Subsalt imaging using secondary scattered waves
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Summary
We propose an approach to perform migration and imaging using secondary scattered waves. Secondary scattered waves are extracted through a back propagation plus cross-correlation approach. The surface shot gathers are thus transformed to subsurface shot gathers with sources located at some subsurface scatterers (gathers of scattering sources). Numerical examples confirm the validity of this secondary scattered wave retrieval approach. Migration and imaging from these secondary scattered wave gathers improves the final image. We apply this secondary scattered wave retrieval approach to SEG-EAGE salt model and improve the image of subsalt faults.

Introduction
Traditional one-way wave equation based migration and imaging methods are formulated to use primary scattered waves from subsurface reflectors. Multiples (including surface-related and internal multiples) and secondary scattered waves are often abandoned or treated as noises. From the point of view of wave propagation and time reversal back propagation, all these kinds of waves contribute to the final image if they are properly handled during migration and imaging. Reverse-time migration is supposed to handle all these waves but requires formidable computation. One-way/one-return wave equation based migration methods (Thomson, 2005; Wu et al., 2008) have been proved to be quite efficient to obtain sharp images of subsurface structure. However, the acquisition system in practice limits the illumination of primary reflections, leaving some ‘shadow zones’, e.g., some part of the subsalt region (Xie et al., 2006). Nevertheless, secondary scattered waves and multiples have potential to improve the illumination due to their complex but different-from-primary wave path (Cao and Wu, 2008; Malcolm et al., 2009). Diffracted waves, mainly first scattered waves, can also be separated from data to obtain clear images of subsurface structure whose scale is smaller or comparable to the wavelength such as pinchouts (Khaidukov et al., 2004; Bansal and Imhof, 2005; Fomel et al., 2007; Zhu and Wu, 2008).

Secondary scattered waves help to image some poorly-illuminated areas such as subsalt regions. This requires strong scatterers in the medium, e.g., sharp edge points on salt boundaries. On the one hand, these scatterers serve as new subsurface sources, emitting waves to illuminate ‘shadow zones’ where primary transmitted waves can not reach. On the other hand, these scatterers serve as ‘transporters’, changing the direction of primary reflected waves to allow surface receivers to record signals from subsurface structures. As an example of secondary scattering sources (Figure 1a), primary waves travel along the black trajectories and secondary scattered waves travel along the red trajectories. If a smooth velocity model is used during migration, back propagation of secondary scattered waves will travel along the red solid arrow (Figure 1b), which results in no contribution to the real image. Another advantage of these secondary scattered waves is that they can be recorded even for small aperture as long as the time record is long enough. Take the SEG-EAGE model for an example: the reflection signals from the sub-salt structure are scattered by the sharp edges of the salt. These secondary scattered waves from sharp edges could possibly be recorded though the direct reflected waves can not reach the receivers due to limited aperture.

![Figure 1: Behavior of secondary scattered waves from subsurface scattering sources during (a) forward modeling and (b) migration and imaging.](image-url)

Methodology
We propose a method similar to interferometric Green’s function retrieval (Schuster et al., 2004; Snieder et al., 2006; Wapenaar and Fokkema, 2006; Wapenaar et al., 2008) to transform the surface data to a new data set. These new data sets form a new acquisition system using subsurface scattering sources. The receivers are located either along the surface or at some subsurface space points. In this article, the new acquisition system contains subsurface scattering sources and surface receivers. Our approach mainly includes wave field downward extrapolation and cross-correlation. In the first step, we downward continue the source-side wave field to subsurface scatterers for each shot. In the second step, we cross-correlated the downward continued source-side wave field with the corresponding receiver-side records, resulting in cross-correlograms for each shot. In the third step, we stack all the cross-correlograms related to one pair of subsurface scatterer and surface receiver. The stacking results in one secondary scattered wave trace as if recorded at the receiver from a subsurface source located at the scatterer. We assemble all these secondary scattered wave
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traces for migration and imaging. The more accurate the velocity model, the better the proposed approach works.

Based on scattering theory, the backscattered waves $D_s$ from subsurface structure are formulated as (Wu, 1996; Wu, 2007)

$$D_s(x_s, x_r, \omega) = \int k^2 G_s(x, x_s, \omega)V(x) G_s(x, x_r, \omega)dx . \quad (1)$$

$G_s(x, x_s, \omega)$ is the frequency domain Green’s function in background medium between source $x_s$ and receiver $x_r$. $V(x)$ is the velocity function. Migration and imaging is regarded as a focusing procedure to the data (scattered wavefield)

$$I(x, \omega) = \int_{x_s} dx \hat{G}_s(x, x_s, \omega) \hat{C}_B(x, x_s, t) G_s(x, x_r, \omega)dx . \quad (2)$$

The asterisk stands for complex-conjugate. Migration focuses the surface receiver data to subsurface point. If the migration velocity model is accurate, the source-side wavefield and receiver-side wavefield arrive the subsurface scattering point simultaneously. Constructive interference of the zero-lag cross-correlation between the extrapolated source-side wavefield and receiver-side wavefield results in images of subsurface heterogeneities/discontinuities.

