One-return boundary element method and salt internal multiples
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Summary
A one-way and one-return boundary element method (BEM) is proposed to calculate the primary transmission/reflection arrivals for layered model or inclusion model with strong velocity contrast. The primaries are obtained by decoupling the interactions between the “top” and “bottom” of the inclusions defined for a given source-receivers configuration. Then the primaries are subtracted from the whole seismic records to obtain the internal multiples generated by the inclusion such as salt bodies. As numerical examples, the primary reflections from top and bottom interfaces of a salt-lens model and the multiples between these two interfaces are calculated. This method is applied to SEG-EAGE model to predict the internal multiples for acoustic wave in 2D case. The results demonstrate that the proposed method is efficient to obtain the primary transmission/reflections generated by internal interfaces with strong velocity contrast.

Introduction
One-way and one-return (multiple-forward-single-backscattering approximation) propagator has shown its high efficiency in modeling the primary transmission/reflection signals and has been widely used in seismic modeling and imaging (e.g. Wu, 1994, 1996; Xie and Wu, 2001; Wild and Hudson, 1998; Wu and Wu, 2006; Wu et al., 2007). However, one return modeling fails when the velocity perturbation of the heterogeneity increases up to more than 40% of the background velocity. For the SEG-EAGE model, the velocity perturbation may be more than 200%. The high velocity contrast lead to strong internal multiples, which can significantly contaminate the reflection signals from structures beneath the inclusion. However, what the internal multiples behave in full seismograms and their effects on seismic imaging using primary reflections are still not clear. How to efficiently eliminate the effects of these multiples is still a challenging problem.

The boundary element method is widely implemented in full seismic wave field modeling. It is efficient in solving the forward modeling problems especially for the rough topographical scattering and for the inclusion model with strong velocity contrast. For a review on the method and recent progress in this area see Bouchon and Sánchez-Sesma (2007). Our goal is to develop a technique of modeling primary reflections in heterogeneous media with strong contrast inclusions using boundary element method. For stratified media, we can partially decouple the interactions between some elements in the full wave boundary element method and hence obtain the primary transmission/reflection waves. As for the salt dome model, we can divide the dome into top and bottom parts with respect to a given source-receivers geometry by adding two artificial interfaces. It is therefore possible to decouple the interaction between the upper interface and the lower interface so that only the primary transmission/reflection waves are obtained. By subtracting these transmission/reflection waves from the full wave field, we can obtain the multiples purely generated by interaction between the upper and bottom interfaces of the salt dome. These synthetic multiples can be used to predict the multiples in real data. By combining the advantage of BEM and one-way/one-return propagator together, the one-way BEM is expected to be able to efficiently handle the internal interface scattering problem and predict the salt-body multiples.

One-way and one-return Boundary Element Modeling
With the aid of free space Green’s function, the partial differential equation for elastic wave can be transformed into a boundary integral equation for homogeneous media or a volume integral equation for inhomogeneous media. For piecewise homogeneous earth model, based on the representation theorem (Aki and Richards, 1980), the boundary integral equation for the displacement in each domain satisfies:

\[ c(r)u(r) + \int \left[ u(r')\Sigma(r, r') - t(r')G(r, r') \right] dI(r') = f(\omega, r^2)G(r, r^2) \]

(1)

where \( u(r) \) is the displacement vector, \( t(r) \) is the traction vector. The coefficient \( c(r) \) generally depends on the local geometry of the boundary. \( G(r, r') \) and \( \Sigma(r, r') \) are the fundamental solutions (Green tensors) for displacement and traction, respectively. Usually the right hand side of the equation can be viewed as the incident wave. By discretizing the boundaries into linear elements and interpolating the wave field by shape functions, we can express the boundary integral equation in matrix form (Sánchez-Sesma, 1991; Fu and Mu, 1994; Fu and Wu, 2000, 2001; Ge et al., 2005). Integrating out the integral for each linear element along the boundary, finally we obtain a
boundary value problem to be solved in the following matrix form:

\[
\begin{align*}
\mathbf{H} \mathbf{U} + \mathbf{G} \mathbf{T} &= \mathbf{F} \\
\text{where } \mathbf{H} \text{ and } \mathbf{G} \text{ are the matrix made up of the integrals over Green's functions on each element.} \quad \mathbf{U} \text{ and } \mathbf{T} \text{ are the matrices made up of the unknown displacement and traction on each node on the boundary (Ge et al., 2005).}
\end{align*}
\]

To illustrate the concept of one-way and one-return boundary element method, consider 2D steady-state elastic wave propagation in a simplified three layered model with free surface (Figure 1 (a)). The source is located in the domain \( \Omega \) and receivers are put in the third domain \( \Omega_3 \). Here we aim to develop an interface-by-interface approach to obtain the primary and secondary waves through \( \Gamma_3 \) in domain \( \Omega_3 \) and the primary reflected waves on the surface in the domain \( \Omega_1 \).

