Illumination analysis using local exponential beamlets

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Summary
In this paper, we develop an efficient method of directional illumination analysis in the local angle domain using local exponential beamlets. To get the local angle information, the wave field is decomposed by local exponential beamlets, which is implemented by a combination of local cosine and sine transforms. As the local cosine/sine transforms have fast algorithms, this method has much better efficiency than the Gabor-Daubechies frame method. We calculated the directional illumination map and the acquisition dip response (ADR) for the 2D SEG/EAGE salt model to demonstrate the validity of our method.

Introduction
Illumination analysis in the target area is a powerful tool to study the influences of acquisition aperture and overlaying structure to the image quality. Many techniques of predicting illumination intensity distributions under certain acquisition geometries are based on ray tracing modeling (Berkhout, 1997; Muerdter et al., 2001a, 2001b, 2001c; Schneider, 1999; Bear et al., 2000). Recently, techniques to obtain the localized angle domain information for the frequency domain wave field based on beamlet decomposition or local slant stack have been developed and applied to directional illumination analysis (Wu and Chen, 2002, 2003, 2006; Xie and Wu, 2002; Xie et al., 2006). The Gabor-Daubechies frame decomposition is complete but not orthogonal, and therefore has redundancy in the representation. Local slant stacks for wavefield decomposition are also computation demanding. The LCB decomposition, on the other hand, is orthogonal and fast algorithms of local cosine/sine transforms exist with order O(N) computation efficiency. However, local cosine beamlets have always two lobes in symmetry with respect to the vertical axis. This lack of uniquely defined direction-localization prevents its use for directional illumination analysis. In this work, we develop a decomposition method using local exponential beamlets, which are linear combinations of local cosine and sine bases. Taking the advantages of availability of local cosine coefficients during the LCB migration process, this method can provide the local angle information by only an extra orthogonal transform with O(N) efficiency. As numerical example, we apply the method for the illumination analysis of 2D SEG/EAGE model and compared with Gabor-Daubechies frame method. Finally, we discuss its efficiency and the potential use for image amplitude correction.

Local cosine/sine bases and local exponential functions

The Balian-Low theorem indicates that orthogonal bases with good frequency localization generated by Gauss windowed exponential functions do not exist. Nevertheless, local cosine/sine bases was successfully constructed by Coifman and Meyer (1991) (see also Mallat, 1999), which consist of cosines/sines multiplied by smooth, compactly supported bell functions. In fact these orthonormal bases are formed by linear combinations from a tight frame of local exponentials (Daubechies et al., 1991; Aucher, 1994). In this paper, we will propagate the wavefield using orthonormal bases for the propagation efficiency. Then we can go back to the tight frame representation with local exponentials for complete local angle-domain information. Due to the availability of LCB coefficients during wave propagation using the LCB propagator, we need only an extra orthogonal decomposition by LSB to get the coefficients of LEB, resulting in high efficiency of both propagation and local angle domain decomposition.

The local cosine/sine basis elements can be characterized by position $\bar{x}_n$, the nominal length of window $L_n = \bar{x}_{n+1} - \bar{x}_n$, and wavenumber index $m$

$$b_{m}^{(c)}(x) = \frac{2}{L_n} B_n(x) \cos \left( \pi \left( m + \frac{1}{2} \right) \frac{x - \bar{x}_n}{L_n} \right)$$

and

$$b_{m}^{(s)}(x) = \frac{2}{L_n} B_n(x) \sin \left( \pi \left( m + \frac{1}{2} \right) \frac{x - \bar{x}_n}{L_n} \right)$$

where $B_n(x)$ is a bell function which is smooth and supported in the compact interval $[\bar{x}_n - \varepsilon, \bar{x}_{n+1} + \varepsilon']$ for $\bar{x}_n + \varepsilon \leq \bar{x}_{n+1} - \varepsilon'$, with $\varepsilon$, $\varepsilon'$ as the left and right overlapping radius, respectively. We define

$$g_{m}^{(c)}(x) = b_{m}^{(c)}(x) + ib_{m}^{(s)}(x) = \frac{2}{L_n} B_n(x) \exp \left( i \frac{\pi}{L_n} \left( m + \frac{1}{2} \right) \frac{x - \bar{x}_n}{L_n} \right)$$

$$g_{m}^{(s)}(x) = b_{m}^{(c)}(x) - ib_{m}^{(s)}(x) = \frac{2}{L_n} B_n(x) \exp \left( -i \frac{\pi}{L_n} \left( m + \frac{1}{2} \right) \frac{x - \bar{x}_n}{L_n} \right)$$

where $\frac{\pi}{L_n} \left( m + \frac{1}{2} \right)$ is the local wavenumber. We will call $g_{m}^{(c)}(x)$ and $g_{m}^{(s)}(x)$ as right- and left-propagating
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local exponential beamlets. Collectively, \( g_{m}^{(s)}(x) \) and \( g_{m}^{(r)}(x) \) form a tight-frame of redundancy 2.

