Mapping Directional Illumination and Acquisition-Aperture Efficacy by Beamlet Propagators
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Summary
Directional illumination and acquisition-aperture efficacy are evaluated through wave equation-based beamlet wavefield propagation. Beamlet decomposition with Gabor-Daubechies frame provides localizations in both space and direction of wavefields. The image conditions in beamlet domain and mixed domain are introduced, and the acquisition-aperture efficacy matrix and dip-response function are derived. As numerical examples, we calculate the directional illumination maps and acquisition-aperture efficacy maps for the SEG-EAGE 2D salt model, and further investigate their influences on the prestack migration image quality.

Introduction
Advances in seismic technology, particularly prestack depth migration, have made significant improvement in providing reliable high-resolution seismic images for complex structures. However, there remains a need to better understand various factors affecting image quality, such as acquisition geometry at surface including the source and receiver apertures, approximation approach used in migration procedure, the influence of overlaying structures to the target area, etc. Illumination analysis in the target area is a powerful tool to study the influences of acquisition aperture and overlaying structures. In the past, most techniques predicting illumination intensity distributions under certain acquisition geometries are based on ray tracing modeling (Muerdter et al., 2001a; 2001b; 2001c; Schneider, 1999; Bear et al., 2000), some authors used FD (finite difference) modeling to consider the wave phenomena. Although ray tracing is fast and inexpensive and its results can be easily sorted into CRP gathers, common offset bands, or common reflection angle bands to understand illumination attributes, the resulted illumination map may bear large errors in complex areas due to the high frequency approximation involved and the singularity problem of ray theory in complex regions. This may result in limited accuracy in modeling (Hoffmann, 2001). On the other hand, FD modeling is the most flexible and accurate method, but its application to practical use is obviously limited by its high computation cost. Another limitation of the conventional illumination analysis is the lack of reliable information on directional illumination. FD is a space domain solution of the wave equation. It can only provide the total illumination at any point and the illumination direction of the wavefield is lost. On the other hand, the ray method seemingly can provide angle-dependent illumination at any point. However, this “over-precise” directional illumination map does not reflect the real behavior of the wavefield since it violates the Heisenberg uncertainty principle: the position and direction of a wavefield can not be accurately predicted at the same time.

In addition, the ray direction is very sensitive to the fine structures of the medium and may bear large errors in complex regions. In order to have reliable directional illumination, we need to have a wave-theory based method which has both space and direction localizations satisfying the uncertainty principle.

Recently, Wu et al. (Wu et al., 2000; Wu & Chen, 2001) developed a wave equation-based beamlet wave propagation and migration method. Beamlet decomposition provides localizations in both space and direction (local wavenumber) of the wavefield, which makes it a natural tool to analyze the directional illumination distributions. In this study, illumination variations and reflector dip-response for the given acquisition geometry and overlaying structures are evaluated through beamlet wavefield decomposition and propagation using Gabor-Daubechies (G-D) frame atoms with the SEG-EAGE 2D salt model as an example. The influences of these factors on the final prestack image quality are also investigated.

G-D beamlet decomposition and propagation
For a 2D model, G-D beamlet decomposition of wavefield can be expressed as (Wu et al., 2000; Wu and Chen, 2001):

\[ u(x, z, \omega) = \sum_{m} \sum_{n} u_1(\tau_x, \tau_z, \omega) g_{mn}(x, z, \omega) \]  

(1)

where \( u_1(\tau_x, \tau_z, \omega) \) are beamlet coefficients, \( \omega \) is the circular frequency, \( g_{mn}(x, z, \omega) \) are G-D frame atoms:

\[ g_{mn}(x) = e^{-i\pi \tau_x} \frac{\xi_{\tau_x} + \Delta \tau_z}{\pi} \]  

(2)

where \( \tau_x = n\Delta_x, \tau_z = m\Delta_z \) with \( \Delta_x, \Delta_z < 2\pi \), and \( g(x) \) is a Gaussian window function. We see that each beamlet (in this case a G-D frame atom) is a windowed plane wave, which has both space localization (\( \tau_x \): window position) and direction localization (\( \tau_z \): local wavenumber).

Beamlets can be propagated by propagators in beamlet domain (Wu et al., 2000; Wu and Chen, 2001):

\[ u_{i+\Delta z}(\tau_x', \tau_z', \omega) = \sum_{m} \sum_{n} p(\tau_x', \tau_z', \tau_x, \tau_z, \omega) u_i(\tau_x, \tau_z, \omega) \]  

(3)

where \( p(\tau_x', \tau_z', \tau_x, \tau_z, \omega) \) is the propagator which includes the free propagator in the background media (with local reference velocities for each window) and the phase correction operator accounting for velocity perturbations. At each step, the wavefield in space domain then can be reconstructed by the so-obtained beamlets (using \( z \) instead of \( z+\Delta z \) for simplicity):

\[ u(x, z, \omega) = \sum_{i} \sum_{j} u_i(\tau_x', \tau_z', \omega) g_{j}(x) \]  

