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Introduction

The data files contain the in-situ temperature data and sediment core data on which the geothermal heat flux measurement was based. This document details the sources of uncertainty in this measurement. We present visualizations of the full temperature records and the equilibration time window from the geothermal probe deployments. We also include the Particle Size Analyzer (PSA) grain size distributions for depth intervals throughout the sediment core.

We provide justifications and references for the values reported in Table 1 for the magnitude and spatial footprint of elevated geothermal heat flux by different mechanisms.
We also offer an analysis of the sensitivity of the critical geothermal heat flux values to model inputs.

**Text S1. Candidates for GHF variability at Whillans Ice Stream.**

A satisfactory explanation of GHF at the Whillans Ice Stream should account for the observed magnitude and length scale of variability. The observations indicate that GHF varies from \(88 \pm 7 \text{ mW m}^{-2}\) to \(285 \pm 80 \text{ mW m}^{-2}\) over a distance of 100 km. We seek explanations that: (a) are capable of generating GHF in excess of 200 mW m\(^{-2}\), (b) have a footprint capable of generating spatial variability in GHF along a distance of 100 km.

We start with the analytic solution for geothermal heat production given by Fox Maule et al. [2005] from which a map of geothermal heat flux for Antarctica was generated. The 1-D steady-state heat equation including conduction and radiogenic heat production is:

\[
k \frac{\partial^2 T}{\partial z^2} = -A(z) \quad (1a)
\]

\(k\) is the thermal conductivity and \(A(z)\) is the rate of radiogenic heat production as it varies with depth. Density and thermal diffusivity are assumed constant with depth. Radiogenic heat production rate in the upper crust is defined as an exponentially decreasing function of depth [Swanberg, 1972]:

\[
A(z) = A_s \exp \left(-\frac{z}{h}\right) \quad (2)
\]

where \(h\) is the e-folding depth of radiogenic heat production, and \(A_s\) is the radiogenic heat production rate at the surface.

The solution to equation (1) given boundary temperature conditions at the surface \((T_s)\) and at the Curie depth, \(L\), \((T_L)\) is

\[
T(z) = T_s + \frac{A_s h^2}{k} \left( 1 - \exp \left( -\frac{z}{h} \right) \right) + \frac{z}{h} \left( T_L - T_s - \frac{L h^2}{k} \left( 1 - \exp \left( -\frac{z}{h} \right) \right) \right) \quad (3).
\]

The heat flux is given by

\[
q(z) = -k \frac{\partial T}{\partial z} \quad (4).
\]

The surface heat flux is

\[
q(z = 0) = -\frac{k (T_L - T_s)}{L} - A_s h + \frac{A_s h^2}{L} \left( 1 - \exp \left( -\frac{z}{h} \right) \right) \quad (5).
\]
We use the same parameters chosen by the Fox Maule et al. model (Table S2) unless indicated otherwise. At WGZ, where the Curie depth is 25 km, the steady-state solution for GHF is 79 mW m\(^{-2}\).

1.a Thermal conductivity variations

The crust of the WARS beneath the WAIS is believed to be comprised of Cenozoic volcanic rocks associated with extension, 70-90% of which are basalt [LeMasurier, 1990]. Thermal conductivities of volcanic and plutonic rocks generally fall within the range 1.5 - 4.5 W m\(^{-1}\)°C\(^{-1}\). Low porosity volcanic rocks such as basalts have a mean and S.D. of 2.9 ± 0.7 Wm\(^{16}\)C\(^{-1}\) [Clauser and Huenges, 1995]. This bedrock is draped by a layer of sediments that is generally <1 km thick [Anandakrishnan and Winberry, 2004; Muto et al., 2013]. The thermal conductivity of sediments we measured was 1.77 ± 0.15 W m\(^{-1}\)°C\(^{-1}\); which is broadly consistent with the observed \(k\) range for low porosity sedimentary rocks, 2.4 ± 0.6 Wm\(^{16}\)C\(^{-1}\) [Clauser and Huenges, 1995]. The analytical solution of Fox Maule et al. requires the choice of a uniform \(k\) from the Curie depth to the top of the crust. Since GHF at the surface is influenced by \(k\) of both basalt and overlying sediments, we consider 3.5 W m\(^{16}\)C\(^{-1}\) to be an upper bound on \(k\) for the uniform model.

