Dreamlet prestack depth migration using Local Cosine Basis and Local Exponential Frames

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Summary

Dreamlet migration using Local Cosine Basis (LCB) for spatial decomposition and Local Exponential Frames (LEF) for temporal decomposition is further developed and tested. Based on the beamlet local perturbation theory, dreamlet method is formulated with a local background velocity and local perturbations for each space window of the wavefield decomposition. The background dreamlet propagator based on LCB and LEF are obtained analytically. The numerical test using the 2D SEG/EAGE A-A' and SmaartJV Sigsbee2A model demonstrate the accuracy and imaging quality of this method. The application in target oriented migration is discussed and tested through numerical examples.

Introduction

Efforts have been made to develop wavefield decomposition and extrapolation methods with localization in both space and direction. Localized propagators can be easily tailored for local heterogeneities and to specific directions (Steinberg, 1993; Steinberg and McCoy, 1993; Wu et al., 2000; Wu and Chen, 2002; Chen et al., 2006; Wu et al., 2008). However, these local propagators have only space-direction localization, but without time-frequency localization. Recently, curvelet transform (Candès & Donoho., 2002; Candès & Demanet, 2005), which is a complete space-time localization, has been applied to wave propagation and seismic imaging using a map migration method (Douma and De Hoop, 2007; Chauris and Nguyen, 2008).

Dreamlet method (Wu et al., 2008; Wu et al., 2009) extended the frequency domain beamlet method to a time-space domain wave propagation using time-space wavelets, in which the $x - \xi$ localization uses the Local Cosine Basis (LCB) (Coifman and Meyer, 1991), and the $t - \omega$ localization adopts the Gabor Frame (Wu and Chen, 2006; Chen et al., 2006) or the local exponential tight frame (Auscher, 1994; Wu and Mao, 2007; Mao et al., 2007). In this paper, we further detailed the theory and method, and demonstrate the performance of the dreamlet method on synthetic data of complex structures and its potential for imaging in the compressed domain. First the local harmonic bases are introduced and the dreamlet propagator based on LCB and LEF is presented. As numerical examples, SEG/EAGE and SmaartJV Sigsbee2A model have been used to test the algorithms. The good imaging results demonstrate the accuracy and validity of the approach. In the last section, we apply dreamlet for the target oriented migration and show some example of numerical test.

Local cosine/sine bases and local exponential frames

The local cosine basis element can be characterized by position $\bar{x}_n$, interval (the nominal length of the window) $L_n = x_{n+1} - x_n$, and wavenumber index $m$ as follows ($m = 0, \ldots, M-1$, M denotes the total sample points of the interval)

\begin{align}
\hat{b}^{(+)}_{mn}(x) &= \sqrt{2/L_n} B_n(x) \cos(m + \frac{1}{2} \frac{x - x_n}{L_n}) \quad (1) \\
\hat{b}^{(-)}_{mn}(x) &= \sqrt{2/L_n} B_n(x) \sin(m + \frac{1}{2} \frac{x - x_n}{L_n}) \quad (2)
\end{align}

Where $B_n(x)$ is a bell function which is smooth and supported in the compact interval $[\bar{x}_n - \varepsilon, \bar{x}_{n+1} + \varepsilon']$, where $\varepsilon, \varepsilon'$ as the left and right overlapping radius. The local exponential frames define as follows

\begin{align}
g^{(+)}_{mn}(x) &= b^{(+)}_{mn}(x) + i b^{(-)}_{mn}(x) = \sqrt{2/L_n} B_n(x) \exp(i \pi x / L_n) \quad (3) \\
g^{(-)}_{mn}(x) &= b^{(+)}_{mn}(x) - i b^{(-)}_{mn}(x) = \sqrt{2/L_n} B_n(x) \exp(-i \pi x / L_n) \quad (4)
\end{align}