![Figure 2: Schematic cartoon of secondary scattered wave retrieval. Considering one subsurface scattering point $x_d$ and another spatial point $x_B$, the wave field emanated from one surface source $x_s$ is scattered by $x_d$. This scattered waves pass $x_d$, and arrive at surface receivers $x_B$ as secondary scattered waves (Figure 2). The secondary scattered waves in surface record is the convolution result among three Green’s functions: one between the surface source and $x_d$, one between $x_d$ and $x_B$, and the other is between $x_B$ and surface receivers. If we can recover the forward modeling wavefields at $x_d$ and $x_B$, the deconvolution between these two wavefields will result in Green’s function between them. We can achieve the wavefield reconstruction through backpropagation. And deconvolution is replaced by cross-correlation since only phase information is concerned and cross-correlation is more stable than deconvolution. The resultant cross-correlograms contain redundant phase information of the Green’s function. Similar to seismic interferometry, if we stack all these cross-correlograms, the constructive interference will recover the Green’s function. The backpropagation and cross-correlation approach is expressed as $G(x_d, x_r, \tau) = \int dx \hat{C}_B(x, x_d, \omega) \hat{C}_B(x, x_r, \omega). \quad (3)$](http://example.com)

The inner integral stands for the back propagation of the recorded wavefield to the subsurface point $x_d$. The second integral means summation of the resultant cross-correlograms between the source-side wavefield at subsurface point $x_d$ and back-propagated wavefield at point $x_B$. The retrieved Green’s function can be expressed in time domain as in (3) or stay in the frequency domain. To what degree this approximation holds depends on the accuracy of the velocity model, surface data aperture and the propagator used for wave field downward continuation. Equation (3) provides the basis for us to retrieve the Green’s function (for secondary scattered waves) among any two spatial points inside the medium for full aperture data. If $x_d$ is located at one surface receiver point, no back propagation is required. We can further reduce equation (3) into

$$G(x_d, x_r, \tau) = \int dx \hat{C}_B(x, x_d, \omega) D_s(x, x_r, \omega). \quad (4)$$

If there is no physical point scatterer at point $x_d$, instead only a receiver at this point, the retrieved Green’s function will be that of a ‘virtual’ source. The corresponding radiation aperture of a virtual source is mainly controlled by real source distribution (aperture) on the surface. In contrast, the advantage of secondary scattering sources is that the derived radiation aperture is mainly controlled by the nature of the scatterer, and can be much larger than the surface aperture. These retrieved Green’s functions between subsurface physical scatterers and surface receivers are exactly the secondary scattered waves in the surface data attributed to these scatterers.

Validity of secondary scattered wave retrieval

Numerical experiments are done to investigate the validity of proposed approach to retrieve Green’s function between subsurface scatterers and surface receivers. The velocity model (Figure 3) contains a homogeneous background with four point scatterers (specified as $0$, $1$, $2$ and $3$) and one $45$-degree-dip inclined fault. The four scatterers are located at $(5.0, 0.8)$ km, $(5.8, 0.8)$ km, $(5.0, 1.6)$ km and $(4.2, 1.6)$ km, respectively. The two ending points of the fault are located
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at (3.4, 1.6) km and (4.2, 2.4) km. The background velocity is 2.0 km/s. The perturbation of the fault and the scatterers are 100% the background with velocity 4.0 km/s. The sources and receivers are put on the surface. Finite difference method is used to generate the surface shot gathers for migration and imaging. A 10 Hz ricker wavelet is used in the forward modeling. The source-receivers geometry will be specified in each experiment.

Finite difference method is used to generate the surface shot gathers for migration and imaging. A 10 Hz ricker wavelet is used in the forward modeling. The source-receivers geometry will be specified in each experiment.