Figure 2.a shows the GHF envelope for 15% variability in \(k\) from the 2.8 W m\(^{16}\)C\(^{-1}\) value used in the Fox Maule et al. model for parameters given in Table S2. Given an upper bound on \(k\) of 3.5 W m\(^{16}\)C\(^{-1}\), the upper bound on GHF is 107 mW m\(^{-2}\) for 25 km thick crust.

A lateral contrast in thermal conductivity can perturb the spatial distribution of GHF through thermal refraction. This contrast in thermal conductivity may be due to a relatively flat sedimentary surface and buried basement relief. As an end member scenario, we consider a step transition in basement elevation, such as that associated with a vertical fault; a more gradual change in basement elevation would be associated with less GHF focusing. Gravity data collected at WGZ are consistent with the presence of a fault with 750 m vertical offset [Muto et al., 2013]. We evaluate this case with the analytic solution for a step contrast in thermal conductivity [Carslaw and Jaeger, 1947]. We assume that the underlying bedrock on both sides of the step has the same thermal conductivity, which is twice that of the sediment layer above. In this case, GHF is enhanced by 30 mW/m\(^2\) at the surface of the uplifted bedrock on the side of the fault with the thin sediment cover, decaying with distance from the crust to within 5% of the background flux within 1 km (Figure S6). Any sediment cover of the uplifted block would dampen the GHF contrast at the sediment-ice interface and increase its spatial extent by diffusion. Fit to the gravity data suggests 1.2 km of sediment cover above the uplifted block.

**Conclusion:** Fails Criterion 1; variations in thermal conductivity are insufficient to generate GHF >200 mWm\(^{-2}\). Passes Criterion 2; variations in thermal conductivity can affect GHF over a spatial scale less than 100 km.

1.b Crustal radiogenic heat production
The sensitivity analysis of Hasterok and Chapman [2011] revealed that the uncertainties in the radiogenic heat production of the upper crust exceeded the uncertainties in the radiogenic heat production in the lower crust and lithospheric mantle. Therefore, for this section, we focus on the uncertainty in radiogenic heat production in the upper crust. In Fox Maule et al.’s published estimate for Antarctic GHF, $A_s$ is assigned a value of 2.5 $\mu$Wm$^{-3}$. This value is representative of radiogenic heat production in granitoids, $2.520 \pm 2.155$ $\mu$Wm$^{-3}$ [Vilà et al., 2010]. We consider Fox Maule et al.’s estimate of 79 mW m$^{-2}$ to represent a moderate plutonic case and we use the 90th percentile radiogenic heat production rate for granitoids, 4.6 $\mu$Wm$^{-3}$ [Vilà et al., 2010]. This results in an upper bound on GHF above a cooled pluton of 91 mW m$^{-2}$ (Figure S8). Thus, even if there were a highly radiogenic granitic body beneath SLW, the radiogenic heat production would be insufficient to raise GHF to >200 mW m$^{-2}$.

Variations in rock type that result in variations in crustal radiogenic heat production rates can occur on small spatial scales down to the sub-kilometer scale [Carson et al., 2014]. However, as the Fox Maule et al. model assumes an exponential radiogenic profile, the calculated variability of 12 mW m$^{-2}$ is representative of spatial scales of 10s km.

**Conclusion:** Fails Criterion 1; WARS geology makes it unlikely that GHF exceeds 200 mW m$^{-2}$ due to radiogenic sources in the crust or upper mantle. Passes Criterion 2; radiogenic heat production rates can vary on a spatial scale <100 km.

