Fig. 6. Same as Figure 5, the scattering patterns of velocity-type and impedance-type scattering.

Pattern will be the vector sum of those patterns. In a similar manner to (5), (6), the scattered field can be decomposed into an impedance-type and a velocity-type,

\[ sU_P = \frac{V \omega^2}{4\pi \rho_0^2} \frac{1}{r} e^{-i\omega(t-\frac{r}{c_0})} \left( \frac{\delta Z}{Z_o} \left( \cos \theta - \frac{\rho_0}{\rho} \sin 2\theta \sin \phi \right) - \frac{\delta \beta}{\beta_0} \left( \cos \theta - \frac{\rho_0}{\rho} \sin 2\theta \sin \phi \right) \right), \]  

(10)

\[ sU_{IM} = -\frac{V \omega^2}{4\pi \beta_0^2} \frac{1}{r} e^{-i\omega(t-\frac{r}{\beta_0})} \left( \frac{\delta Z}{Z_o} \left( \sin \theta + \cos 2\theta \sin \phi \right) - \frac{\delta \beta}{\beta_0} \left( \sin \theta - \cos 2\theta \sin \phi \right) \right), \]  

(11)

and

\[ sU_{UF} = \frac{V \omega^2}{4\pi \beta_0^2} \frac{1}{r} e^{-i\omega(t-\frac{r}{\beta_0})} \left( \frac{\delta Z}{Z_o} + \frac{\delta \beta}{\beta_0} \right) \cos \theta \cos \phi \].

(12)

Figure 5 gives the scattering patterns for impedance type and velocity type scattered fields. Some examples of resultant scattering pattern are shown in Figure 6.

For both P and S-wave incidences, converted waves (P-S or S-P) have only side lobes (with respect to the incident direction), while nonconverted scattered waves always have main lobes along the incident direction (either forward or backward direction). The impedance type scattered field will have no forward lobe, and the velocity type field will have no back lobe.

Scattering Characteristics of Elastic Waves by an Elastic Inclusion: II. Elastic Wave Scattering beyond Rayleigh Scattering

R. S. Wu, M.I.T.

Beyond Rayleigh scattering, the equivalent forces of the scattered fields can no longer be regarded as a point source. It is shown in this paper that the scattered far-field can be obtained as a product of two factors. One is that of elastic wave Rayleigh scattering, the other is a scalar wave scattering factor for the finite body, which we call "volume interference factor," of which we derive here the explicit expressions for a spherical inclusion. The general scattering pattern will depend on different combinations of density and elastic-constant perturbations and also on the ratio of wavelength to the size of the inclusion. Some examples are given to show the general characteristics.

Beyond Rayleigh scattering, when the size of the inclusion becomes comparable to the wavelength, the equivalent forces of scattering by an inclusion can no longer be regarded as a point source. The phase differences of the incident field at different parts of the inclusion and of the scattered field from different parts of the inclusion can no longer be ignored. Nevertheless, if the total scattered field is still much weaker than the incident field, the Born approximation can still be a useful tool for calculating the scattered field and deriving the scattering characteristics.

Suppose the density excess \( \delta \rho \) and the excesses of the elastic constants \( \delta \lambda \) and \( \delta \mu \) have the same spatial distribution within the inclusion. Using the Fraunhofer approximation, we derive the scattered far field for a finite volume elastic inclusion as follows

\[ rU_P(x) = rU_P(x)\Theta_1(\delta), \quad rU_S(x) = rU_S(x)\Theta_2(\delta), \]

\[ sU_P(x) = sU_P(x)\Theta_1(\delta), \quad sU_S(x) = sU_S(x)\Theta_2(\delta), \]  

(1)

Fig. 1. Volume interference factors of S-S scattering for different frequencies, where \( a \) is the radius of the sphere.