First, let us set up the equations to solve the full wave problem. In the first domain \( \Omega_1 \), according to (1), we obtain

\[
\mathbf{c}(\mathbf{r}) \mathbf{u}(\mathbf{r}) + \int \left[ \mathbf{u}(\mathbf{r'}) \Sigma(\mathbf{r}, \mathbf{r'}) - \mathbf{t}(\mathbf{r'}) \mathbf{G}(\mathbf{r}, \mathbf{r'}) \right] d\mathbf{r'} = \mathbf{f}(\mathbf{r'}, \omega) \mathbf{G}(\mathbf{r}, \mathbf{r'}) \quad \mathbf{r} \in \Omega_1
\]

Similarly, in the second domain we obtain

\[
\mathbf{c}(\mathbf{r}) \mathbf{u}(\mathbf{r}) + \int \left[ \mathbf{u}(\mathbf{r'}) \Sigma(\mathbf{r}, \mathbf{r'}) - \mathbf{t}(\mathbf{r'}) \mathbf{G}(\mathbf{r}, \mathbf{r'}) \right] d\mathbf{r'} + \int \left[ \mathbf{u}(\mathbf{r'}) \Sigma(\mathbf{r}, \mathbf{r'}) - \mathbf{t}(\mathbf{r'}) \mathbf{G}(\mathbf{r}, \mathbf{r'}) \right] d\mathbf{r'} = 0 \quad \mathbf{r} \in \Omega_2
\]

And in the third domain we have

\[
\mathbf{c}(\mathbf{r}) \mathbf{u}(\mathbf{r}) + \int \left[ \mathbf{u}(\mathbf{r'}) \Sigma(\mathbf{r}, \mathbf{r'}) - \mathbf{t}(\mathbf{r'}) \mathbf{G}(\mathbf{r}, \mathbf{r'}) \right] d\mathbf{r'} = 0 \quad \mathbf{r} \in \Omega_3
\]

Combined with the continuity condition of displacement and traction along the interface, an equation array in matrix form can be deduced:

\[
\begin{align*}
\mathbf{H} \mathbf{U} + \mathbf{G} \mathbf{T} &= \mathbf{F} \\
\mathbf{H} \mathbf{U}_1 + \mathbf{G} \mathbf{T}_1 + \mathbf{H} \mathbf{U}_2 + \mathbf{G} \mathbf{T}_2 &= 0 \quad \mathbf{r} \in \Gamma_1 \quad \& \quad \mathbf{r} \in \Gamma_2 \\
\mathbf{H} \mathbf{U}_3 + \mathbf{G} \mathbf{T}_3 &= 0 \quad \mathbf{r} \in \Gamma_3
\end{align*}
\]

Here, \( \mathbf{G} \) and \( \mathbf{H} \) are the matrices containing the integrals over elements with respect to Green functions for displacement and traction on the \( i \)th domain respectively.

And \( \mathbf{U}_i, \mathbf{T}_i \) are the matrices made up of the unknown displacement and traction at the nodes on the \( i \)th interface.

The above formulation is a unified representation for both the acoustic and elastic cases. For the acoustic case, the boundary conditions require the continuity of pressure and vertical particle-motion across the boundary. Therefore, \( \mathbf{u} \) and \( \mathbf{t} \) become scalar quantities \( p \) and \( t = \frac{1}{\rho} \frac{\partial p}{\partial n} \); In the case of 2D elastic \( \mathbf{SH} \) wave, \( \mathbf{u} \rightarrow \mathbf{u}_y \) and \( t = \frac{\mu}{\rho} \frac{\partial u_y}{\partial n} \). The corresponding Green’s functions are treated in the similar way. For the 2D \( P-SV \) and 3D elastic cases, the formulation is directly applicable. The system of equation (6) will give a full wave solution, in which the wave field on \( \Gamma_1 \) is coupled with the wave field on \( \Gamma_2 \) so that the multiple reflections are accurately modeled. In the following we will try to decouple the interaction between these two boundaries and derive the one-way transmitted waves and one-return reflected waves following the idea of De Wolf approximation (De Wolf, 1971; Wu, 1994, 1996; Wu et al., 2007).