First we will investigate the accuracy of retrieved Green’s function. We use local cosine beamlet (LCB) propagator (Wu et al., 2008) to extrapolate the source-side wave field to scatterers. Sources are put along the surface between 3.0 and 7.0 km with a spatial interval 0.02 km. Receivers are fixed from 1.0 to 9.0 km along the surface. There are 201 shots in total. At the bottom of the Figure 4 shows the comparison between exact Green’s function (red) and retrieved Green’s function (blue) between scatterer 0 and one surface receiver at (5.8, 0). The exact Green’s function is calculated using finite difference method by recording the wave field at the receiver point (5.8, 0.0) with a source at the scattering point (5.0, 0.8). Each Green’s function is normalized with the maximum amplitude following the direct arrival. They match very well with each other except the direct arrival. The upper part of Figure 4 shows cross-correlograms from each shot. The retrieved Green’s function is the zero-time-lag stacking result of all these cross-correlograms. Thanks to interference, the main contribution to the retrieved Green’s function comes from the shots related to the stationary phase points (Snieder et al., 2006). Figures 5a and 5b show comparison between the whole retrieved shot gather and that from finite difference calculation. We can see that arrival times of the events following direct arrival match very well both in phase and relative amplitude.

Next we will investigate the image from migration of retrieved secondary scattered wave gathers. One-way wave equation based LCB migration method is used to perform the migration. Figure 5c shows the image from migration of total 201 shot gathers along the surface. Figure 5d shows the image from one retrieved secondary scattered wave gather (Figure 5b) with source at scatterer 0 and receivers at the surface. The two images agree well with each other.

Imaging with secondary scattered waves

In this section we will show the application of secondary scattered waves to improve imaging. We use the same constant velocity model. The surface data include 51 shots located between 3.0 and 4.0 km with a spatial interval 0.02 km. The receiver array is fixed from 1.0 to 5.0 km. No direct reflected waves from the inclined fault could reach the receivers from optical ray theory. Figure 6a shows the image from LCB migration of these 51 shot gathers. The four scatterers and two ending points of the fault are well imaged. The fault is completely missing in the image. This is due to fact that one-way wave equation based migration and imaging method only takes advantage of the primary reflections/scattered waves. The primary reflections from the fault can not reach the receivers with this source-receivers geometry. Nevertheless, these primary reflections are scattered, travel in diverse directions and reach the receivers as secondary scattered waves.

We retrieve the secondary scattered waves between the four scatterers and surface receivers for migration and imaging. Figure 6b shows the stacked image from migration of these four retrieved second scattered wave gathers. The upper part of the fault is clear seen on the image. The emergence of the fault is attributed to the secondary scattered waves from the four scatterers: The direct waves emitted from
surface sources are scattered by these scatterers; the scattered waves are then reflected by the fault and travel back to surface receivers. Green’s function retrieval focuses the scattered waves to the scattering points such that they serve as new physical sources. Meanwhile the secondary scattered waves are enhanced during focusing so that they could be used to obtain sharp images. The images from each retrieved shot gather are shown in Figure 6c, 6d, 6e and 6f. Although the artifacts are diverse on each image, there is one consistent image — the upper part of the fault.

![Figure 6: Images from migration of (a) 51 shot gathers along surface, (b) four retrieved shot gathers and individual retrieved shot gather with source located at (c) scatter 0; (d) scatter 1; (e) scatter 2 and (f) scatter 3. Red arrows indicate the fault.](image)

Our next example is the benchmark SEG-EAGE salt model. The sharp edges of the salt body are natural scattering sources. Our target is one subsalt fault within the red ellipse in Figure 7a. Shown in Figure 7b is the image from migration of the total 325 surface shots data. We can see that the target fault is missing due to the poor illumination of primary waves. We select some points on the boundary of the salt body as subsurface scattering sources and retrieve the secondary scattered wave gathers. Migration and imaging with these new data results in the image shown in Figures 7c-f with selected scatterers located at grid point (838,77), (832,78), (830,79) and (824, 79), respectively. Note that one consistent fault image demonstrated that the subsalt fault in the shadow zone can be illuminated by secondary scattering sources distributed on the sharp edges of the salt body.

Conclusions

We proposed to do migration and imaging with secondary scattered waves. These secondary scattered waves have potential to improve the illumination in ‘shadow zones’ of primary reflections such as in subsalt regions. These secondary scattered waves are extracted by transforming the surface shot gathers to subsurface shot gathers with sources located at scatterers along the sharp salt boundaries. The cross-correlation of the backpropagated waves removes the primaries and enhances the secondary scattered waves. Numerical experiments confirm the validity of Green’s function retrieval for the scattering sources and show the improvement of image, especially in the shadow zones of the primaries by the migration of these retrieved secondary scattered wave gathers.

![Figure 7: (a)SEG-EAGE salt model, (b) image from migration of surface shot gathers and images from secondary scattering wave gathers with scattering source located at (c) (838,77), (d) (832,78), (e) (830,79) and (f) (824, 79).](image)

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Reference


