Target oriented 3D acquisition aperture correction in local wavenumber domain

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Summary

The acquisition aperture correction in the local-angle domain can greatly improve the image quality, especially in sub-salt areas. However, to calculate the image in the local-angle domain is time-consuming due to the computationally demanding angle-domain decomposition. Based on an efficient local-wavenumber domain decomposition of wavefields using local exponential frames, a fast method is proposed to obtain the image in the local-wavenumber domain. In a similar way, the amplitude correction factor in the local-wavenumber domain can be also obtained. With the image matrix and amplitude correction factor matrix in the local-wavenumber domain, an efficient method of acquisition aperture corrections can be applied. This method can be used for target oriented aperture correction for 3D model. Through the tests on the 3D SEG salt model, this target oriented aperture correction method shows great potential on improving the image quality, especially for sub-salt areas.

Introduction

The theory and method of true-amplitude imaging has been developed based on high-frequency asymptotic theory (ray theory) and is traditionally carried out through Kirchhoff prestack depth migration. Since the amplitude corrections have to be done in the angle-domain, some effort has been tried to extract common-angle image (CAI) gathers for wave-equation based migration methods from offset related angle gathers or shot related angle gathers (Mosher et al., 1997; Rickett and Sava, 2002). Wu et al. (2004) proposed an amplitude correction method in local angle domain for acquisition aperture effect of wave-equation based migration methods. Numerical examples showed significant improvement in both the total strength of the images and angle-dependent reflection amplitudes, which demonstrated the significance of aperture correction in true-reflection imaging.

Recently developed techniques, such as beamlet decomposition (Wu et al. 2000; Wu and Chen, 2002, 2006a) or local slant stack (Xie & Wu, 2002; Xie et al., 2006), can decompose the frequency domain wavefield into localized beamlets containing direction information. It opened the way to process information in space local-angle domain for wave-equation based methods, such as the directional illumination analysis (Wu and Chen, 2002, 2006a; Xie & Wu, 2002; Xie et al., 2006) and the acquisition aperture correction in the local-angle domain (Wu et al., 2004).

The beamlet propagation and imaging methods proposed by Wu et al. (2000) have been developed using the Gabor-Daubechies frame (GDF) (Wu et al., 2000; Wu and Chen, 2001; 2006b) and the local cosine basis (LCB) (Wu et al., 2000; Wang and Wu, 2002; Wang et al., 2003; Wu et al., 2003; Luo and Wu, 2003). The GDF beamlet has the local direction information available during the propagation process so that acquisition aperture correction (Wu et al., 2004) and other angle-related filtering can be easily incorporated into the migration process, but the GDF beamlet propagator is computationally more expensive than the LCB method due to the redundancy in the representation. However, LCB beamlets always have two lobes in symmetry with respect to the vertical axis. This lack of uniquely defined directional localization prevents its use for many operations in the local angle domain. For the wavefield in space-frequency domain, the local slant stack method (Xie & Wu, 2002; Xie et al., 2006) can decompose the wavefield into local plane waves containing angle information. However, this decomposition is time-consuming. For example, the computation time to get the image in local dip-angle domain is about five times of that to get the image in space domain for 2D case. As a result, this acquisition aperture correction in local angle domain is very hard to use in 3D case and to handle the huge amount data in industrial migration processing.

The newly developed local exponential frame (LEF) beamlets decomposition method can decompose the wavefield into local wavenumber domain with uniquely defined direction information (Mao and Wu, 2007, 2009). It is implemented by a combination of local cosine and sine transforms. As the local cosine/sine transforms have fast algorithms, this decomposition is efficient. If taking the advantage of the availability of local cosine coefficients during the LCB propagation process, this method can provide the local direction information by only an extra orthogonal transform with O(N) efficiency.

Since it is much more efficient to get the wavefield in the local-wavenumber domain than in the local-angle domain, we propose a fast acquisition aperture correction method with partial imaging in local wavenumber domain. Firstly a wave field decomposition method in local-wavenumber domain (local exponential beamlets decomposition) is briefly summarized. The partial images and the corresponding amplitude correction factors in the local-angle domain and the total strength image after corrections in the local dip domain for the 3D SEG 45 shots model data are calculated by this method. Finally, we give some slices of the image after aperture correction.
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2D local exponential frames

The local cosine basis constructed by Coifman and Meyer (1991) (see also Mallat, 1999) can be characterized by position \( \xi_n \), the nominal length of window \( L_n = x_n - x_{n+1} \), and wavenumber index \( m \)

\[
\psi_n^m(x) = \frac{2}{L_n} B_n(x) \cos \left( \frac{m + \frac{1}{2}}{L_n} x - \xi_n \right),
\]

where \( B_n(x) \) is a bell function which is smooth and supported in the compact interval \([\xi_n - \epsilon, \xi_{n+1} + \epsilon']\) for \( \epsilon, \epsilon' \) as the left and right overlapping radius, respectively. In the same manner, the local sine transform can be defined.

In order to have uniquely specified direction in wave propagation, we extend the local exponential frame (Mao and Wu, 2007) to the 2D case as follows

\[
g_{mnpq}(x) = \frac{2}{L_x} B_m(x) \exp \left( \frac{\pi}{L_x} \left( x - x_m \right) \right),
\]

where \( x = (x, y) \), \( x_m = (x_m, y_m) \) and \( x_{mnpq} = (\xi_m, \eta_n) \).

