean residual of 13 mm/yr (10% of the data). However the model does tend to overpredict the data to the northwest while underpredicting to the southeast, with the peak model deficiency near the eastern edge of the south moat fault. Once again, this is likely due to unmodeled deformation due to seismicity across the south moat.

All the above data types have components of unmodeled deformation that are spatially and temporally correlated with the four M4.6 − 4.9 right-lateral strike-slip earthquakes along the south moat in late November 1997. It is interesting to note that for all periods within the model VE1, at a radial distance of 6 km, corresponding to the distance to the south moat fault, the pressure source always created a small positive force in the radial direction and near normal to the south moat, $\sigma_{33} = 0.05 − 0.25$.
MPa. This force alone is not large enough, and of the wrong direction to allow for largely dilatational earthquakes observed on the south moat by Dreger et al. [2000]. It would be fruitful to additionally assess the deformation associated with this seismicity. However, because the model developed here is axisymmetric, it is unable to directly account for such deformation. Though it is beyond the scope of this paper, it would be plausible to first model the south moat seismicity, remove its effect from the data and then model the volcanic source. The shortfall with this method, however, is that a component of the volcanic source deformation will also affect the seismic results. Thus, it is likely best, though time intensive, to perform a full three dimensional model that can jointly assess both sources, as done analytically for an elastic half-space by Langbein [2003]; Battaglia et al. [2003a].

5 Conclusions

A single radially symmetric prolate spheroid pressure source within a thin 0.5 to 1 km viscoelastic shell with a relatively simple and small magnitude pressure history, well matches the observed four dimensional time-dependent geodetic data, temporally continuous GPS and EDM and spatially continuous InSAR, from the 1995 to 2000, including the 1997-98 inflation episode at Long Valley caldera. The vertically dipping prolate spheroid, twice as tall as wide, at 37.687°N, 118.915°W and 6 km depth and the viscosity and thickness of the surrounding layers are constrained by previous studies. However, the source volume, was constrained here to require the long-term pressure increase between 1978 and 2000 to stay below the lithostatic load, as well as agree with observed seismic and deep borehole data. This paper represents the first attempt to geodetically constrain the source volume of the volcanic deformation source at Long Valley, a critically important value for assessing the internal pressures that cause deformation and possible future eruptions. For the time period studied, the viscoelastic model required only a modest pressure increase of about 14 MPa, far lower than the lithostatic load, about 100 MPa. The pressure grew slowly between 1995 and early 1997 before rapidly accelerating until Nov 22nd, 1997, the onset of major seismic moment release along the south moat. At that point, the pressure continued to grow but at a considerably lower rate before decaying in early 1998. For a purely elastic model with the same geometry and rigidity, the maximum pressure change necessary to describe the 1995 through 2000 inflation is around 40 MPa, significantly lowers the necessary pressures. Though the model describe here is certainly non-unique, it provides a considerable advance over purely elastic models in defining the time-dependent nature and the pressures necessary to

Figure 9: Deformation and model fit to GPS and EDM data: [left] Vertical, radial and transverse components of modeled (blue, red and green) and measured (dashed line) displacement at continuous GPS sites CASA and KRAK shown on figure 2. [right] Baseline length changes between the base station, CASA, and individual reflector stations, shown in figure 4. EDM stations CASA and KRAK are similar in location to GPS stations with the same name. EDM baselines shot across the resurgent dome (red) closely fit model prediction (dashed lines). However, sites measured across the south of the resurgent dome (blue) and around the seismically active south moat, poorly match model prediction. Note visible changes in deformation, especially on tangential component of GPS deformation at KRAK and EDM deformation to station TILLA, correspond to peak in south moat seismicity.
create the observed deformation at Long Valley Caldera.

References