(4)

\[ = \sum_{j} e^{i\pi \tau_z} \sum_{i} g(x-\tau_x') u_i(\tau_x', \tau_z, \omega) \]

It is seen that at each space location the field can be recovered by superposing the contributions of all the
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windowed plane waves from all the neighboring windows. We can also have a partial reconstruction (mixed domain wavefield: local phase – space):

\[ u(x, z, \vec{e}, \omega) = e^{j \omega \vec{e} \cdot \vec{r}} \sum g(x-\vec{r}) k_r(\vec{r}, \vec{e}, \omega) \]  (5)

We call \( u(x, z, \vec{e}, \omega) \) a local plane wave which is a superposition (weighted average) of windowed plane waves of the same local wavenumber from all neighboring windows. The local wavenumber \( \vec{e} \) corresponds to a local incident angle \( \vec{e} = \sin^{-1}(\vec{e} \cdot v(x))/\omega \) (\( v(x) \) is the velocity at \( x \)).

Single-source DI (Directional Illumination) map and group-source DI map

For a single-source (point source, beam source or plane source), we put a unit-strength source at the surface and propagate the field to the image space. Then we partially reconstruct the beamlet domain field to get the local plane waves \( u(x, z, \vec{e}, \omega) \). For the illumination problem, only the intensity of amplitude is concerned. From computation efficiency point of view, we consider the main energy of the source wavefield around the dominant frequency \( \omega_0 \). So for the DI – map, we calculate and display the average amplitudes of the local plane waves within a small frequency band around \( \omega_0 \),

\[ DI(x, z, \vec{e}) = \frac{1}{\omega_0 - \omega} \sum u(x, z, \vec{e}, \omega) \]  (6)

The DI – map can be also calculated by summing up the local plane waves for all frequencies or for a specific frequency band. By summing up the DI – maps of individual sources, the DI – map of group sources or all the sources of an acquisition can be calculated as well. Taking point sources (shot domain) as an example:

\[ DI(x, z, \vec{e}) = \sum_{j=1}^{N_S} DI(x, z, \vec{e}_j, j) \]  (7)

where \( x_S \) is the source position on the surface, and the DI – map is for \( N_S \) sources. For group beam sources or group plane sources, the summation procedure is similar.

Local image matrix: image condition in beamlet domain and mixed domain

We use point source as an example. For each point source, the forward-propagated wavefield can be decomposed locally

\[ u^f(x, z, \omega) = \sum u^f_r(\vec{r}, \omega) g_r(x) \]  (8)

and the received scattered wavefield at each receiver can be back-propagated to the image space and decomposed locally

\[ u^s_r(x, z, \omega) = \sum u^s_r(\vec{r}, \omega) g_r(x) \]  (9)

Here the superscripts \( S \) and \( R \) refer to the point source and the point receiver (of source \( S \) located on the surface, respectively. Substitute \( u^f_r(x, z, \omega) \) and \( u^s_r(x, z, \omega) \) into the image condition,

\[ \sum_{j} u^f(x, z, \omega) \sum_{j} u^s(x, z, \omega) \]

\[ = \sum_{j} \sum_{i} e^{j \omega \vec{e}_i \cdot \vec{r}_j} \sum_{i} u^f_r(\vec{r}_i, \omega) \] \[ \sum_{j} u^s_r(\vec{r}_i, \omega) g_r(x) = \sum_{j} \sum_{i} M_{ij} g_r(x) \]  (10)

We call \( M_{ij} \) the image matrix in beamlet domain. It is the image produced by the incident windowed plane wave in \( l \)th window with \( j \)th wavenumber and the scattered windowed plane wave in \( q \)th window with \( p \)th wavenumber. Summing up the contributions from all the neighboring windows, we get the image matrix in mixed domain: local plane-wave image matrix

\[ I(x, z, \omega) = \sum_{j} \sum_{i} e^{j \omega \vec{e}_i \cdot \vec{r}_j} L_{ij}(\vec{e}_j, \vec{e}_p, x, \omega) \]  (11)

The local plane-wave image matrix \( L_{ij}(\vec{e}_j, \vec{e}_p, x, \omega) \) with the local incident plane wave to get the apparent local scattering matrix:

\[ S_{ij}(\vec{e}_j, \vec{e}_p, x, \omega) \]  (12)

The goal of imaging/inversion is to recover the real local scattering matrix. If both the source and receiver arrays have infinite apertures, the apparent local scattering matrix will reflect the real scattering (or reflection) property of the corresponding local structure. In the ideal situation, it will be identical with the real local scattering matrix. However, due to the limited apertures of source and receiver arrays, combining with the propagation effects of the overlaying structures, \( S_{ij}(\vec{e}_j, \vec{e}_p, x, \omega) \) may severely deviate from the real local scattering matrix.