First we decouple the interaction between \( \Gamma_1 \) and \( \Gamma_2 \), which is to neglect the reflected waves from \( \Gamma_2 \) when calculating the transmitted waves on \( \Gamma_1 \). With the drop of \( \mathbf{U}_2 \) and \( \mathbf{T}_2 \) in (6), we obtain an equation array for the wave field on \( \Gamma_1 \):

\[
\begin{align*}
\mathbf{H} \mathbf{U}_1 + \mathbf{G} \mathbf{T}_1 &= \mathbf{F} \\
\mathbf{H} \mathbf{U}_1 + \mathbf{G} \mathbf{T}_1 &= 0
\end{align*}
\]

Once the displacement and traction on interface \( \Gamma_1 \) are obtained, the wave field at any point inside domain \( \Omega_1 \) or domain \( \Omega_2 \) can be calculated by substituting \( u_1 \) and \( t_1 \) into equation (3). So we can calculate the incident wave \( u_2^{inc} \) and \( t_2^{inc} \) on interface \( \Gamma_1 \). Now we turn to domain \( \Omega_2 \) and domain \( \Omega_3 \) with interface \( \Gamma_2 \). The derivation will be similar to that for \( \Gamma_1 \) as in (7) with the incident wave calculated by the representation integral along \( \Gamma_1 \):

\[
\begin{align*}
\mathbf{H} \mathbf{U}_2 + \mathbf{G} \mathbf{T}_2 &= \mathbf{U}_2^{inc} \\
\mathbf{H} \mathbf{U}_2 + \mathbf{G} \mathbf{T}_2 &= 0
\end{align*}
\]

Once \( \mathbf{U}_2 \) and \( \mathbf{T}_2 \) are solved, the wave field at any point in domain \( \Omega_2 \) and domain \( \Omega_3 \) can be calculated. If there are more interfaces, we can proceed with the similar procedure iteratively until to the domain where receivers are located or the bottom of the model. By substituting the displacement and traction on the boundary into equation (3), we can obtain the primary transmitted wave at those receivers.
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In summary, our approach can be described as follows:
First, we calculate the transmitted wave through $\Gamma_1$ by applying the full-wave BEM to domain $\Omega_1$ and domain $\Omega_2$ with interface $\Gamma_1$, ignoring the effect of interface $\Gamma_2$. The second step is to propagate the output wave field to the interface $\Gamma_2$ as the incident wave. Finally we obtain the transmitted wave through interface $\Gamma_2$ in domain $\Omega_1$ by solving the two-domain boundary value problem involving domain $\Omega_2$ and domain $\Omega_1$ with interface $\Gamma_2$. Each time we only need to solve a much smaller matrix associated with the current interface rather than a much larger full rank matrix in full-wave BEM. However, this technique decouples the wave field interaction between interface $\Gamma_1$ and $\Gamma_2$, thus eliminates the multiples between $\Gamma_1$ and $\Gamma_2$, and can only obtain the primary transmitted waves. Therefore we call this technique 'one-way' boundary element modeling.

Next we consider the primary reflections by one-return approximation. In the same spirit of neglecting multiples, the backscattered waves at each interface is picked up and propagated to the surface by the one-way BE propagator as formulated above. In our three layer model, first we pick up the reflected waves on $\Gamma_2$. The reflected (backscattered) fields $U_r^G$, $T_r^G$ are obtained by subtraction of the incident field from the total field. Then we can calculate the transmission of $U_r^G$ and $T_r^G$ at $\Gamma_1$ by solving equation (7) with different right-hand terms:

$$
\begin{align*}
\mathbf{H}_1 \mathbf{U} + \mathbf{G}_1 \mathbf{T} &= \mathbf{0} & \mathbf{r} \in \Gamma_1, \\
\mathbf{H}_1 \mathbf{U} + \mathbf{G}_1 \mathbf{T} &= \mathbf{U}_{inc}^{(1)} & \mathbf{r} \in \Gamma_1,
\end{align*}
$$

where $\mathbf{U}_{inc}^{(1)}$ is the incident wave on $\Gamma_1$ due to the back scattered wave from $\Gamma_2$.

