Imaging in Compressed domain using Dreamlets
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Summary
The objective of this paper is to further investigate the theory and algorithm of wave propagation and imaging in the dreamlet domain for direct application of compressed seismic data. In our former work, dreamlet shows great application potential in efficient representation of seismic data. Based on the combination of dreamlet compression and wave propagation theory, we discuss one-way dreamlet wave propagation and imaging in terms of data processing in the compressed domain, that allows us to process only a few percentage of the decomposition coefficients of the whole data set to get the accurate imaging result. By using a 5-layer velocity model as numerical example, we show that the dreamlet coefficients of the seismic data is also decreasing with the depth of migration in the receiver side, meanwhile the imaging quality using the compressed data remains in a similar accuracy.

Introduction
Working directly on the compressed seismic data can dramatically reduce the storage and transfer of huge seismic data sets. The key is to find an optimum decomposition scheme which can sparsely represent the seismic data and the propagator matrix, and to work out a theory of wave propagation directly in the decomposition domain. Due to the sparse representation of seismic data using Curvelets (Candès & Donoho., 2002; Candès & Demanet, 2005), research work has been done using curvelets for data compression and wavefield extrapolation (Lin & Herrmann, 2007, 2008). Meanwhile, as a complete space-time localization atom, Curvelet has also been applied to wave propagation and seismic imaging using a map migration method (Douma & De Hoop, 2007; Chauris & Nguyen, 2008), in which high-frequency asymptotic approximation was invoked to propagate the curvelets in smoothly inhomogeneous media. However, for complicated structures with high-contrast inclusions, such as those involved with salt domes, the high-frequency asymptotic methods have very limited success. During the last decade, efforts have been made to develop wavefield decomposition and extrapolation methods with localization in both space and direction. Localized wave propagators can be easily tailored for local heterogeneities and to specific directions (Steinberg, 1993; Steinberg and McCoy, 1993; Wu et al., 2000; Wu and Chen, 2002; Chen et al., 2006; Wu et al., 2008). However, these local propagators have only space-direction localization, without time-frequency localization. Therefore, it does not offer an optimum compression on seismic data. Research have been done to test the effects using adaptive local cosine/sine bases to 2D seismic data compression (Wang & Wu 1998, 2000) , which is proved to be an efficient method providing high compression ratio as well as preserving seismic data information. Wu, Wu and Geng (2008) extended the frequency domain beamlet into a time-space domain dreamlet wave propagation using time-space wavelets, in which the $x-\xi$ localization uses the local cosine basis (LCB) (Coifman and Meyer, 1991), and the $t-\omega$ localization adopts the local exponential tight frame (Auscher, 1994; Wu and Mao, 2007; Mao et al., 2007).

In this paper, we further discuss wave propagation and imaging in compressed dreamlet domain. First we discuss the sparse representation of wavefield data using dreamlets and directly extend this sparse representation into wavefield compression method. With these preparation, we will also discuss the high compression ratio property of dreamlet during wave propagation in the localized $(t-\omega,x-\xi)$ domain. We use a five-layer velocity model as numerical example to demonstrate the validity and explore the advantageous of imaging in the dreamlet domain.

Sparse representation and Wavefield compression using Dreamlets
Using the tensor product of drumbeat and beamlet as the $t-f-x-\xi$ dreamlet atom, the wavefield in the time-space domain or in the frequency-wavenumber domain can be decomposed into a representation in the domain of time-frequency-space-wavenumber.

$$u(x,t) = \sum_{\tau} \sum_{\omega} \sum_{\xi} \sum_{\omega} d_{\tau}(x) g_{\omega}(t) h_{\omega}(x)$$

where $d_{\tau}(x)$, $g_{\omega}(t)$ and $h_{\omega}(x)$ are dreamlet, drumbeat and beamlet atom, respectively (Wu et al., 2008).

Since the introduction of time-axis localization in dreamlet, the most significant coefficients in dreamlet domain seems tend to be localized in the upper-left corner of each local-time-frequency-space-wavenumber block, which provided a sparse representation and also provide the possibility of
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obtaining high compression ratio after applying dreamlet decomposition to wavefield data, shown as in Figure 1.

