Problem set #1. Heat problems
(Due in class on April 11th)

Please be neat and organized in what you hand in. For example, it would be a good idea to put a box around your final answer, so that the grader can find it. We do not want to see all your scratch work. Once you have found a way to the answer, rewrite it in an orderly fashion so that we can follow your steps. When plots are requested, do your best to use a please use a computer. Programs like Excel, Kaleidagraph, Matlab and Gnuplot are relatively easy to make plots from.

Warm-up (so to speak)

Radiation
1. Radiation balance. At what distance form the sun would the black-body temperature of a hypothetical planet be exactly 0°C ( = 273°K)? Assume that the albedo of this planet is 0.3, or roughly that of Earth. (You may want or need to look up pertinent astronomical scales in the Harte handout.)

2. If the eccentricity, $e$, of the earth's orbit about the sun is 0.017 ($e = 1 - \frac{R_{\text{min}}}{R_{\text{mean}}}$), what is the expected amplitude of the variation in arriving solar radiation flux, $Q_o$, over the course of a year? Specifically, tell me the value of $(Q_{\text{omax}} - Q_{\text{omin}})/Q_o$. (Note that eccentricity is the orbital parameter that varies with about 100 kyr period, in which there is so much power in the climate variability within the last half a million years...)

Conduction
3. Heat flow in the Basin and Range. In the Basin and Range province of the western US the heat flow at the earth's surface is considerably enhanced. In a 300 m deep borehole in Owens Lake in the eastern California portion of the Basin and Range, the temperature increased by 18°C from the surface to the base of the borehole. The conductivity of the materials is $4 \text{ W m}^{-1} \text{ °K}^{-1}$. What is the heat flux (also sometimes called the heat flow) at the earth surface? Please express this first in W/m², and then in heat flow units (HFU, where $1 \text{HFU} = 41.84 \text{ mW/m}^2$). Note that the worldwide average heat flow is 1.67 HFU.

4. Thermal profile in and beneath an ice sheet. Consider a portion of the Antarctic ice sheet that is 2 km thick (in places it is significantly more than this). The mean annual surface temperature is -50°C. The heat flux is 54 mW m², which is a decent average for Antarctica. The conductivity of ice is $2.2 \text{ W m}^{-1} \text{ °K}^{-1}$, and that of the underlying bedrock is $3.5 \text{ W m}^{-1} \text{ °K}^{-1}$.
   a) Calculate and then plot the steady state geotherm for this location, taking the temperature profile down into the underlying rock by 2 km. Assume no heat production takes place.
   b) At what depth into the bedrock does the temperature of the rock rise above freezing point?
5. Contribution of radioactive heat sources to surface heat flow. Consider the case we treated in class, of exponentially distributed radioactive heat sources. The surface heat flow is measured to be 54 mW m\(^{-2}\), and the measured heat production rate due to radioactive heat sources, \(A\), is \(9.6 \times 10^{-10}\) W kg\(^{-1}\). (note the units -- this is heat production per unit mass, not per unit volume) Take the characteristic depth, \(D\), over which the heat production rate falls off to \(1/e\) of its surface value, to be 8 km. The rock is granite and has a density of 2700 kg m\(^{-3}\).

a) Please estimate the mantle heat flow in this location.

b) What fraction of the heat flow in this location comes from the mantle, and what fraction comes from the decay of radioactive elements?

c) If the mean annual surface temperature is 10 °C, and the thermal diffusivity of the entire rock column is 1 mm\(^2\) s\(^{-1}\), please plot the geotherm down to 30 km depth. You will need to know that the heat capacity \(c = 1000\) J kg\(^{-1}\) °C\(^{-1}\).

6. Periodic surface temperatures.

a) Estimate the depth to which frost penetrates the ground at a location where the temperature ranges between -10 °C and 25 °C. Assume the temperature history at the surface is sinusoidal, and that the water content of the soil is low enough that we can ignore the latent heat associated with the phase change of water. Assume the thermal diffusivity of soil, \(\kappa\), is 0.8 mm\(^2\) s\(^{-1}\).

b) At this depth, how far out of phase (by what fraction of a year) will the temperature be relative to the surface. In other words, the maximum frost penetration will likely occur after the minimum in surface temperature; I am asking how much later this will occur.