Accuracy improvement for super-wide angle one-way waves by wavefront reconstruction

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Summary

To overcome the angle limitation of regular one-way wave propagation methods, we propose to extend the capability of one-way propagators by a wavefront reconstruction method which combines and interpolates the two orthogonally propagated one-way wavefields. The proposed method has accurate super-wide angle (greater than 90°) propagation and can model turning waves. The method has the potential to be used in imaging steep subsalt reflectors and overhanging salt flanks. Numerical tests demonstrated the validity of the method.

Introduction

Although significant progress has been made in improving the wide-angle accuracy of one-way wave propagation methods (Collins and Westwood, 1991; Ristow and Ruhl, 1994; Wu, 1994, 1996; Wu and Xie, 1994; Xie and Wu, 1998; Grimmerben et al., 1998; Jin and Wu, 1999a, b; Jin et al., 1999, 2000; Huang et al., 1999a, b; Xie et al., 2000; De Hoop et al., 2000; Le Rousseau et al., 2000; Han and Wu, 2005; Thomson, 2005), the accuracy of wide-angle waves for strong contrast media is still a serious problem and put a practical limitation on applying these methods to steep reflector imaging, especially in complex subsalt regions. The other more fundamental limitation of the one-way methods is the difficulty of handling turning waves and therefore renders the method of little use in imaging overhanging salt flanks.

In this work we propose to extend the capability of one-way propagators by a wavefront reconstruction method which combines and interpolates the two orthogonally propagated one-way wavefields to rebuild the distorted wavefront to good accuracy. The reconstruction will be conducted iteratively and the wavefront expansion can be accurately simulated well beyond 270° (135° single-side). Numerical examples of impulse response in c(z) medium and two-layered medium demonstrate the good accuracy of super-wide angle waves and the modeling of turning waves. The efficiency of the algorithm and potential of applications are discussed.

Reconstruction of accurate wavefront using two orthogonally propagated one-way waves

One-way wave equation has a preferred direction for wave evolution (propagation). In this paper we focus only on the two-dimensional case. In Cartesian coordinate system, the preferred direction is either the z-axis or a horizontal direction in the 2D case. In exploration geophysics, normally z-axis is the preferred direction. In such a case, the large-angle waves, e.g. the waves with propagating angles exceeding 70°, inevitably carry some errors both in phase and amplitude. However, large-angle waves with respect to z-axis become small-angle waves to x-axis. Therefore, we propose to use a wavefront reconstruction method combing two orthogonally propagated waves for good accuracy in super-wide angle ranges.

Assume two one-way equations in x and z directions:

\[ \frac{\partial u(x, z)}{\partial x} = P_x u(x, z) \]
\[ \frac{\partial u(x, z)}{\partial z} = P_z u(x, z) \]

where \( u(x, z) \) is the wavefield, \( P_x \) and \( P_z \) are the one-way propagators in z- and x-directions respectively. The formal solution for one step forward propagation can be written as

\[ u(x, z + \Delta z) = P_z^1[u(x, z)] \]
\[ u(x + \Delta x, z) = P_x^1[u(x, z)] \]

where \( \Delta x \) and \( \Delta z \) were incorporated into the propagators in the right-hand of equation (2). Now we consider the one-way propagation in x-direction having M steps forward, and in z-direction having N steps forward with respect to a reference point \( x_n = (x_n, z_n) \):

\[ u(x_n + m \Delta x, z_n + n \Delta z) = P_x^m[u(x_n, z_n)] \quad n = 1, \ldots, N \]
\[ u(x_n, z_n + n \Delta z) = P_z^n[u(x_n, z_n)] \quad m = 1, \ldots, M \]

We reconstruct the accurate wavefield by taking a weighted average of the two one-way wavefields. The wavefield reconstructed will be recorded as the accurate wavefield for modeling or imaging. The wavefront reconstructed will be used for further one-way propagation (see Figure 1). We know that the one-way propagation is exact in the forward direction and very accurate for small-angle waves, especially for good one-way propagators. Therefore, at any location we put heavy weight on the one-way solution which propagates small-angle waves at that point and light weight on the other one-way solution which propagates large-angle waves. This is to say that we set up a weighting scheme according to the local propagating angles:

\[ u(x_n + m \Delta x, z_n + n \Delta z) = w_x(\theta_x)P_x^m[u(x_n, z_n)] + w_z(\theta_y)P_z^n[u(x_n, z_n)] \]

where \( \theta_x \) and \( \theta_y \) are the weights functions for the one-way fields along x-axis and along z-axis, respectively. M and N determine how often wavefront reconstruction is conducted. N=M=1 is the case...
of stepwise continuous reconstruction; on the other hand, \( N=N_x, M=M_x \) corresponds to a simple scheme of weighted average without updating the wavefront, where \( N_x \) and \( M_x \) are the total sampling numbers of the model in \( z \)- and \( x \)-directions. Updating the wavefronts are important for modeling turning waves, refracted and diffracted waves in strongly heterogeneous media.

\[
\theta = \tan^{-1}\left( \frac{c_{0}/h_1}{c_{ref}} \right) \]

\( \theta \) is the propagating angle which is the angle between the propagating direction and the preferred direction. \( \theta_c \) is the cut angle of the Hanning window.

