Scattering of elastic waves in 2-D VTI media
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Summary
The principal objective of this paper is to investigate the elastic wave field scattering in the 2-D transversely isotropic media with a vertical symmetry axis (VTI). We extend the elastic thin-slab method for elastic wave propagation from the isotropic media to the VTI media. The elastic thin-slab approximation is a dual-domain formulation for the wide-angle forward scattering and backscattering of the elastic wave field. The elastic property of a horizontal elastic thin-slab is decomposed into a homogeneous background medium with corresponding perturbations. We introduce anisotropy parameters into the elastic perturbations and obtain scattered anisotropic displacement and stress fields. The scattered stress fields act as subsource which are induced by the interaction of VTI perturbations with the background wavefield. We derive the formulation for the anisotropic scattered displacement of the qP and qSV wavefields in the 2-D elastic media. The simulation results of a simple model show that the anisotropic perturbations generate scattered waves. Numerical results also show that this method works well for complex VTI media such as the modified SEG/EAGE salt model.

Introduction
Scattering theory is well-known for seismic modelling and imaging (Aki and Richards, 1980; Wu, 1989; Wu, 1994; Wu, 1996). A historical review and recent progress in this research direction is presented in details by Wu (2003). Currently, the research of wave scattering methods is mainly for isotropic media. Extension of one-way propagation of scattered wave to anisotropic media has great potential in anisotropic modelling and imaging. In this paper we formulate the elastic wave scattering in the VTI media. By definition of homogeneous isotropic background and VTI perturbations, we derive the scattering stress and displacement fields contributed by each elastic parameter and density in a thin-slab. The formulae are represented in the dual-domain (spatial- and wavenumber-domain). The scattered wavefield is calculated in spatial domain, but wave propagates in the wavenumber domain. To test the scattering method, we present an example with a complex model. The model is constructed by modifying the complex SEG/EAGE Salt model from the isotropic media to the VTI media while keeping its original complex features. The multi-component primary scattering fields of both qP-wave and qSV-wave are recorded as shot gathers. Good capability of the algorithm for complexly structured media with very strong perturbations is illustrated with the results.

Method

Wave scattering in general anisotropic media
The total wave field consists of the incident field and scattered field, i.e. \( u(x) = u^0(x) + U(x) \). The parameters of the elastic media can decomposed as a background medium and perturbations, \( \rho(x) = \rho_0 + \delta \rho, c(x) = c_0 + \delta c \). The scattered displacement field for a thin-slab in dual-domain can be represented as (Wu, 1996),

\[
U(z^*, K_T) = \int_{z'}^z dz \int d^2 x_T [k \rho \omega^2 u_0(x_T, z) + \nabla \cdot \left( \delta c \cdot \epsilon^0(x_T, z) \right)] \cdot G_0(K_T, z^*; x_T, z)
\]

where \( z', z (z' < z) \) are slab entrance and exit of the wave fields, \((x_T, z)\) is any position in the slab, \( z^* \) is vertical position in vertical direction, \( K_T \) is wave number in tangential direction, \( \omega \) is angle frequency, \( \epsilon^0 \) is forward strain field, and Green’s function \( G_0 \) is scattered wave field from \( z \) to \( z^* \) in homogeneous background media,

\[
G_0(z^*, K_T; z, K_T) = G_0^p(z^*, K_T; z, K_T) + G_0^s(z^*, K_T; z, K_T)
\]

with

\[
G_0^p(z^*, K_T; z, K_T) = \frac{ik^p k^p}{2 \rho_0 \omega \gamma^p} k^p e^{ik^p r}
\]

\[
G_0^s(z^*, K_T; z, K_T) = \frac{ik^s k^s}{2 \rho_0 \omega \gamma^s} (I - \hat{k}^s \hat{k}^s) e^{ik^s r}
\]

where I is the unit dyadic, \( k_p = \omega / \alpha \) and \( k_s = \omega / \beta \), \( \alpha \) and \( \beta \) are velocities of P- and S-waves.

Wave scattering in VTI media
In this section, we derive the formula of scattered wavefield for VTI media from the general formula (1).