Where $\pm \pi x / L_n = \pi (m + 1/2) / L_n$ is the local wavenumber. When local exponential frames are used to decompose the space axis, $g^{(+)}_{mn}(\xi)$ and $g^{(-)}_{mn}(\xi)$ are taken as right and left propagating local exponential beamlets, respectively. Correspondingly, when LEB are used to decompose the time axis, $\hat{g}^{(+)}_{mn}(t)$ and $\hat{g}^{(-)}_{mn}(t)$ are taken as the positive and negative frequency drumbeats. Collectively, $g^{(+)}_{mn}$ and $g^{(-)}_{mn}$ form a tight-frame of redundancy 2.
Dreamlet prestack depth migration using Local Cosine Basis and Local Exponential Frames

The coefficients of the local harmonics of a function \( f(t) \) can be calculated by projection

\[
\begin{align*}
  f(t) &= \sum_{m \nu} \left[ < f, g_m^{(\nu)}(t) > g_m^{(\nu)}(t) + < f, g_m^{(\nu)}(t) > g_m^{(\nu)}(t) \right] \\
  &= \sum_{m \nu} \left[ f_m^{(\nu)} b_m^{(\nu)}(t) + f_m^{(\nu)} b_m^{(\nu)}(t) \right]
\end{align*}
\]

and \(<,>\) stands for inner product.

Since local cosine transform and local sine transform have fast algorithm, the coefficients corresponding to LEF can be calculated as follow

\[
\begin{align*}
  f_m^{(\nu)} &= \frac{f_m^{(\nu)} - f_m^{(\nu)}}{4} \\
  f_m^{(\nu)} &= \frac{f_m^{(\nu)} + f_m^{(\nu)}}{4}
\end{align*}
\]

where \( f_m^{(\nu)} \) and \( f_m^{(\nu)} \) are the coefficients of LCB and LSB decomposition of function \( f(t) \).

**Dreamlet decomposition implement using LCB and LEF**

Using a tensor product of the LEF as the time-frequency atom and LCB as the space-wavenumber atom, the superposition of the phase space atoms

\[
\sum_{\nu} \sum_{m \nu} \left[ < f, g_m^{(\nu)}(t) > g_m^{(\nu)}(t) + < f, g_m^{(\nu)}(t) > g_m^{(\nu)}(t) \right]
\]

is the Fourier transform of \( f(t) \). We keep the bell shape same for all time windows, we have

\[
\begin{align*}
  g_m^{(\nu)}(t) &= \frac{2}{\sqrt{T_n}} B(t) e^{i\omega_m(t-\hat{\tau}_a)} \\
  g_m^{(\nu)}(t) &= \frac{2}{\sqrt{T_n}} B(t) e^{-i\omega_m(t-\hat{\tau}_a)}
\end{align*}
\]

With \( B(t) = (\hat{\tau}_d, \hat{\tau}_d, \hat{\tau}_d) \) where \( \hat{\tau}_m(x) \) is same as equation (1) and \( g_m^{(\nu)}(t) \) and \( g_m^{(\nu)}(t) \) use as drumbeat atoms

\[
\begin{align*}
  g_m^{(\nu)}(t) &= \frac{2}{\sqrt{T_n}} B(t) e^{i\omega_m(t-\hat{\tau}_a)} \\
  g_m^{(\nu)}(t) &= \frac{2}{\sqrt{T_n}} B(t) e^{-i\omega_m(t-\hat{\tau}_a)}
\end{align*}
\]

Where \( \pm \omega_m = \pm \omega (m + \frac{1}{2}) / T_n \), \( \hat{\tau}_a \) is the time moment and \( T_n \) denotes the time interval. Because the LCB are orthonormal basis and LEF are tight frame, the dual dreamlet atoms are the same as the original atoms.