**1.c Crustal thickness**

Magnetic field strength is used to infer the depth to the Curie (580°C) isotherm. The depth to this isotherm generally varies between 15 and 40 km [Fox Maule et al., 2005]. Using the analytic solution of Fox Maule et al. for the GHF given an exponential solution for the radiogenic heat production in the crust, we solve for the difference in GHF due to crustal thickness variations (Figure S5). The difference between a 15 km crust and a 40 km crust results in GHF of 120 and 60 mWm$^{-2}$, respectively.

The spatial resolution of the magnetic model is ~40 km [Fox Maule et al., 2005]. The minimum distance over which crustal thickness thinned from 40 km to 15 km according to this model was ~140 km [Fox Maule et al., 2005]. The maximum variability in crustal thickness estimated by seismic velocity structure is 41 to 18 km over a distance of ~130 km [Chaput et al., 2014]. Thus this scale of crustal thickness change in West Antarctica occurs over spatial scales greater than 100 km.

**Conclusion:** Fails Criterion 1; crustal thicknesses observed in the WARS influence GHF by <60 mW m$^{-2}$. Fails Criterion 2; major crustal thickness changes occur on a spatial scale >100 km.

**1.d Erosion**
Glacial erosion acts to thin the crust, resulting in vertical advection of heat and enhanced thermal gradients at the surface. The maximum mean erosion rate in West Antarctica is 58.8 m Myr\(^{-1}\) based on the maximum estimated total erosion since 34 Myr [Wilson et al., 2012].

We evaluate the influence of erosion on GHF using the analytical solution of Mancktelow and Grasemann [1997] (Table S3). The steady state temperature equation now includes an advection term:

\[
k \frac{\partial^2 T}{\partial z^2} + u \rho c_p \frac{\partial T}{\partial z} = -A(z) \quad (1b)
\]

where \(u\) is the erosion rate, \(\rho\) is the density of the crust set to 2850 kg m\(^{-3}\), and \(c_p\) is the heat capacity set to 1200 J kg\(^{-1}\) °C\(^{-1}\).

Based on the maximum mean erosion rate calculated, we conclude that erosion enhances GHF by <3%. Even if erosion were to be an order of magnitude higher, it cannot explain GHF that is >200 mW m\(^{-2}\).

The spatial scale of erosion beneath the West Antarctic Ice Sheet is controlled by the width of ice streams, which varies from 10 to 200 km [Rignot et al., 2011]. The Whillans Ice Stream is ~100 km across where it flows into the Ross Ice Shelf [Rignot et al., 2011]. Erosion is not expected to vary significantly between WGZ and SLW given that they both lie beneath the Whillans Ice Stream.

**Conclusion:** Fails Criterion 1; maximum GHF is ~80 mWm\(^{-2}\). Passes Criterion 2; erosion is capable of increasing GHF on a spatial scale <100 km.

**1.e Lithospheric extension**

Lithospheric stretching vertically compresses isotherms, enhancing the thermal gradient. On average over the past 105 Myr, the WARS has laterally extended at a rate of ~1% Myr\(^{-1}\) [Trey et al., 1999]. Higher rates of extension might exist for short periods along parts of the WARS [Decesari et al., 2007] and adjacent to the Transantarctic Mountains [Cande et al., 2000]. In order to evaluate the effect of rifting on GHF in the WARS, we follow the approach of Lachenbruch and Sass [1978].

\[
k \frac{\partial^2 T}{\partial z^2} + \frac{h \varepsilon^2}{\beta^2} \frac{\partial T}{\partial z} = -A(z) \quad (6)
\]

Extension is accounted for through parameter \(\beta\). \(\beta^2 = \kappa(s\rho c_p)^{-1}\), where \(s\) is the horizontal areal strain rate caused by extension. Parameters are the same as those used to solve for the geotherm with erosion but the effect of erosion is not included.