Scattered stress fields
The second term at right hand of formula (1) can be considered as an equivalent traction force induced by the elastic perturbations \( \delta c \). For elastic media, elastic parameters can be decomposed into isotropic background and anisotropic perturbations. The background tensor
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can be written as,
\[
c_0 = \begin{bmatrix}
\begin{array}{cccccc}
c_{11} & (c_{33} - 2c_{44}) & (c_{33} - 2c_{44}) & 0 & 0 & 0 \\
(c_{33} - 2c_{44}) & c_{11} & (c_{33} - 2c_{44}) & 0 & 0 & 0 \\
(c_{33} - 2c_{44}) & (c_{33} - 2c_{44}) & c_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & c_{11} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{11} & 0 \\
0 & 0 & 0 & 0 & 0 & c_{11}
\end{array}
\end{bmatrix}
\]

where , $c_{33} = \lambda + 2\mu$, $c_{44} = \mu$. The perturbation tensor is
\[
\delta c = \begin{bmatrix}
\begin{array}{cccccc}
\delta c_{11} & (\delta c_{33} - 2\delta c_{44}) & \delta c_{13} & 0 & 0 & 0 \\
\delta c_{33} & \delta c_{11} & \delta c_{13} & 0 & 0 & 0 \\
\delta c_{44} & \delta c_{13} & \delta c_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & \delta c_{11} & 0 & 0 \\
0 & 0 & 0 & 0 & \delta c_{11} & 0 \\
0 & 0 & 0 & 0 & 0 & \delta c_{11}
\end{array}
\end{bmatrix}
\]

The perturbation-induced stress field, $\delta \sigma$, includes stress fields of qP-, qSV- and qSH-wave modes: $\delta \sigma_{11}$, $\delta \sigma_{33}$, $\delta \sigma_{13}$ and $\delta \sigma_{44}$, where $n$ indicates the wave mode. For 2-D heterogeneous VTI media and ignoring the qSH-wave, we write the stress fields as
\[
\delta \sigma_{11} = \delta c_{11}(\nabla \cdot \mathbf{u}^0 - \mathbf{u}^0 \cdot \mathbf{e}_z),
\]
\[
\delta \sigma_{33} = \delta c_{33}(\mathbf{u}^0 \cdot \mathbf{e}_z),
\]
\[
\delta \sigma_{13} = \delta c_{13}(\mathbf{u}^0 \cdot \mathbf{e}_z),
\]
\[
\delta \sigma_{44} = \delta c_{44}(\mathbf{u}^0 \cdot \mathbf{e}_z),
\]
\[
\delta \sigma_{06} = 0,
\]
where $n = p$ for qP-wave, $n = s$ for qSV-wave, $\mathbf{e}_j$, $j = x, z$ is the unit vector, $\mathbf{u}^0 \cdot \mathbf{e}_j$, $j = x, z$ is the displacement and $\mathbf{e}_z$ is the strain tensor in the background media.

Scattered displacement fields

The scattered displacement field contributed by the density perturbation is same as that in isotropic media (Wu, 1996),
\[
U_p(K_x, z^*) = \int_{x} dz \int dx \delta(p(x, z)\omega^2 \mathbf{u}^0(x, z) \cdot \mathbf{G}_0(K_x, z^*; x, z)
\]
(5)
The elastic tensor induced scattering field is represented as
\[
U_{sc}(K_x, z^*) = U_{sc13}(K_x, z^*) + U_{sc33}(K_x, z^*) + U_{sc13}(K_x, z^*) + U_{sc44}(K_x, z^*),
\]
where
\[
U_{scm}(K_x, z^*) = U_{spcm}(K_x, z^*) + U_{spcm}(K_x, z^*) + U_{spcm}(K_x, z^*),
\]
where $m = 11, 33, 13, 44$. $U_{spcm}(l, n = p, s)$ indicates scattered l-wave from background n-wave, which computed by the following equation,
\[
U_{scp}(K_x, z^*) = \int_{x} dz \int dx (\nabla \cdot \mathbf{g}^{\text{sc}}_0(K_x, z^*; x, z)
\]
According to, Equations (3), (4), (5) and (6), we can calculate the scattered displacement fields. For example, the displacement of qP- and qSV-waves scattered by $c_{11}$ can be calculated by the following equations,
\[
U_{scp13}(K_x, z^*) = \int_{x} dz \int dx e^{iK_p(x-z)} f dx e^{ik_p x}
\]
(\nabla \cdot (\delta c_{11}(\mathbf{v}^0 - \mathbf{u}^0 \cdot \mathbf{e}_z)) \cdot (K^p k^p)
\]
\[
U_{scp33}(K_x, z^*) = \int_{x} dz \int dx e^{iK_p(x-z)} f dx e^{ik_p x}
\]
(\nabla \cdot (\delta c_{11}(\mathbf{v}^0 - \mathbf{u}^0 \cdot \mathbf{e}_z)) \cdot (K^p k^p)
\]
\[
U_{scp13}(K_x, z^*) = \int_{x} dz \int dx e^{iK_p(x-z)} f dx e^{ik_p x}
\]
(\nabla \cdot (\delta c_{11}(\mathbf{v}^0 - \mathbf{u}^0 \cdot \mathbf{e}_z)) \cdot (K^p k^p)
\]
Examples

