Summary

The super-wide angle one-way method has been proposed to overcome the angle limitation of regular one-way wave propagation methods. In the method, a technique of wavefront reconstruction was used to combine the two orthogonally propagated one-way wavefields. This paper is focused on the accuracy improvement of the wave back propagation from the receiver array. The proposed method can model turning waves well and consequently can be employed in imaging overhanging salt flanks. Numerical tests demonstrated the validity of the method.

Introduction

Although quite a few methods have been proposed to improve the wide-angle accuracy of one-way wave propagators (Collins and Westwood, 1991; Ristow and Ruhl, 1994; Wu, 1994, 1996; Xie and Wu, 1998; Grimbergen et al., 1998; Jin et al., 1998, 2002; Huang et al., 1999a, b; Xie et al., 2000; De Hoop et al., 2000; Le Rousseau et al., 2001; Han and Wu, 2005; Thomson, 2005), the accuracy of wide-angle waves for strong contrast media is still a serious problem and put a practical limitation on applying these methods to steep reflector imaging.

The super-wide angle one-way method (Wu and Jia, 2006) has been developed to extend the capability of one-way propagators by a wavefront reconstruction method which combines and interpolates the two orthogonally propagated one-way wavefields to rebuild the distorted wavefront to good accuracy. In this paper, a more complex reconstruction scheme is developed for the propagation from receiver arrays to improve the accuracy and reduce the artifacts. The new scheme employs the wavefield gradients to determine the weighting function for the two orthogonally propagated wavefields. Numerical examples of impulse response demonstrate the good accuracy of the weighting scheme and the application in modeling turning waves, compared with the regular one-way method. We also show the good performance of this method on imaging the salt dome with overhanging flanks.

Wavefront reconstruction for array source case

One-way wave equation has a preferred direction for wave propagation. In Cartesian coordinate system, the preferred direction is either the z-axis or a horizontal direction in the 2D case. For a certain preferred direction, the large-angle waves carry some errors both in phase and amplitude. However, large-angle waves with respect to z-axis become small-angle waves to x-axis and vice versa. Therefore, we can combine two orthogonally propagated waves for good accuracy in super-wide angle ranges, i.e. for angles larger than 90° to the preferred direction.

Here we reconstruct the wavefield by taking a weighted average of the two one-way wavefields. The wavefront reconstructed will be used for further one-way propagation (see Figure 1). We know that the one-way propagation is very accurate for small-angle waves. Therefore, at any location we put heavy weight on the one-way solution which propagates small-angle waves at that point and light weight on the other one-way solution which propagates large-angle waves. The final wavefield is determined by the summation of the two weighted wavefields. We use the Hanning function to calculate the weights:

\[
 w(\theta) = \begin{cases} 
 1 & \theta \leq \theta_c \\
 [1 - \sin^2((b\theta - a)\pi / 2 - \theta_c)]^{(b - 1)/2} & \theta_c < \theta \leq \pi/2 - \theta_c \\
 0 & \theta > \pi/2 - \theta_c 
\end{cases},
\]

in which \(\theta_c\) is the cut angle of the Hanning window, \(a\) and \(b\) are two constants, and \(\theta\) is the wave propagation angle which can be determined by different methods. In the following, we propose a method using the gradient calculated at the reconstruction point for the weighting function.

The gradient of the wavefield at a given point determines the energy flow direction at that point which can be used to specify the wave propagation direction for the weight calculation. It gives

\[
 P^G = -\nabla P = -\begin{pmatrix} \frac{\partial P}{\partial x} \\ \frac{\partial P}{\partial z} \end{pmatrix},
\]

where \(P\) is the pressure in the wavefield. Thus we have

\[
 \theta = \tan^{-1} \left( \frac{\partial P}{\partial z} / \frac{\partial P}{\partial x} \right). 
\]

To calculate the two components of the gradient, one simple method is the forward difference approximation. The other way is deriving the applicable expressions of the gradient from the equations of the one-way wave propagators. In the experiments below, the second method will be employed.

The weight function is a key point to the wavefront reconstruction method. The accuracy of the weights depends on the calculation of the wavefield gradients. We can use either horizontal screens (i.e. downward one-way wave propagation) or vertical screens (i.e. horizontal one-way wave propagation) to obtain the wavefield gradients.
Imaging salt flanks by super-wide angle one-way method

Since neither of these two one-way propagators is accurate at large propagation angles, the final wavefield gradient at a given point should be determined by weighted summation of the gradients obtained from the two orthogonal one-way wave propagations. We can still use the Hanning function shown in eq. (1) to implement the weighting on wavefield gradients, which gives

\[ \mathbf{P}^\text{d} = w \mathbf{P}^\text{d} + (1 - w) \mathbf{P}^\text{h}, \]

where \( \mathbf{P}^\text{d} \) and \( \mathbf{P}^\text{h} \) are the wavefield gradients obtained from downward and horizontally propagated wavefields, respectively. Once the wavefield gradient is determined, the propagation angle can be obtained immediately and the weighting function used for wavefront reconstruction will also be available.