A wavefield in time-space domain can be represented as superposition of the phase space atoms

\[
\begin{align*}
  w(x,t) &= \sum_{\nu} \sum_{m \nu} \sum_{\nu} \left[ < a, d_m^{(\nu)} > d_m^{(\nu)}(x,t) + < a, d_m^{(\nu)} > d_m^{(\nu)}(x,t) \right] \\
  &= \sum_{\nu} \sum_{m \nu} \left[ < a \hat{c}, d_m^{(\nu)} > d_m^{(\nu)}(x,t) + < a \hat{c}, d_m^{(\nu)} > d_m^{(\nu)}(x,t) \right]
\end{align*}
\]

One-way propagator in the dreamlet domain

The evolution of dreamlets must observe the wave equation. After propagation, a dreamlet is no longer a dreamlet, and is spreading into other cells in the localized phase-space. The redecomposition of the distorted dreamlet into new dreamlets forms the propagator matrix elements:

\[
\begin{align*}
  P_{\nu,\nu}' &= \left\{ (d_m^{(\nu)} + d_m^{(\nu)}) \right\} \left\{ (d_m^{(\nu)} + d_m^{(\nu)}) \right\} \\
  &= \left\{ d_m^{(\nu)} \right\} \\
  &= \left\{ d_m^{(\nu)} \right\} + \left\{ d_m^{(\nu)} \right\}
\end{align*}
\]

\( P \) stands for the one-way propagator.

In the following, we use the analytic solution of wave equation in the frequency-wavenumber domain in a homogeneous background for the derivation of dreamlet propagator in the background media under the local perturbation theory.

In this case the dreamlet propagator (13) can be decomposed for four parts:

\[
\begin{align*}
  P_{\nu,\nu}' &= \left\{ (d_m^{(\nu)} + d_m^{(\nu)}) \right\} \left\{ (d_m^{(\nu)} + d_m^{(\nu)}) \right\} \\
  &= \left\{ d_m^{(\nu)} \right\} \\
  &= \left\{ d_m^{(\nu)} \right\} + \left\{ d_m^{(\nu)} \right\}
\end{align*}
\]

From the frequency-wavenumber domain, taking \( P_{\nu,\nu}' \) as example, it can be explicitly derived as

\[
\begin{align*}
  P_{\nu,\nu}' &= \frac{1}{2\pi} \int \left\{ d_m^{(\nu)} \right\} \left\{ d_m^{(\nu)} \right\} \\
  &= \left\{ d_m^{(\nu)} \right\}
\end{align*}
\]

Where \( d_m^{(\nu)}(\omega) \) is the Fourier transform of \( g_m^{(\nu)}(t) \) and \( P_{\nu,\nu}'(\omega,\omega,\omega) = \frac{1}{2\pi} \int \left\{ d_m^{(\nu)}(\omega) \right\} \left\{ d_m^{(\nu)}(\omega) \right\} \) is the beamlet propagator. \( \omega = \omega m^{(\nu)} - p^2 = \sqrt{(\omega_c)^2 - p^2} \) is the vertical wavenumber and \( p \) is the horizontal slowness.

We keep the bell shape same for all time windows, we have

\[
\begin{align*}
  g_m^{(\nu)}(\omega) &= \frac{2}{\sqrt{T_n}} \hat{\nu}^{\nu}_m \hat{\nu}^{\nu}_m B_m(\omega + \omega_n)
\end{align*}
\]

and \( B_\nu(\omega) \) is the time window at \( \hat{\tau} = 0 \).

After some simple derivation, the equation (17) becomes

\[
\begin{align*}
  P_{\nu,\nu}'(\omega,\omega,\omega) &= \frac{1}{\sqrt{T_n}} \hat{\nu}^{\nu}_m \hat{\nu}^{\nu}_m B_m(\omega + \omega) B_m(\omega + \omega) B_{\nu}^{\nu}(\omega,\omega,\omega)
\end{align*}
\]