The weighted summation for reconstruction can be done either in space domain or in wavenumber domain. More efficiently it should be performed in local angle domain (beamlet domain) using beamlet propagators (e.g., Wu et al., 2000). In the local space domain, the propagating angles are determined by the local updating geometry. In the local angle domain, the propagating angle has a direct relation with the local wavenumber. We preferred to adopt a weighting scheme in local angle domain. There is an additional advantage of doing weighting in angle domain. Since the weighting in effect is a high-angle filtering, it is much more stable than the weighting in space domain. The weighting summation can be also applied to other one-way propagators, such as the GSP (generalized screen propagators) (Wu, 1994, 1996; Jin et al., 1998, 2002; Huang et al., 1999a, b; Xie and Wu, 1998). In this work, we test the scheme using the wide-angle Padé GSP algorithm (Xie and Wu, 1998). Different weight functions have been tested (see Figure 2). We found the Hanning type weighting gives the best results. In this scheme, the field with propagating angle less than 22.5° will be kept untouched, and that greater than 67.5° will be discarded. The field with mid-angles will be weighted accordingly.

**Impulse responses in different media**

In this section several numerical examples will be shown using the method mentioned above and the comparison with the FD method will be discussed. For simplicity, in this section we employ \( N=M=N_x = N_z \) in equation (4) to see the effect of simple weighted sum of two orthogonally propagated GSP one-way fields. The first model in Figure 3(a) is a \( c(z) \) model with moderate gradient of velocity variation, i.e. \( c=c_0 + |z-z_s|c_0/h_1 \). For all the models mentioned below, \( c_0=2 \text{km/s}, h_1=2.55 \text{km} \) and \( x_s=z_s=2.55 \text{km} \). The source is a single point source and the dominant frequency of the wavelet is 30Hz. The modeling results are given as the snapshots for \( t=0.6, 0.75, 0.9, 1.05 \) and 1.2s. We see that the one-way modeling with wavefront reconstruction can simulate wide-angle propagating waves quite accurately. Due to the help of one-way propagation in the lateral direction \( (x \)-direction), excellent accuracy can be achieved for propagating angles well beyond 90°. In the next section we will see that this feature extends the one-way method to a new area of modeling turning waves.

The model for Figure 4 is a two-layered model with 50% velocity contrast. The depth of the interface is \( z=h_1-h_2 \). The anisotropic model is \( c=c_0 + |z-z_s|c_0/h_1 \). Except the reflected waves, the wide-angle direct waves and transmitted waves are all well modeled.
One-way wave propagation with wavefront reconstruction

We now discuss the computational efficiency of the methods compared with the regular GSP or other one-way methods. If we use the global propagator such as the GSP or FFD methods, the weighted one-way method with wavefront reconstruction will take roughly twice the computation time as the regular one-way methods. However, if we use the localized propagators such as the beamlet propagators (Wu et al., 2000), the efficiency of the wavefront reconstruction method should increase, since the wavefront reconstruction near the source region do not need a full length calculation in the lateral direction. Compared with the full-wave finite difference method, the computation efficiency of the proposed method is still advantageous.

Turning wave simulation

The wavefield reconstruction method using the weighted average of two one-way wavefields can be used to model turning waves. We designed a c(z) model with large velocity gradient to test the performance of the method for turning waves. Figure 6 shows the snapshots and the surface records. As shown in Figure 6(c), turning waves can be seen clearly for the far offsets and the traveltimes of the turning waves become less than those of the direct arrivals. In contrast, for the regular one-way method with preferred direction along the z-axis, turning wave modeling is a great obstacle.

Conclusions

From simple numerical examples, it is shown that the proposed one-way wave propagation method by combining two orthogonally propagated one-way waves with wavefront reconstruction can overcome the angle limitation of regular one-way methods. The proposed method has accurate super-wide angle (greater than 90°) propagation and can model turning waves. The method has the potential to be used in imaging steep subsalt reflectors and overhanging salt flanks.

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![Fig. 6 modeling turning waves by wavefield reconstruction method](image-url)

(a) $c = c_0$

(b) $c = c_0 + 2|x - z_s|c_0/h_1$

(c) $c = c_0 + 2|x - z_s|c_0/h_1$

The horizon coordinate is reduced traveltim and the vertical is trace number on the surface (the point source is in the middle). The direct arrivals are weakened to render the signals of turning waves more clearly.