The SEG/EAEG Salt model is a complex model with strong velocity perturbations, complex structure and faults. We modify it from the isotropic media to the anisotropic media and generate anisotropic scattered elastic wavefield. Keeping its original distribution of the compressional velocity, we introduce anisotropic parameters for the VTI model. The grid size of the model is 150 in depth and 645 in horizontal, and dx = dz = 80ft. We keep the salt body isotropic while modify its surrounding sediment and faults into the VTI media. The shear wave velocity is linear function of the compressional velocity, $\beta(x, z) = 0.6\alpha(x, z)$. The anisotropy of the model is characterized by Thomsen’s parameters which in this model are linear functions of the perturbations of the compressional velocity, i.e., $\epsilon(x, z) = 0.8\alpha(x, z) - \sigma_{\text{min}} / \sigma_{\text{max}}$,
$\gamma(x, z) = 0.64\alpha(x, z) - \sigma_{\text{min}} / \sigma_{\text{max}}$. The maximum values of the $\epsilon$ and $\delta$ are 0.31 and 0.39 respectively, and the their minimum values are 0. Figure 1 shows the VTI model for prestack imaging where the Figure 1(a) is the compressional velocity, the Figure 1(b) is the shear wave velocity, The Figure 1(c) is anisotropy $\delta$ and the Figure 1(d) the anisotropy $\epsilon$. Some shot-gathers are simulated based on the modified VTI salt model. The x-components and z-components of both the primary scattered compressional wave (P - P wave) and shear wave (P - SV wave) are shown in Fig. 2. Fig. 2(a) is x-component of scattered compressional wave (P - Pp), Fig. 2(b) is z-component of the compressional wave (P - Pp).Fig. 2(c) is x-component of shear wave (P - SVx) and Fig. 2(d) is z-component of shear wave (P - SVz wave). A Ricker wavelet is used as a compressional source which is located at 16kft of the model surface. The peak frequency of the wavelet is 8Hz. Recording length is 5.0s and sampling rate is 8ms. The scattered P - P wave has stronger energy than scattered P - S wave. The scattered energy is mainly from the boundary of the salt body which has strong contrasts in velocities and anisotropy. x-component and z-component of the full wavefield are as showed in Fig. 2(c) and Fig 2(f). Multi-scattering and direct arrivals are not recorded in this experiment.
Discussion and conclusions

In this paper we present a 2-D thin-slab method for elastic wave scattering in VTI media. Elastic scattering of stress and displacements excited by elastic parameters and density of the media is formulated in the elastic thin-slab. Scattering wavefield including mode-conversion is generated by the interaction of the background wavefield with perturbations. Scattering waves are propagated by elastic one-way propagators. The scattered wavefield excited by every perturbation acts as a subsource and propagates in every direction. In the example, we simulated elastic wavefield in 2-D complex VTI media. This method can also be applied to elastic wavefield imaging in VTI media, depth migration based velocity and anisotropy analysis. Extension from 2-D to 3-D is straightforward.

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References


Fig. 1: The modified SEG/EAEG Salt model in terms of compressional velocity, anisotropy $\delta$ and anisotropy $\epsilon$. The shear velocity at every point of the model is set equal to half of the compressional velocity. The heterogeneous anisotropy parameters have linear relations with respect to perturbation of compressional velocity: $\delta(x, z) = 0.80(\alpha(x, z) - \alpha_{\text{min}})/\alpha_{\text{max}}$, $\epsilon(x, z) = 0.64(\alpha(x, z) - \alpha_{\text{min}})/\alpha_{\text{max}}$. But the salt body keeps isotropic.
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Fig. 2: A shot-gather of the Scattered wavefield calculated with the modified SEG/EAGE salt model shown in Fig. 1. The source is located at 16 kft at the surface. Sample rate is 8 ms and sample number is 750. The peak frequency of Ricker wavelet is 8 Hz.