Derived in a similar way

\[
\begin{align*}
  P_{\nu,\nu}'(\omega,\omega,\omega) &= \frac{1}{\sqrt{T_n}} \hat{\nu}^{\nu}_m \hat{\nu}^{\nu}_m B_m(\omega + \omega) B_m(\omega + \omega) B_{\nu}^{\nu}(\omega,\omega,\omega)
\end{align*}
\]

\[
\begin{align*}
  P_{\nu,\nu}'(\omega,\omega,\omega) &= \frac{1}{\sqrt{T_n}} \hat{\nu}^{\nu}_m \hat{\nu}^{\nu}_m B_m(\omega + \omega) B_m(\omega + \omega) B_{\nu}^{\nu}(\omega,\omega,\omega)
\end{align*}
\]
Dreamlet prestack depth migration using Local Cosine Basis and Local Exponential Frames

Here $P^{(+)}_{\mu\nu}, P^{(-)}_{\mu\nu}, P^{(-+)}_{\mu\nu}$ and $P^{(-)}_{\mu\nu}$ are the kernel of the dreamlet propagator in the background media.

**Dreamlet prestack depth migration**

To demonstrate the performance of dreamlet propagator, first, we apply the method on the 2D SEG/EAGE A-A’ salt model. The acquisition has 325 shots with left-hand-side receivers and the maximum number of receivers for one shot is 176. The original velocity model has 1200 samples in horizontal extend and 150 samples in depth, both the sample intervals are 80ft, as shown in Figure 1.

The prestack depth migration result using dreamlet is shown in Figure 2. The boundary of the salt body and sharp edges are clearly and correctly imaged. Most of the subsalt structures and the base straight line are also well imaged.

The benchmark data Sigsbee2A from the SMAART consists 500 shot gathers with 348 receivers per shot. The time sample interval is 8ms with 1500 samples per trace. The migration velocity model shown in Figure 3 has 2133 samples in horizontal with interval 37.5 feet and 1200 samples in depth with interval 25 feet.

Figure 4 shows the migrated image. The sediments, the two rows of point diffractors and the baseline reflector at the bottom of the model are all well imaged.

One advantage of dreamlet decomposition method is its high compression ratio and the potential for imaging in the
compressed domain (Wu et al., 2008; Wu et al., 2009). In figure 4 is the dreamlet coefficients number in each depth during the migration process, the red line is for the receiver side and the blue line for source. During the migration process, the average compression ratio on the receiver side is around 10 and the number of the dreamlet coefficient does not increase during migration. The coefficient number on the source side is increasing with depth but gradually becomes saturated and still keeps a higher average compression ratio.

In both of the model, we apply the automatic gain control on the time series, that is to say, the recorded seismic data is multiplied by the square root of time. The number of the velocity in the background propagator is 50, equally spaced from the minimum to the maximum velocity in each model.

**Target-oriented migration using dreamlet**

One benefit of the localization on time of dreamlet is its application on target oriented migration and imaging. For a specific target area, its reflected signal on the surface is only in a few time windows in the traces. Only migrate these target related portion of data can not only accelerate the migration process but also get a better image in the target area. On the Sigsbbe2A model, we fix our target beneath the salt body, indicating by the box in figure 4. We calculate the first arrival travelt ime and pick the time windows around it in the traces. The migration result using these portion of data is shown in figure 6 (b) and the same area in figure 4 is cut out for comparison.

The fault and the reflectors in the target oriented result are clearer than in the full migration result and the computation cost reduced tremendously.

**Conclusions and discussion**

In this paper, we present the dreamlet migration formulation using LCB for space-direction localization and LEF for the time frequency localization and numerical examples using SEG/EAGE A-A’ and Sigsbee models demonstrate the accuracy and the quality of the method. One benefit of dreamlet imaging is that the localization on time axis can be used for target oriented migration and imaging. We can pick the target reflected signal in the recorded seismic data to image the specific structures, such as the structure beneath the salt body. Preliminary numerical test of the target oriented migration show the feasibility and potential of this method.

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Dreamlet prestack depth migration using Local Cosine Basis and Local Exponential Frames


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