Wave field decomposition in local wavenumber domain

The frequency domain wave field \( u(x, z, \omega) \) at depth \( z \) can be decomposed into local exponential beamlets with windows along the x-axis

\[
u(x, z, \omega) = \sum_{m} \sum_{s} \left[ u_{m}^{(s)}(\vec{x}_{m}, \vec{z}_{m}, \omega) + u_{m}^{(r)}(\vec{x}_{m}, \vec{z}_{m}, \omega) \right] g_{m}^{(s)}(x) + u_{m}^{(r)}(\vec{x}_{m}, \vec{z}_{m}, \omega) g_{m}^{(r)}(x)
\]

(4)

where \( g_{m}^{(s)}(x) \) and \( g_{m}^{(r)}(x) \) are the local exponential decomposition beamlets. \( u_{m}^{(s)}(\vec{x}_{m}, \vec{z}_{m}, z) \) and \( u_{m}^{(r)}(\vec{x}_{m}, \vec{z}_{m}, z) \) are the coefficients for the corresponding right- and left-propagating local exponential beamlets respectively, located at space window \( \vec{x}_{m} \) and wavenumber window \( \vec{z}_{m} \), \( <, > \) stands for inner product.

If we use LCB propagator, the LCB coefficients are already available during wave propagation. After an extra LSB decomposition, the coefficients corresponding to local exponential beamlets can be calculated as follow

\[
u_{m}^{(s)} = \frac{u_{m}^{(s)}(z) - i u_{m}^{(r)}(z)}{4}
\]

(5)

\[
u_{m}^{(r)} = \frac{u_{m}^{(s)}(z) + i u_{m}^{(r)}(z)}{4}
\]

(6)

where \( u_{m}^{(s)}(x) \) and \( u_{m}^{(r)}(x) \) are the complex coefficients of LCB and LSB decomposition of the wave field.

Now we rewrite the equation (4)

\[
u(x, z, \omega) = \sum_{m} \sum_{s} u_{m}^{(s)}(\vec{x}_{m}, \vec{z}_{m}, \omega) g_{m}^{(s)}(x) + u_{m}^{(r)}(\vec{x}_{m}, \vec{z}_{m}, \omega) g_{m}^{(r)}(x)
\]

\[
u(x, z, \omega) = \sum_{m} \sum_{s} \exp(i \vec{z}_{m} \vec{x}) \sum_{r} u_{m}^{(s)}(\vec{x}_{m}, \vec{z}_{m}, \omega) \frac{z}{L_{e}} B_{s}(x) \exp(-i \vec{x} \vec{z})
\]

(7)

Then we can have a partial reconstruction (mixed domain wave field in local phase space)

\[
u^{(s)}(x, z, \vec{z}_{m}, \omega) = \exp(i \vec{z}_{m} \vec{x}) \sum_{r} u_{m}^{(s)}(\vec{x}_{m}, \vec{z}_{m}, \omega) \frac{z}{L_{e}} B_{s}(x) \exp(-i \vec{x} \vec{z})
\]

(8)

We see that either \( u^{(s)}(x, z, \vec{z}_{m}, \omega) \) or \( u^{(r)}(x, z, \vec{z}_{m}, \omega) \) is a local plane wave, which is a superposition (weighted average) of two windowed plane waves of the same local wavenumber from the overlapping windows.

For the local plane wave of local wavenumber \( \vec{z}_{m} \), the corresponding propagating angle is

\[
u_{m}^{(s)} = \arcsin \left( \frac{v(x, z)}{\vec{z}_{m}} \right)
\]

(9)

Where \( \nu_{m}^{(s)} \) is the local incident angle with respect to the vertical, and \( v(x, z) \) is the velocity at \( (x,z) \).

Illumination analysis in the local angle domain

For a given acquisition geometry, the frequency-space Green’s function from source \( s \) to subsurface point \( (x,z) \) can be decomposed at the image region to a summation of local wavenumber components. That is

\[
u(x, z, \omega) = \sum_{\nu} G(x, z, \theta, s, \omega)
\]

(10)

Where \( G(x, z, \omega) \) is the frequency-space Green’s function and \( G(x, z, \theta, s, \omega) \) is its local-angle-component at \( \theta \). Similarly, the frequency-space Green’s function from subsurface point \( (x,z) \) to receiver \( r \) can be decomposed

\[
u(x, z, r, \omega) = \sum_{\nu} G(x, z, \theta_{r}, s, \omega)
\]

(11)

Where \( G(x, z, r, \omega) \) is the frequency-space Green’s function and \( G(x, z, \theta_{r}, r, \omega) \) is its local-angle-component at \( \theta_{r} \).