Provided with the weight functions, the wavefront reconstruction for array source case can be operated as shown in Figure 1. We first propagate the wavefield downward completely to get both the downward wavefield and its gradients. For the horizontal propagator, the wavefield will be obtained by combining the downward propagated wavefield at each step with the weights calculated from both wavefields. This scheme is also valid for the point source case and can be used for the wave back propagation from the receiver array in imaging problems.

**Figure 1:** wavefront reconstructions for the array source case
(a) reconstruction of the right propagated wavefield; (b) reconstruction of the left propagated wavefield. The grid number is \( M \times N \). The source array is located at nodes \( M_L \sim M_R \), shown as a green line. The shading areas have reconstructed wavefields and the dashed lines denote the areas with wavefields to be reconstructed immediately.

**Figure 2:** velocity model and weight distribution
(a) velocity model with the source marked on the surface. The unit of the velocity is km/s; (b) weight values of the horizontally propagated wavefield.

**Impulse responses in a salt model**

The weighting summation can be applied to most of the one-way propagators. In this work, we test the scheme using the wide-angle Padé GSP algorithm (Xie and Wu, 1998).

The impulse response of wavefront reconstruction for the point source case has been discussed in the previous work (Wu and Jia, 2006), using simple weighted sum of two orthogonally propagated GSP one-way fields. Here we will show a few numerical examples with the new scheme developed above. Figure 2(a) shows the velocity model in which the salt bulk with overhanging flanks is surrounded by the background \( c(z) \) media. The source is a single point source and the dominant frequency of the wavelet is 10Hz. For the scheme in which the wavefield gradients need to be calculated, the key point is the accuracy of the weights determined by eq. (3). Figure 2(b) shows the weight values of the horizontally propagated wavefield when using this scheme to generate the impulse response. The weight values are dependent of the frequency. The corresponding frequency is 10Hz for the map in Figure 2(b).

The modeling results are given as the snapshots for \( t=1.4, 1.9 \) and 2.4s. The wavefront reconstruction method has a great advantage over the regular GSP method when the propagation angle is extremely large. This is shown in Figure 3. We see that with the help of one-way propagation in the lateral direction (x-direction), excellent accuracy can be achieved for propagating angles beyond 90º.
Imaging salt flanks by super-wide angle one-way method

![Image](80x608 to 280x711)

**Figure 3:** impulse responses for the salt model  
(a) regular GSP method; (b) wavefront reconstruction method in which eq. (3) is used to calculate the weights.

![Image](82x502 to 278x602)

![Image](86x354 to 274x457)

![Image](89x248 to 271x346)

In order to see the advantage of the wavefront reconstruction method in modeling wide-angle waves more closely, the wavefronts obtained by three different methods are compared at the same time in Figure 4. The model is a \( c(x,z) \) model in which the velocity varies linearly in both horizontal and vertical directions. The wavefront from the regular one-way method has been distorted significantly at large propagation angles (see the inner wavefront in Figure 4a). On the other hand, the new propagator models the wavefront much better (see the outer wavefront in Figure 4a). Figure 4(b) shows the phases picked from the snapshot. Note that the phase corresponding to the new propagator is almost the same as that of the FD method while the regular one-way propagator results in remarkable errors.

Imaging salt dome flanks by wavefront reconstruction method

As we have mentioned above, the new propagator can model the turning waves more accurately than the regular one-way method. This advantage indicates that the new method is very promising in the applications of imaging steep subsalt reflectors and overhanging salt flanks. In this section an example will be taken to show the good performance of the method in imaging salt flanks.

The velocity model is shown in Figure 5(a). We use 12 single-ended shot gathers as the input data. These data were generated by the FD method. The sources are distributed evenly on the surface with the interval equal to 50\( \Delta x \), assuming \( \Delta x \) is the grid interval. The number of the receivers for each spread is 397 and the receiver interval is \( \Delta x \).

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**Figure 4:** wavefront improvement by the wavefront reconstruction method  
(a) impulse responses for the regular GSP method and the new super-wide angle method in a \( c(x,z) \) medium; (b) phases picked from the snapshots shown in (a), compared with that of the finite difference method.

**Figure 5:** imaging results for overhanging salt flanks  
(a) velocity model with shots (red points) and receivers for the first shot (green line); (b) regular GSP method; (c) wavefront reconstruction method.
Imaging salt flanks by super-wide angle one-way method

Figure 5(b) shows the stacked image obtained by using regular one-way method. Due to the difficulty in handling turning waves, the overhanging flanks of the salt dome can be hardly imaged. Figure 5(c) shows the image using our new method. The energy on both flanks of the salt has been recovered clearly and correctly.

Conclusions

This work improves the accuracy of the super-wide angle one-way propagator by using wavefield gradients. The method is also applied to seismic imaging for steep salt flanks using turning waves. Numerical examples show that the super-wide angle one-way method can model large angle waves well beyond 90°. The image quality, especially for overhanging salt flanks, has been markedly improved compared with the standard one-way propagation methods.

Acknowledgements

We thank Dr. Xiao-Bi Xie for providing the salt model data and the program help. The supports from the WTOPI (Wavelet Transform On Propagation and Imaging for seismic exploration) Research Consortium and the DOE/BES Project at University of California, Santa Cruz are acknowledged.

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