We use the solution to equation (6) given Dirichlet boundary conditions. Here we depart from Lachenbruch and Sass, who used Neumann boundary conditions.

\[
T(z) = T_S + \left[ T_L - T_S - \frac{A_s h^2}{k} \left(1 - \exp \left(-\frac{L}{h}\right)\right) \frac{\text{erf} \left(\frac{x^2}{2h^2}\right)}{\text{erf} \left(\frac{L^2}{2h^2}\right)} \right]^{1/2} + \frac{A_s h^2}{k} \left(1 - \exp \left(-\frac{z}{h}\right)\right) \quad (7)
\]
$k$ is the thermal conductivity, $L$ is the thickness of the crust. When there is no extension ($\beta = 0$), this solution is identical to that derived by Fox Maule et al. [2005]. We use the same Dirichlet boundary conditions as before. Thus GHF at the surface is:

$$q(z = 0) = \left( T_L - T_S - \frac{A_0 h^2}{k} \left( 1 - \exp\left( -\frac{L}{h} \right) \right) \right) \left( -\frac{z}{\beta^2} \right) \text{erf}\left( \frac{L}{2\beta^2} \right)^{-1/2} + \frac{A_0 h}{k}$$ \hspace{1cm} (8)

At WGZ, where the crustal thickness is thought to be 25 km [Fox Maule et al., 2005], an extension rate of 1% Myr$^{-1}$ raises GHF from 79 to 81 mW m$^{-2}$ (Table S4), a difference that is insufficient to explain the high GHF at THW or SLW. Even with extension of 4% Myr$^{-1}$, GHF would be increased by $\leq$10 mW m$^{-2}$ (Table S4).

The width of zones of highly extended lithosphere from gravity data is $\geq$75 km [Decesari et al., 2007]. We assume that the width of the extended zone beneath the Whillans Ice Stream falls within the same range. Thus, lithospheric extension may be capable of increasing GHF at scales $<$100 km.

**Conclusion:** Fails Criterion 1; extension is insufficient to explain GHF $>$200 mWm$^{-2}$. Passes Criterion 2; extension may generate GHF variability at a spatial scale $<$100 km.

**1.f Vertical fluid flow through sediments**

Groundwater flow through sedimentary basins is a significant component of the hydrological inputs at the base of Siple Coast ice streams [Christoffersen et al., 2014]. We analyze the effect of vertical advection of porewater through sediment using an analytical model with a uniform vertical pressure gradient and uniform hydraulic conductivities in a 100 m thick layer [Bredehoeft and Papadopoulos, 1965]. Given lithostatic stress conditions (~20 Pa m$^{-1}$) and the observed hydraulic conductivity of a till sample collected upstream of this site (1e-10 m s$^{-1}$) [Tulaczyk et al., 2001], fluid flow in the sediments will be too slow to significantly modify GHF (Table S5). If the hydraulic conductivity were two orders of magnitude higher, GHF could be enhanced by 10% by upward fluid flow. Thus, fluid flow through overlying sediments is insufficient to enhance GHF to $>$200 mW m$^{-2}$.

**Conclusion:** Fails Criterion 1; vertical fluid flow through sediments is incapable of generating GHF $>$200 mW m$^{-2}$.

**Text S2. Vertical velocity profiles in ice.**

The analytic solution to the heat equation used in this study assumes that the vertical velocity of ice decreases linearly with depth. Measured velocity profiles appear to be quasi-linear [Paterson, 1976]. However, at some ice rises, the vertical velocity is nonlinear due to low driving stresses and the nonlinear stress dependence of effective viscosity [Raymond, 1983; Zumberge et al., 2002; Kingslake et al., 2014]. Some observations also indicate that the vertical velocity profiles become more linear on the flanks of ice rises [Zumberge et al., 2002; Kingslake et al., 2014].
The use of a nonlinear vertical velocity profile (with z-exponent greater than 1) results in a smaller $G_{\text{crit}}$ value than a linear vertical velocity profile. We use the linear vertical velocity profile to derive $G_{\text{crit}}$ because it provides an upper bound on $G_{\text{crit}}$.