Figure 1: Dreamlet coefficients distribution in one local time-frequency-space-wavenumber block. As showing in the figure, the most significant coefficients are concentrated in the upper-left corner. We can just pick the most important coefficients localized in the dash black line to get a compressed representation of the data.

We can easily found that even in the area circled by the dash line in Figure 1, there are still many small coefficients. That is to say, having this sparse coefficient matrix, we need to define a proper threshold, which can lead us to the high compression ratio as well as obtain a good reconstruction result.

Here, the compression ratio (CR) is defined as

\[
CR = \frac{\text{Uncompressed Data Size}}{\text{Compressed Coefficients Size}}
\]  

Figure 2 shows the comparison of compression ratios by different decomposition methods under different thresholds. We can see that the beamlet decomposition is the least efficient one, since there is no \( t-\omega \) localization along the \( \omega \)-axis. Curvelet compression is in the middle. The dreamlet decomposition shows high compression ratios in different thresholds as we discussed above.

Having this high compression ratio feature, dreamlet could be used directly to seismic data storage. We can also use this property to the one-way wave migration theory, which means to do imaging in the compressed coefficient domain.

Evolution of dreamlets and wave propagation in the dreamlet domain

The evolution of dreamlets must observe the wave equation. After propagation, a dreamlet is no longer a dreamlet, and is spreading into other cells in the localized phase-space. Assuming a wave propagator, here a one-way wave propagator, \( \mathbf{P} \), is applied to a dreamlet, the redecomposition of the distorted dreamlet into new dreamlets forms the propagator matrix elements:

\[
\mathbf{P}_{\tau \omega \xi \eta} = \langle d_{\tau \omega \xi \eta} \rangle \left( \mathbf{P} d_{\tau \omega \xi \eta} \right)
\]  

where \( \langle f|g \rangle \) stands for the inner product of \( f \) and \( g \). As in the case of beamlet propagator (e.g. Wu et al., 2000; Chen et al., 2006; Wu et al., 2008), a dreamlet propagator can be derived by the local perturbation theory. The use of local background velocities and local perturbations results in a two-scale decomposition of dreamlet propagators: a background propagator for large-scale structures and a local phase-screen correction for small-scale local perturbations. The perturbation operator for the dreamlet propagator will be similar to the case of beamlet propagator. Here we show only the derivation of the background propagator.

In the following, we use the analytic solution of wave equation in the frequency-wavenumber domain in a homogeneous background for the derivation of dreamlet propagator in the background media under the local perturbation theory. For notation simplification, we use \( d_4 \) to denote the four parameterized \( d_{\omega \xi \eta \tau} \) and \( d_4 \) for \( d_{\omega \xi \eta \tau} \).

From the frequency-wavenumber domain, (4) can be explicitly derived as
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\[ \mathbf{P}_{\tau \tau \tau \tau}^{(a)}(\omega) = \{ \mathbf{d}_t \mid \mathbf{P} \mathbf{d}_t \} = \int \int d\xi d\omega \mathbf{P}_\xi(\omega) \mathbf{P}_\omega(\omega) \mathbf{P}_\tau(\tau) \mathbf{P}_\tau(\tau) \]

\[ = \frac{1}{(2\pi)^3} \int d\omega \int d\xi d\omega \mathbf{P}_\xi(\omega) \mathbf{P}_\omega(\omega) \mathbf{P}_\tau(\tau) \mathbf{P}_\tau(\tau) \]

\[ = \frac{1}{(2\pi)^3} \int d\omega \int d\xi \mathbf{P}_\xi(\omega) \mathbf{P}_\tau(\tau) \mathbf{P}_\tau(\tau) \]

where

\[ \mathbf{P} = \mathbf{I} \mathbf{A} \mathbf{A} \mathbf{I} \]

is the vertical wavenumber and \( p \) is the horizontal slowness.