In order to evaluate the local angle domain illumination energy distribution for a given acquisition system in a velocity model, we sum up the contributions from all the sources at the image point, for each local angle. That is

\[
u_{\nu} \equiv \sum_{\nu} G(x, z, \nu_{\nu}, s, \omega)
\]

(12)

where \( \nu_{\nu} \equiv \sum_{\nu} G(x, z, \nu_{\nu}, s, \omega) \) is the directional illumination map.

Then we can get the total illumination by the summation of all the angles

\[
u_{\text{total}}(x, z, \omega) = \sum_{\nu} \sum_{\nu} |G(x, z, \nu_{\nu}, s, \omega)|^{2}
\]

(13)

In order to evaluate the aperture and propagation effects of the given acquisition geometry on energy distribution for a specific pair of incident/receiving angles, we use unit impulse as source at both source and receiver points for the
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Entire acquisition configuration. Similar to the procedure of DI mapping, we sum up contribution of the Green’s functions for each incident/receiving pair to get the acquisition aperture efficacy (AAE) matrix, which neglects the detailed wave interface pattern and considers only the energy distribution of the acquisition configuration. Then the AAE matrix at point \((x,z)\) is defined as

\[
E(x,z,\theta_r,\theta_s,\omega) = \sum_x G(x,z,\theta_r,\theta_s) G(x,z,\theta_r,\theta_s) \]

where \(\theta_r\) is the reflector normal angle (migration-dip angle) to the vertical and \(\theta_s\) is the reflection angle with respect to the normal, which is shown in Figure 1. The relationship between \((\theta_r,\theta_s)\) and \((\theta_r,\theta_s)\) is

\[
\theta_n = (\theta_r + \theta_s) / 2 \\
\theta_i = (\theta_r - \theta_s) / 2
\]

Figure 1. Definition of dip-angle and reflection-angle

Note that reflector-normal is opposite to the migration-dip in direction, but \(\theta_n\) is equal to the dip-angle (the angle between X-direction and the dip direction).

We can further reduce the acquisition efficacy matrix at each point to a function of reflector dip by summing up all the reflected energy for each reflector.

\[
A_j(x,z,\theta_n,\omega) = \sum_{\theta_r,\theta_s} E(x,z,\theta_r,\theta_s,\omega)
\]

Where \(A_j(x,z,\theta_n,\omega)\) is the ADR (acquisition dip response) vector for point \((x,z)\). The value of the ADR map measures the dip-angle response of the acquisition system, including the source and receiver apertures.

Numerical examples

We use both Gabor-Daubechies frame and local exponential beamlets for the directional illumination analysis of the 2D SEG/EAGE salt model, which has 1200 samples with an interval of 80 feet in the horizontal direction and 150 samples with an interval of 80 feet in depth. The minimum velocity is 5000 feet/sec and the maximum velocity is 14700 feet/sec. There are 325 shots with an interval of 160 feet and each shot has 176 receivers in its left-side with an interval of 80 feet. From comparisons of the directional illumination maps (Figure 2 and 3) and acquisition dip responses (Figure 4), we see the similarity in quality of analysis by the new method compared to the more expensive Gabor-Daubechies frame decomposition.

Figure 2. Directional illumination (DI) maps using the Gabor-Daubechies frame decomposition: (a) DI for angle=-30\(^\circ\); (b) DI for angle=30\(^\circ\); (c) Total illumination
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Figure 3. Directional illumination (DI) maps using the local exponential beamlet decomposition: (a) DI for angle=-30°; (b) DI for angle=30°; (c) Total illumination

Figure 4. Comparison of acquisition dip response (ADR) for dip-angle = 0°: (a) Using the Gabor-Daubechies frame method; (b) Using Local exponentials

Then let’s discuss the computation efficiency. For G-D frame or other frame decomposition, we need a FFT computation for each window. The number of windows in the representation depends on the redundancy ratio of the frame. On the other hand, for local exponential beamlets decomposition, we only need an extra orthonormal LST transform which has an order of $N_w \log_2 N_w$ computation, where $N_w$ is the window length and $N_r$ is the record length. In this example, the local exponential method is about 3 times faster than G-D frame method.

Conclusions

We developed an efficient method of directional illumination analysis in the local angle domain using local exponential beamlets, which form a tight-frame of redundancy 2. Although local exponentials are not orthogonal bases, however, due to the availability of LCB coefficients during wave propagation using the LCB propagator, we need only an extra orthogonal decomposition to get into the local angle domain. As the local cosine/sine transforms have fast algorithms, this method has much better efficiency than the Gabor-Daubechies or other frame methods. Numerical examples of directional illumination map and the acquisition dip response (ADR) for the 2D SEG/EAGE salt model illustrated the validity and efficiency of the new method. For the further study, we will use this method for image amplitude correction.

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