Seismic image resolution: numerical investigation of role of migration imaging operator
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Summary

The resolution of seismic migration imaging is often given as some function of the data aperture, the size of the Fresnel zone at the image location, the frequency of the data used in the migration and the illumination of the target region. Traditional resolution analysis is based on calculation of a Point Spreading Function (PSF) in the wavenumber domain from the scattering wavenumbers at the image point. This spectral representation can be converted into a space-domain PSF by Fourier Transform. However, other issues influence image resolution or distort migrated images. For example, the velocity model is generally not reliably known and variations of the model from the true structure can have a significant impact on image focus, thus reducing resolution. Another factor that limits resolution is the type and accuracy of the propagator used in imaging. To compare the resolution of images obtained when using ray-based Kirchhoff migration with that obtained from wave equation migration, we developed a 2D heterogeneous model that can be numerically simulated. Poststack migrations of exploding reflector data obtained for the model show that the PSF are influenced by the migration operator and are not necessarily the same as expected from the range of scattering wavenumbers estimated using a simple analysis approach.

Introduction

The resolution of seismic migration imaging is an important factor to consider in both acquisition design, data processing, and in image interpretation. Beylkin (1985) presented a formulation based on the generalized Radon transform and Born scattering that has provided a basis for most subsequent investigations of the resolution of seismic imaging. The formulation allows the estimation of seismic resolution at each point within an image based on survey design including source-receiver geometry and Earth model. Lecomte and Gelius (1998) and Gelius et al (2002) provide an informative discussion of the Beylkin approach and show that the predicted resolution, or Point Spreading Function (PSF) is a function of the range of scattering wavenumber at an image point. 

\[
R(x, x_0) = \int e^{iK_s(x-x_0)}dK_s
\]

where \(R\) is the resolution as a function of \(x\) for point \(x_0\), and \(K_s\) is the scattering wavenumber. Note that the integration is only over the range of scattering wavenumbers resulting from the acquisition geometry and frequency content of the data. Scattering wavenumber, also referred to as exchange wavenumber, is the difference between the wavenumber of the incident wave and the scattered wave at the image point. While this formulation is useful, it does not account for practical effects such as the signal-to-noise ratio of data or fold, which strongly influence image quality and hence resolution (Gibson and Tzimeas, 2002). Nevertheless, it has provided a useful tool for survey design (von Seggern, 1991; Gibson and Tzimeas, 2002; Lecomte et al. 2003) and deconvolving the image blur caused by limited resolution (Sjöberg et al, 2003).

Figure 1. Heterogeneous model used for the resolution investigation. The exploding source point is shown by a star.

Data processing will also have a significant influence on image resolution. For example the use of an incorrect velocity model not only degrades an image but leads to poor image resolution. Use of an unreliable raytracer in ray-based Kirchhoff migration imaging will degrade image resolution. It is expected that using a more reliable migration operator, such as one based on the wave equation rather than ray theory, should lead to better image resolution. Thus, acquisition-based resolution analysis is unlikely to lead to a complete estimation of resolution.

Xie et al (2005) have investigated image resolution of wave-equation imaging using the Beylkin (1985) approach adapted to wave-equation imaging. They use a wave-equation migration operator to recursively backpropagate source and receiver wavefields down to a target depth where a local analysis is done to decompose the wavefield into the wavenumber domain at a given image point. This gives the range of scattering angles at the image point, which can be used to estimate the image resolution.
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Wu et al. (2005) provide a new look at image resolution based on inverse theory. They develop a formulation for estimating the image resolution at a given point in an image that is a function of not only data acquisition but also the migration operator. This allows us to take account of how reliably and completely the migration operator reproduces the data. For exploding reflector data, they show that the Point Spread Function (PSF) in the wavenumber domain can be expressed as:

\[ R(K, x_0) = \int d\omega \int d^2 x \, \mathcal{G}_I(\omega, x, x_0) \mathcal{G}_M(\omega, x_0) \]

and in the space domain as:

\[ R(x, x_0) = \int d\omega \int d^2 x \, \mathcal{G}_I(\omega, x, x_0) \mathcal{G}_M(\omega, x_0) \]

where the resolution function is in the wavenumber domain, \( \omega \) is angular frequency, \( \mathcal{G}_M \) is the modeling Green function representing the true earth operator, and \( \mathcal{G}_I \) is the imaging operator. Note that \( R \) is a function of the data and the migration operator in this formulation.