We provide an example of the difference between these two analytic solutions corresponding to parabolic and linear vertical velocity profiles at Siple Dome [Engelhardt, 2004]. Siple Dome is thought to have been approaching steady-state for at least 2 ka [Price et al., 2007]. Vertical velocities are observed to be parabolic at the drill site [Zumberge et al., 2002]. All parameters are chosen to match the analysis of Engelhardt (2004); notably, the thermal conductivity does not depend on temperature. This parabolic function results in lower gradients in temperature at depth than the linear function (Figure S10). Thus, the parabolic function is associated with lower $G_{\text{crit}}$ values: 66 mW m$^{-2}$ for the Siple Dome site vs. 80 mW m$^{-2}$ for the linear case. If the measured GHF, 69 mW m$^{-2}$, is correct, then basal melting would occur when the ice temperature reaches steady-state for a parabolic vertical velocity profile. The difference between two plausible vertical velocity profiles can make the difference between a frozen and thawed bed when GHF is close to $G_{\text{crit}}$. 


Figure S1. Cross plot of co-located GHF estimates for West Antarctica derived from a seismic similarity model [x-axis, Shapiro and Ritzwoller, 2004] and a magnetic crustal thickness model [y-axis, Fox Maule et al., 2005]. 1:1 line in red shown for reference.
Figure S2. Full temperature record for two geothermal probe deployments at WGZ. Temperature data are plotted for the three sediment thermistors for (a) the first deployment and (b) the second deployment. These data have been corrected based on probe calibration in a stirred water bath, as described by Fisher et al. [2015].
Figure S3. Comparison of thermistor measurements. Temperature data from the three thermistors of the geothermal probe for (a) a period when the sensors were being lowered through the borehole during the first deployment and (b) a period when the sensors were in the sub-ice shelf cavity during the second deployment. Data shown was already corrected using calibration coefficients from lab experiments, and additional shifts were applied on the basis of differences shown above. Dotted rectangle denotes period of temperature stability used to define temperature shifts for Sensors 092, 098, and 423 relative to Sensor 418. In the second deployment, the bottom water (BW) thermistor temperatures are shown, which record the same temperature as Sensor 418, ~1.5 m above Sensor 418.
Figure S4. Grain size histograms. Distribution of grain sizes in sediment core samples, measured with a laser-diffraction particle size analyzer, after sieving to remove most of the grains >1mm in diameter (see Methods in main text). Color is scaled such that blue represents samples near the top of the core and red represents samples at the deepest part of the core.

Figure S1. Coarse grain size photograph. (a) Photograph of grains with diameters between 1 and 2 mm. (b) Photograph of grains with diameters greater than 2 mm. Both populations shown were derived from a sample taken at 40 cm depth in core WGZ-GC-1.
**Figure S2.** The effect of vertical velocity on ice temperature. Temperature measurements from the Siple Dome borehole [Engelhardt, 2004] (black dots) vs. the analytical solution assuming a linear vertical velocity profile (black line) and the analytical solution assuming a parabolic vertical velocity profile (blue line).

**Figure S7.** Sensitivity of $G_{\text{crit}}$ to three independent variables. Contours of $G_{\text{crit}}$ plotted as a function of ice thickness and surface temperature for accumulation rates of (a) 20 cm yr$^{-1}$ and (b) 50 cm yr$^{-1}$. Contours are at 10 mW m$^{-2}$ intervals.
**Figure S8.** The effect of a lateral contrast in thermal conductivity on GHF. GHF at the top of a lateral contrast in thermal conductivity of 2:1 maintained in a layer 750 m thick. The solid line marks the solution. Dashed line marks vertical discontinuity coincident with the contrast in thermal conductivity.