It is quite straightforward to apply this interface-to-interface method to layered media. However, to extend this method to models with inclusion, extra care must be taken. Here, we take a simple inclusion model (Figure 1 (b)) as an example. The inclusion could be a salt dome. We separate boundary of the salt dune into top part $\Gamma_1$ and bottom part $\Gamma_3$ based on the shape of inclusion and the source-receivers configuration. We also add flat artificial interfaces $\Gamma'$ and $\Gamma''$, extending to infinite. Now we divide the whole model into three domains: 1) $\Omega_1$ with boundary $\Gamma_1$, $\Gamma'$ and $\Gamma''$; 2) $\Omega_2$ with boundary $\Gamma_1$ and $\Gamma_3$; 3) $\Omega_2$ with boundary $\Gamma_2$, $\Gamma_3$ and $\Gamma''$. We can apply the one-way and one-return boundary element method to this inclusion model to calculate the primary transmitted and reflected waves through this inclusion without most of the internal multiples. Attention should be paid to the calculation of the integrals over Green functions on artificial interface $\Gamma'$ and $\Gamma''$, where the Rayleigh integral of the second type (Morse and Feshback, 1953; Wu, 1989, 1994) is used.

This one-way and one-return boundary element method is expected to efficiently handle the scattering problem due to internal interfaces with strong velocity contrast. In the next section, we will show some numerical examples to show the validity of this method.

Figure 1: The layered model (a) and inclusion model (b) used to illustrate the concepts of one-way and one-return boundary element method for seismic wave forward modeling. Triangles indicate the receivers. The star indicates the source.

Numerical Example

In this section, numerical examples are presented to test the feasibility of the one-way/one-return boundary element method. Here we only calculate primary reflection arrivals for the acoustic wave in 2D case.

Our first model contains a lens embedded in an otherwise homogeneous media. The parameters for the lens model are $V_i = 2.5 km/s$, $\rho = 2.0 g/cm^3$ for the outer homogeneous domain and $V_e = 5.0 km/s$, $\rho = 2.5 g/cm^3$ for the lens inclusion. Artificial interfaces are added at the edge to separate the domains of interaction. The sketch of the model and geometry of the source and receivers are shown in Figure 1 (b). The full wave response is shown in Figure 2 (a). Primary reflections from the top interface and the bottom interface are shown in Figure 2 (b) and (c) respectively. We could see that not only the arrival time but the amplitude of the primary events by one-way boundary element method fully agree with those by full boundary element method. Finally, we can subtract the primary reflections due to the interfaces from the full wave field to obtain the multiples generated between the top and bottom interfaces of the lens, which is shown in Figure 2 (d). There are still scattering waves messed with the multiples due to truncation points.

To test the validity and capability of this one-way and one-return boundary element method, we applied this one-return BEM to a modified SEG-EAGE model. The salt dome is assumed to be homogeneous embedded in a homogeneous background medium. The velocity and density of the salt dome are $V = 4.5 km/s$ and ...
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\[ \rho = 2.2 \ km/s \] For the medium outside the dome, \( V = 2.5 km/s, \rho = 2.0 km/s \). The snapshot of the full wave field for this simplified model is shown in Figure 3. The source is located at (6.0, 0.0), and the receivers are put at the same depth with an interval of 25m. Here we use a Ricker wavelet with center frequency at 5Hz. The synthetic full acoustic arrivals, primary reflections from top interface, primary reflections from bottom interface and internal multiples are shown in Figure 4. Note that the internal multiples are amplified by a factor of ten for the comparison with the primary arrivals.

Conclusion

An interface-to-interface one-way boundary element modeling method is developed to calculate the primary reflection/transmission arrivals for layered earth model and inclusion model. Numerical examples show that not only the arrival time but the amplitude of primary arrivals are accurately modeled in comparison with the events in full wave seismograms. Internal multiples due to the interaction between the interfaces could be obtained by subtracting these primary arrivals from the full wave synthetics. The method is accurate and efficient for layered model, and can handle strong-contrast interface. It can also be applied to inclusion models such as the models with salt domes.

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Figure 2: Synthetic seismograms for reflected waves for the lens model. (a) By full-wave BEM. (b) Primary reflections from the top interface. (c) Primary reflections from the bottom interface. (d) Internal multiples (amplified by a factor of ten).

Figure 3: Snapshot of wave field for the modified SEG-EAGE model and geometry of the source (star) and receivers (triangles). Note the internal multiples (yellow arrows) generated by the left part of the salt.

Figure 4: Synthetic seismograms for reflected waves for the modified SEG-EAGE model. (a) By full-wave BEM. (b) Primary reflections from the top interface. (c) Primary reflections from the bottom interface. (d) Internal multiples (amplified by a factor of ten).
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