We wish to investigate the resolution of an image obtained in a medium containing small scale heterogeneity, whose structure is known. We choose a medium whose scale of heterogeneity is the same as the scale of the dominant wavelength of the imaging wavefield to allow us to investigate the effect of the migration propagator accuracy on the image resolution. We thus investigate imaging in one realization of a 2D randomly heterogeneous medium. The medium is described by a Gaussian-type autocorrelation function (Sato and Fehler, 1998). The background (average) velocity of the medium is 3500 m/s, the correlation length is 100 m and the fractional velocity fluctuation is 5%. The medium is shown in Figure 1.

The scalar acoustic finite difference code described in Fehler et al. (2000) was used to generate exploding-reflector synthetic seismograms for an exploding reflector source located at the position of the star in Figure 1. To make the illuminating wavefield have dominant wavelength that is on the same scale as the scale of the medium heterogeneity, we use a 35 Hz Ricker wavelet source. Figure 2 shows the surface receiver gather generated for the lower source point.

The exploding reflector shot gathers were migrated using the ray-based Kirchhoff migration imaging code described by Fehler et al (2002). Data were migrated using only first arrivals and no corrections were made for ray amplitudes or phases. Traveltimes were calculated using a wavefront construction algorithm. Figure 2 shows traveltimes calculated using the raytracing algorithm in comparison with the trace data. The traveltimes predicted by the raytracing scheme appear to be quite reliable although the trace data contain significant later arrivals that will not be used in first-arrival migration.

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migrated wavefield for velocity heterogeneity.

Figure 3 shows the migrated image obtained using exploding reflector data generated in a homogeneous medium. The migration was performed using a homogeneous velocity model. The migrations obtained using the wave equation and the Kirchhoff methods are visually identical.

Figure 4 shows the result obtained from using the wave equation imaging approach to image in the heterogeneous medium. The correct velocities were used during the migration. The resolution is comparable to that shown in Figure 3, obtained for the homogeneous medium. Figure 5 shows the image obtained using a first-arrival ray-based Kirchhoff migration approach. The size of the central dark spot in the image is comparable to that obtained by the wave equation approach; however, the diffraction pattern radiating from the image point has significant amplitude at locations well away from the image point.

Point Spread Functions

Figure 6 shows the range of scattering wavenumbers calculated using raytracing for the homogeneous medium. We assumed that the data have a bandwidth of 17 – 70 Hz, roughly that expected for a 35 Hz Ricker source. This plot is equivalent to the angular spectrum of the PSF for the homogeneous medium that is consistent with Equation (1). The spectrum of scattering wavenumbers is nearly identical to a wavenumber spectrum of the image in Figure 3, which confirms the relation between the wavenumber spectrum of the resolution and the scattering wavenumber spectrum for this simple case.

Figure 7. Angular spectrum of the PSF for wave-equation migration in the heterogeneous medium.
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The angular spectra of the PSFs for the images in heterogeneous media can be estimated by taking the 2D Fourier Transform of the images since there was only one source point when generating the exploding reflector dataset.

Figure 7 shows the angular spectrum of the PSF for the wave-equation migration approach. Note that the spectrum is not as rich as the one in Figure 6, which indicates that even wave-equation migration degrades the image resolution compared to what we would expect from that calculated from the acquisition process.

Figure 8 shows the angular spectrum of the PSF calculated for the first-arrival ray-based Kirchhoff migration in the heterogeneous medium. This PSF is clearly not as rich or smooth as the one obtained using wave-equation migration. There is some streaking of the angular spectrum of the PSF, which shows up as linear trends passing through the origin, caused by the existence of preferential scattering angles from the image point for rays that reach the surface, which is due to the heterogeneous velocity structure. This limits the range of scattering wavenumbers at the image point, thus limiting the resolution and clearly indicates why the image in Figure 5 is inferior to the one in Figure 4.

Conclusions

We have performed a numerical investigation of imaging resolution in a complex medium. We have found that the angular spectrum of the Point Spread Function can be a useful tool in assessing imaging resolution. We have shown that the angular spectrum of the PSF is different in real imaging scenarios than its estimate based only on acquisition. We have also shown that the PSF is dependent on the migration operator employed in a given imaging scenario and hence migration imaging resolution is a function of the operator used.

References


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