**Figure S9.** Comparison of measured temperature gradients with thermobarometry studies and analytical solutions. The temperature gradients (mean ± 1 S.D.) measured in shallow subglacial sediments extrapolated downward into the upper 10 km of crustal rocks (WGZ in grey, SLW in red). The analytic solutions for the WGZ geotherm with no extension, 2% Myr\(^{-1}\), and 4% Myr\(^{-1}\) are plotted as solid, dashed and dotted black lines, respectively. T-P conditions derived from granulite inclusions collected in the McMurdo Embayment (McM) and the Transantarctic Mountains (TAM) just south of McMurdo (blue points) [Berg et al., 1989], with an interpreted geotherm (blue line). T-P conditions from spinel-lherzolite and harzburgite xenoliths collected in North Victoria Land (NVL) (black points) [Armienti and Perinelli, 2010]. The dry peridotite solidus is from Hirschmann [2000].

<table>
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<th>Units</th>
<th>Value</th>
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<th>S.D.</th>
<th>S.E.</th>
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Table S1. Errors associated with the GHF measurement.

¹ includes the S.D. of temperature measurements made with the needle probe thermistors.
² The uncertainty in geothermal probe depth does not affect the uncertainty of the geothermal gradient, as the distance between thermistors is fixed.
³ TP-Fit solution S.E. include the geothermal probe thermistor accuracy, the selection of in-situ thermal conductivity and thermal diffusivity values (which can vary locally), and selection of the data window used for processing. The S.E in temperature due to selection of data window is set equal to the difference between the temperature solution using the full analysis window (see methods) and the last third of the analysis window.
⁴ Calculation of S.E. associated with linear regression follows York et al. [2004] and includes TP-Fit solution errors.
⁵ calculated using Gaussian propagation of errors from the thermal conductivity S.E. and the geothermal gradient S.E.
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<thead>
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<td>Surface radiogenic heat production rate</td>
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<td>E-folding depth of radiogenic heat production</td>
<td>$h$</td>
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<td>Surface temperature</td>
<td>$T_s$</td>
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<td>Temperature at base of the magnetic crust*</td>
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* as defined by the Curie depth

**Table S2.** Parameters used in analytical modeling of GHF.

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<th>GHF (mW m$^{-2}$)</th>
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**Table S3.** GHF results for geotherm with erosion. GHF according to the analytical solution of Mancktelow and Grasemann given erosion rates ($u$). The erosion rate of 58.8 m Myr$^{-1}$ corresponds to the maximum estimated total erosion since 34 Myr [Wilson et al., 2012].

<table>
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<th>$s$ (% Myr$^{-1}$)</th>
<th>GHF (mW m$^{-2}$)</th>
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**Table S4.** GHF results for geotherm with extension. GHF according to the analytical solution derived in this study given extension rates ($s$), flux from the mantle ($q$ at $z=L$) and magnetic crustal thickness, $L$, is 25km, as modeled at WGZ [Fox Maule et al., 2005].
Table S5. Effect of vertical fluid advection on GHF. Results of calculation of heat flow under fluid advection following Bredehoeft and Papadopulos [1965]. $K_h$ is the minimum hydraulic conductivity needed to produce the observed temperature gradient under background GHF and lithostatic pressure conditions.

<table>
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<tr>
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<th>GHF, $u=u_0$ (mW m$^{-2}$)</th>
<th>$u_0$ (m yr$^{-1}$)</th>
<th>$K_h$ (m s$^{-1}$)</th>
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Table S6. Vertical conductive heat flux ($q_i$) uncertainties associated with errors in input variables. Errors were evaluated about the mean of each input across the profile in Fig. 3: an accumulation rate of 12 cm yr$^{-1}$, a surface temperature of -21 °C and an ice thickness of 800 m. Total error is the worst-case scenario in which the sign of all input errors are assigned such that combined error is maximized.

Data Set S1. Full temperature records for two geothermal probe deployments at WGZ. Full calibrated temperature data are plotted for three thermistors over two deployments. Calibration is described in Fisher et al. [2015].

Data Set S2. Thermal conductivity measurements and grain size fractions for core WGZ-GC-1.