Acquisition-aperture efficacy matrix

In order to evaluate the effects of the acquisition geometry for a specific area (target area), including the aperture and propagation effects, we put unit impulses at both the source and receiver points for the whole acquisition configuration. The local plane-wave image matrix obtained in this way is called the acquisition-aperture efficacy matrix, or simply acquisition efficacy matrix \( E_{ij}(\vec{e}_j, \vec{e}_p, \omega) \).
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\[ E_s \left[ \varpi , \varpi , x, \omega \right] = \sum \sum G_s (\varpi , \varpi , x, \omega) e^{-i\omega} \left[ \sum \sum G_s^*(\varpi , \varpi , x, \omega) e^{i\omega} \right] \]  

(14)

where \( G \) is the impulse responses or Green’s functions, \( G_s (\varpi , \varpi , x, \omega) \) and \( G_s^*(\varpi , \varpi , x, \omega) \) are the decompositions of \( G \) into beamlet domain. If we relate the \( \varpi \) and \( \varpi \) to the local incident and scattering angles, then \( E_s \left[ \varpi , \varpi , x, \omega \right] \) is the efficacy of the acquisition system to the local scattering measurement. For the ideal case with infinite apertures, it should hold

\[ E_s \left[ \varpi , \varpi , x, \omega \right] = 1 \quad \text{for all } \varpi \text{ and } \varpi \]  

(15)

Therefore, the value of \( E_s \left[ \varpi , \varpi , x, \omega \right] \) is an indication that how the acquisition aperture and the overlaying structure will influence the scattering measurements in the target area. If the local scatterer is a plane reflector, the local scattering matrix or the local efficacy matrix can be simplified and represented by dip-response function. Change \( \varpi \) into a coordinate of \( \varpi \) with

\[ \varpi = \left[ \varpi , \varpi \right] / 2 \]

(16)

where \( \varpi \) is the normal of the dipping reflector and \( \varpi \) is the reflection angle with respect to the normal. Because of the mirror reflection of plane interfaces, we can sum up all the responses for different reflection angles, resulting in an acquisition efficacy as a function of dip angle of local interface

\[ E_s \left[ \varpi , x, \omega \right] = \sum E_s \left[ \varpi , x, \omega \right] \]  

(17)

\( E_s \left[ \varpi , x, \omega \right] \) measures the dip-angle response of the acquisition system, including the source and receiver apertures, at the target area.

Numerical example

We use the SEG-EAGE 2D salt model as an example. In Fig.1, the directional illumination gathers of a single point source and a point source array are shown in the form of “rose diagram”, respectively. At each point in these maps, 11 arrows (petals) are plotted, each of which represent one illumination direction. The lengths of the arrows indicate the relative strength of illumination. The number near each point gives the total illumination strength at that point. As expected, the DI – maps show clearly the directional features of source illumination. The other form of presenting the DI – map is the DI – map album. For different incident angles \( \varpi \), we output the DI – map of the whole acquisition aperture (total 325 shots) \( D(x, z, \varpi) \) as shown in Fig.2. We can see that the oblique illuminations have some “blank areas” under the salt body, especially in the large angle cases (Fig.2c and d). Similar with the DI – map, the ADR (Acquisition-dip-response) can also be outputted in the form of ADR album for different dip angles as given in Fig.3. For the oblique structures, especially for the steep faults, it can be seen that the dip-angle responses have many blank areas (different with the DI map) due to the influences of the acquisition apertures of both sources and receivers and the overlaying salt body structure. In fig.4, the prestack migration image using G-D beamlet propagator, the ADR gathers, and the dip-angle image gatherers of the migrated image are shown in (a), (b) and (c), respectively. The dip-angle image gathers are obtained by summing up the local plane-wave image matrices over frequency, and represented using the dip angles instead of the incident-scattering angle pairs. In the two gather maps, each line represents the response or image of a local reflector with a certain dip direction, and the length of the line is proportional to the response or image strength. Note the dip direction is perpendicular to the reflector normal \( \varpi \). In Fig.4b the ADR distributions become highly nonuniform at the left part of the subsalt area. This is caused by the limited acquisition apertures and the overlaying high-velocity salt body. The right-side rough salt boundary has strong effect on preventing the illumination energy reaching the subsalt area. Compared the dip-angle image gathers with the ADR gathers, we can see their direct correspondence and understand well the quality variation of the depth migration. Fig.4d gives the image after a preliminary aperture correction. It is obvious that the subsalt image quality in Fig.4d is improved considerably compared with Fig.4a. However, some artifacts at the same area remain. It need to be further studied and tested in the future research.

Conclusions

Directional illumination and acquisition-aperture efficacy analysis based on beamlet wavefield decomposition and propagation are proposed and applied to the SEG-EAGE 2D salt model. Beamlet decomposition provides localizations in both space and direction of the wavefield, and is more flexible and accurate compared with the traditional illumination analysis methods. The directional illumination maps and the acquisition-dip-response maps are calculated for the SEG-EAGE model, showing...
considerable influences of acquisition geometry on the image quality of prestack migration. Improved image for the subsalt steep faults is obtained by correcting the acquisition aperture influences.

References


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