Fig. 2. Same as Figure 1, when \( \frac{\omega}{\beta_o} = 10 \).
where \( \hat{U} \) is the corresponding elastic wave Rayleigh-scattering field (Wu, 1983), \( \Theta_{1,4} \) is called volume interference factors:

\[
\Theta_{4}(\theta) = \frac{1}{V} \int_{V} P(\xi)e^{i\mu_{\xi}S_{1}aV(\xi)},
\]

(2)

\[
S_{1} = \frac{1}{\alpha_{0}}\hat{x}_{1} - \frac{1}{\alpha_{0}}\hat{x}, \quad S_{1} = \frac{1}{\alpha_{0}}2\sin\frac{\theta_{1}}{2},
\]

\[
S_{2} = \frac{1}{\alpha_{0}}\hat{x}_{1} - \frac{1}{\alpha_{0}}\hat{x}, \quad S_{2} = \sqrt{\left(\frac{1}{\alpha_{0}}\right)^{2} + \left(\frac{1}{\beta_{0}}\right)^{2} - \frac{2}{\alpha_{0}\beta_{0}} \cos \theta_{1}},
\]

\[
S_{3} = \frac{1}{\beta_{0}}\hat{x}_{1} - \frac{1}{\beta_{0}}\hat{x}, \quad S_{3} = S_{2},
\]

\[
S_{4} = \frac{1}{\beta_{0}}\hat{x}_{1} - \frac{1}{\beta_{0}}\hat{x}, \quad S_{4} = \frac{1}{\beta_{0}}2\sin\frac{\theta_{2}}{2},
\]

(3)

where \( \theta_{1} \) is the angle between the incident direction and the scattering direction, and \( P(\xi) \) is the normalized parameter distribution function,

\[
P(\xi) = \frac{\delta P(\xi)}{\delta \phi} = \frac{\delta \alpha(\xi)}{\delta \lambda} = \frac{\delta \mu(\xi)}{\delta \mu}.
\]

(4)

In fact, volume interference factors are exactly the scalar wave scattering patterns of the inclusion. However, in this case the scattered wave can have different velocity from the incident wave. Thus, we succeeded in decomposing the general elastic scattering into two factors. One is the elastic wave Rayleigh scattering, the other is a scalar wave scattering pattern, which can be calculated at least numerically from (2) if the parameter spatial distribution \( P(\xi) \) is known. In some cases, the volume interference factor can be calculated analytically. As an example, we give the formulas for a homogeneous spherical inclusion:

\[
\Theta_{4}(\theta) = \frac{3}{(\omega S_{a})^{2}} \left[ \frac{\sin \omega S_{a}a}{\omega S_{a}a} - \cos \omega S_{a}a \right],
\]

(5)

where \( a \) is the radius of the sphere and \( S_{a} \) can be calculated from (3). Notice that,

\[
\frac{\sin(\omega S_{a}a)}{\omega S_{a}a} - \cos \omega S_{a}a = \frac{1}{3}(\omega S_{a}a)^{2},
\]

\[
\Theta(\theta) \approx 1, \text{ when } \omega S_{a}a \ll 1.
\]

(6)

This agrees with the Rayleigh scattering.

Figures 1 and 2 give the spatial patterns of the volume factor of \( P-P \) and \( S-S \) scattering (\( \Theta^{P} = \Theta_{4} \) and \( \Theta^{P} = \Theta_{1} \) have the same shape). Figures 3 and 4 give that for converted
waves ($\Theta' = \Theta_2 = \Theta_3$). We can see that, for $P-P$ or $S-S$ scattering the volume interference factor always has a main lobe in the forward scattering direction. When wavelength decreases, the scattered energy will be more concentrated in the forward direction, but the converted waves will diverge to all directions and become smaller compared to the same-mode scattered waves.

The general scattering pattern is a combination of $U(v)$ and $\Theta(v)$, which becomes quite complicated for high frequencies. One example is shown in Figures 5 and 6.

Reference

### Flexural Waves in Floating Ice

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Flexural waves can produce a serious coherent noise problem in the offshore Arctic regions. As a step toward understanding and eliminating this noise, the properties of flexural waves are studied for a simplified 3-layer model. This consists of a homogeneous thin ice layer over water over solid half-space with plane parallel interfaces. The various surface-wave modes that can be excited are identified. The displacements for various surface-wave modes and their relative importance are also derived and calculated for situations typical of the offshore Arctic regions. The expressions for the far field $P$ and $SV$-wave radiation from a circular disc vibrator are derived. The partition of elastic energy into $P$ and $SV$ waves is worked out for a circular disc vibrator. The result shows strong dependence on the frequency of excitation. The implications of this study for seismic reflection profiling in the offshore Arctic regions of Alaska is discussed.