Substitute the drumbeat and beamlet atoms into the equation, resulting in

\[ \mathbf{P}_{\tau \tau \tau \tau}^{(a)}(\omega) = \frac{1}{(2\pi)^3} \int d\omega \int d\xi \mathbf{P}_\xi(\omega) \mathbf{P}_\tau(\tau) \mathbf{P}_\tau(\tau) \]

\[ = \frac{1}{(2\pi)^3} \int d\omega \int d\xi \mathbf{P}_\xi(\omega) \mathbf{P}_\omega(\omega) \mathbf{P}_\tau(\tau) \mathbf{P}_\tau(\tau) \]

\[ = \frac{1}{(2\pi)^3} \int d\omega \int d\xi \mathbf{P}_\xi(\omega) \mathbf{P}_\tau(\tau) \mathbf{P}_\tau(\tau) \]

where

\[ \mathbf{P} = \mathbf{I} \mathbf{A} \mathbf{A} \mathbf{I} \]

is the beamlet propagator and \( D=2 \) for the 2D case, and \( D=3 \) for the 3D case.

Numerical tests

In order to show the sparseness of dreamlet propagators, in Figure 3 we give the spreading in the \((T, \omega, \xi)\) domain of a single coefficient of \( T = 0, \omega = 2\pi \times 39, \xi = 4800 \) (in the middle) and \( \xi = 2\Delta \xi \) after propagation in homogeneous space for 200m. On the left is the dreamlet spreading function; on the right, the beamlet spreading. Since the \( \omega \)-axis is not localized in the case of beamlet decomposition, the coefficient distribution is rather uniform. In contrast, the dreamlet propagator is sparsely distributed because of the wave propagation nature in the \((t, \omega, x, \xi)\) domain.

Figure 4 shows the snapshots of a single dreamlet atom propagation. Because of the symmetrical property of local-cosine basis, the dreamlet has two symmetric lobes of packets.

Figure 3: Comparison of the sparseness of the dreamlet propagator (left) and the beamlet propagator (right). The coefficients below 10-3 have been cut. The coefficients are in logarithm scale with the maximum value as reference.

In comparison, we show the beamlet propagation in Figure 5. The nature of dreamlet in the \((T, \omega, \xi)\) domain can be seen clearly from these snapshots.

Figure 4: Snapshots of a single dreamlet propagation. Because of the symmetrical property of local-cosine basis, the dreamlet has two symmetric lobes of packets, \( \xi = 4\Delta \xi \).

Figure 5: Spreading of beamlet propagation, \( \xi = 4\Delta \xi \).

We use a five-layer velocity model to further investigate the dreamlet imaging method (shown in figure 6). The synthetic data consists of 201 shot gathers with 561 receivers per shot. The lateral and vertical sampling intervals are 25 m and 10 m respectively. The shot interval is 50 m, and the largest offset is 7 km. A ricker wavelet is used as the source time function and the dominant frequency is 17.5 Hz. The time sampling interval is 8 ms and the total recording time is 3.5 seconds.

Figure 6: five-layer velocity model.
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Figure 7: Prestack depth migration results using dreamlets

Figure 7 shows the prestack depth migration results of the dreamlet propagator based on the velocity model in figure 6. The image result shows that all the four interfaces are imaged quite well. We choose the 100th shot as example to take a look of the dreamlet coefficients change during the migration process, shown in figure 8. For the source field, the coefficients grow with the increase of depth, but fluctuating at a certain level, shown as the green dashed line. For the scattered wavefield, we “throw away” the time records above certain time because they have no contribution to the layer below and do not need to propagate further. For the wavefield compression in each depth, we use two different thresholding methods: one is applying the same threshold to compress the wavefield (global threshold compression method) and the other is using different thresholds for different time-space windows (local threshold compression method). The solid blue line in figure 6 is for the global threshold method and the compression ratio is 5.6 on the surface. The image result in figure 5 also used this method. Using local threshold method on the surface, compression ratio is 15.6, and global threshold for the rest can achieve similar image quality, shown as red das-dotted line. Similar migration result using the high compression local threshold method indicates that the compression does not lose information in the data. The black line is for the local threshold method for all the wavefield compression but the image result is not as good. From the test we see that it is possible to find a flexible compression method to achieve a high compression ratio in every depth while keeping good image quality.

Conclusion

Dreamlets, localized both in space-wavenumber and time-frequency domain, can be used for seismic data compression and sparsely represent the one-way propagator matrix. Numerical test results show that during the propagation, the number of dreamlet coefficients decreases with the depth in various degrees. This property is important for the realization of migration in the compressed data domain while achieving good image quality.

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