The local image matrix in the local wavenumber domain

The standard imaging condition in the space domain for a single frequency is in the form of cross-correlation,

\[
I(x) = \frac{\partial}{\partial z} \int A(x_s, x) \bar{G}_z(x, x_s) \bar{G}_z^*(x, x_s) dx_s,
\]

where \( \bar{G}_z \) is the Green’s function used in the imaging process, which could be different from the Green’s function used in the forward modeling; \( \bar{G}_z \) stands for complex conjugate; and the integral is a back propagation Rayleigh integral of the recorded data, in which \( A(x_s, x) \) is the spatial receiver aperture for the given source and \( u_s(x_s, x) \) is the recorded scattered wavefield at the receiver \( x_s \) from the source at \( x \).
Seismic imaging should provide us an estimate of the subsurface reflectivity/scattering strength, which is reflection/scattering angle dependent. Therefore the imaging condition in the space domain \(5\) needs to be extended to the space-local angle domain (or beamlet domain) (Wu and Chen, 2002, 2006b). Then the image obtained at each imaging point is no longer a scalar value but a matrix, called the local image matrix (LIM) \(L(x, \theta_s, \theta_g)\), where \(x= (x, z)\) is the position vector at depth \(z\); \(\theta_s\) and \(\theta_g\) are the source and receiving angles, respectively. The LIM is a distorted estimate of the local scattering matrix (LSM) due to the acquisition aperture limitation and the propagation path effects. LSM is the intrinsic property of the scattering medium and is independent of the acquisition system and free from propagation effects and contains information of the local structure and elastic properties (Wu et al., 2004). The task of true-reflection imaging is to restore the true LSM from the distorted LIM by applying amplitude corrections.

In the original acquisition aperture correction method, the image in the local-angle domain is directly formed using the wavefields in the local-angle domain, which is very time-consuming. Since it is much more efficient to get the wavefield in the local-wavenumber domain than in the local-angle domain, the images in the local-wavenumber domain for a single frequency can be formed for each shot firstly and then summed up to form the image for all shots in the local-wavenumber domain. The imaging condition for a single frequency \(\omega\) in the space-local wavenumber domain can be written as,

\[
\begin{align*}
L(\omega, x, k_s, k_g) &= 2 \sum_{x_s} G_I(\omega, x, k_s; x_s) \\
&= \int_{A(x_s, x)} dG_x \frac{\partial G_I(\omega, x, k_g; x_x)}{\partial \omega} u_s(\omega, x_g; x_s)
\end{align*}
\]

where \(k_s\) and \(k_g\) are the source and receiving wave vectors, respectively, \(G_I(\omega, x, k_s, k_g)\) is the incident beamlet at the imaging point \(\mathbf{x}\) generated by a point source at \(x_s\), and the integral is the back-propagated beamlet at the imaging point \(\mathbf{x}\) from the recorded data. The summed image \(L(\omega, x, k_s, k_g)\) in \(6\) in the local-wavenumber domain can be transferred to the local dip angle domain.

With similar derivation to that obtaining the amplitude correction factor directly from the wavefield in the local-angle domain (Wu et al., 2004), we can get the corresponding amplitude correction factor in the local wavenumber domain for the above single frequency imaging condition \(6\),

\[
\begin{align*}
\left| F_o(x, \mathbf{k}_s, k_g) \right| &= 2 \sum_{x_s} \left| G_I(x, \mathbf{k}_s; x_s) G_F(x, \mathbf{k}_s; x_s) \right|, \\
&= \int_{A(x_s, x)} dG_x \left| G_F(x, \mathbf{k}_g; x_g) \right|^2 \right|^{1/2}
\end{align*}
\]

where \(G_F\) is the Green’s function used in the forward modeling.

With similar strategy to the image in the local-wavenumber domain, the amplitude correction factor \(7\) in the local wavenumber domain can also be interpolated to obtain the amplitude correction factor in the local-angle domain \(F_o(x, \theta_s, \theta_g)\).

With the image matrix \(L(x, \theta_s, \theta_g)\) and amplitude correction factor matrix \(F_o(x, \theta_s, \theta_g)\) in the local-angle domain, we can do the same acquisition aperture corrections as those in the original acquisition aperture correction method, such as the common reflection-angle imaging and the total strength imaging.

**Numerical examples**

To demonstrate the application of the illumination analysis, we calculate a number of numerical examples using the 3D SEG/EAGE salt velocity model. This example simulates the illumination condition of the 45 shot data set. The data set represents a land type acquisition geometry. The grid size for the model is 676, 676 and 210 grids in \(x, y\) and \(z\) axis respectively, with 20m for both of the horizontal and depth interval. Figure 2 shows the location of vertical slices to demonstrate. Figure 3(a) is the velocity model on slice-A, figure 3(b) is the raw image generated by depth beamlet migration, and figure 3(c) shows the result after acquisition aperture correction. Figure 4 demonstrated similar result on slice-B. From these results, we see that the image quality has been greatly improved.  

![Figure 2: Locations of vertical slices for plotting.](image)

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Figure 3: Acquisition aperture correction on slice-A: (a) velocity model; (b) raw image; (c) image after aperture correction.

Conclusions

We extended the fast acquisition-aperture correction method into the 3D case, in which the partial images and the corresponding amplitude correction factors for a given frequency are obtained in the local-wavenumber domain. With the new fast method, the acquisition aperture corrections can be applied to some target area for the 3D case. For the 3D SEG/EAGE model, amplitude correction factor in the local-angle domain matches the migration image very well. The final aperture correction results showed the improvement of the image quality, especially for some sub-salt structures. This target-oriented aperture correction method shows great potential for use in the 3D case.

Figure 4: Acquisition aperture correction on slice-B: (a) velocity model; (b) raw image; (c) image after aperture correction.

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