### Introduction

When using seismic reflection profiling for oil exploration in Arctic regions, it is observed that large amplitude coherent surface waves frequently interfere with desired reflected seismic waves. This is particularly the case for seismic data acquired on the ice in the transition zone one encounters in going onshore to offshore in North Alaska. Coherent noise, primarily flexural waves, are so severe in the area as to render reflection seismic data essentially unusable. It is useful to understand the characteristics of the noise observed in this transition zone so that effective strategies can be devised for data acquisition and processing. This goal will be facilitated by gaining an understanding of the properties of flexural waves. The environment where the flexural wave problem is most severe is a thin ice layer over shallow water. A simplified model, consisting of ice layer over water layer over a rock half-space with plane parallel interfaces, is considered here. The purpose of this study is to determine the nature of surface waves that may be excited in this simplified system and their dependence on ice thickness and water depth. Expressions for the partition of elastic energy from simple sources of seismic waves are presented in this theoretical development. This will be useful in choosing the appropriate source, frequency range, and detector type and array for exploration in the transition zones of Alaska. Furthermore, it will aid in developing seismic processing techniques for suppressing the undesired flexural waves.

### Model

For ease of analysis, the thickness of the ice layer is assumed to be small compared to the seismic wavelengths of interest. With this simplification, the ice layer can be treated using plate theory from elasticity (Ewing et al., 1957). To solve for the seismic wave field in the water layer and the rock, it is convenient to uncouple $P$ and $SV$ waves by expressing the relevant dynamical variables in terms of the divergence $(\Delta = \nabla \times U$ for $P$) and the curl $(W = \nabla U$ for $SV$). It is further assumed that there is cylindrical symmetry in the problem. In the water layer, no shear wave exists (or $W = 0$). Inside the linearly elastic rock, the radiation condition at infinite depth requires that only exponentially decaying displacements exist (or that downgoing waves exist). By matching the boundary conditions on the ice-water interface as well as water-rock interface, it is straightforward to express dynamical variables in terms of the pressure exerted by the circular disc vibrator (Miller and Pursey, 1954). Among the dynamical variables of interest are the displacements $U_1$, and $U_2$, and pressure in the water layer. They may be expressed in terms of the contour integrals over the ray parameter.

The existence of the surface-wave modes requires the existence of nontrivial solutions for the dynamical variables (e.g., $U_1$, $U_2$) in the absence of external excitations. This gives rise to the period equation as well as the modal response for displacements in this simplified 3-layer model. The period equation is a transcendental equation, whose solutions correspond to possible surface-wave modes. Both the phase and group velocities are obtained as a function of frequency. This solution is discussed later for typical situations encountered in the transition zone. The modal response of the medium also gives the displacements as a function of depth.

In addition to the dispersion curve for phase and group velocities, and distribution of displacements for the surface wave modes, it is desirable to know the relative excitation of surface-wave modes compared with $P$ and $SV$ waves. The external pressure source must be specified to do this. Here only the circular disc vibrator exerting uniform pressure on the ice is discussed. From this consideration the far field $P$ and $SV$ waves are obtained by the standard method of steepest descent. Similarly, the relative strengths of each surface mode excited can be determined by evaluating the residue at each pole. For the total power of the seismic energy excited by the vibrator, the radiation admittance method of Miller and Pursey (1955) can be used to evaluate the partition of elastic energy. Moreover, the power radiated into $P$ and $SV$ waves can be found by integrating the square of far field expression over a hemisphere at infinity. The partition of energy follows from these considerations.

### Numerical results for transition zone model

The following set of material parameters are used in the numerical calculations (Ewing et al., 1957):

- $\rho_i = 0.9174$ = density of ice
- $\rho_w = 1.0$ = density of water