Applications of Prestack Beamlet Migration Using Gabor-Daubechies Frames

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Summary

Wave equation-based prestack beamlet migration using Gabor-Daubechies Frames is applied to both directional illumination (DI) and acquisition-aperture efficacy (AAE) analysis and to target-oriented imaging for the SEG-EAGE 2D salt model. Beamlet decomposition of wave fields with Gabor-Daubechies frame bears localization property in both space and direction, which is a great advantage for the studies in the two domains simultaneously. The analytic results of DI and AAE distributions and the high quality of images obtained in a target-oriented way demonstrate the great potential of G-D beamlet migration in local directivity-involved analysis and imaging.

Introduction

Prestack depth migration, which currently becomes a major application of seismic data processing, has made significant improvement in providing reliable high-resolution seismic images for complex structures. However, there remain some issues to be particularly considered. First, it is necessary to better understand various factors affecting image quality, such as acquisition geometry at the surface including both source and receiver apertures, approximation approach used in migration procedure, the influence of overlaying structures to the target area, etc. Second, prestack migration involves a huge amount of computations required in the extrapolation process for the prestack data, especially for 3-D surveys. Therefore, its computation efficiency should be improved largely to meet certain requirement. As for the former issue, illumination analysis in the target area is widely considered as a powerful tool to study the influences of acquisition aperture and overlaying structures. In the past, most techniques predicting illumination intensity distributions under certain acquisition geometries are based on ray tracing modeling (Muerdter et al., 2001a; 2001b; 2001c; Bear et al., 2000), some authors used FD (finite difference) modeling to consider the wave phenomena. However, both ray tracing and FD based illumination analyses can hardly provide reliable information on directional illumination, since the former violates the Heisenberg uncertainty principle, and the latter is a space domain solution of the wave equation with no direction resolution at all. As for the second issue, usually only a few specific structures are of great interest for imaging, therefore target-oriented migration with reasonable computational expenses becomes an attracting topic for current research. Rietveld & Berkhout (1994) proposed an efficient as well as accurate method to migrate prestack data in a target-oriented way by means of controlled illumination, which is a special case of the more general approach of “areal” shot-record migration first introduced by Berkhout (1992). Obviously, target-oriented migration also involves space-direction dual domain analysis since most target structures bear special directivity properties and require reliable direction information to get better image. Recently, Wu et al. (Wu et al., 2000; Wu & Chen, 2001) developed a wave equation-based beamlet migration method. Based on the localization property in both space and direction of wave field beamlet decomposition, they analyzed the directional illumination distributions and performed target-oriented prestack migration using Gabor-Daubechies frames and got encouraging results (Wu and Chen, 2002; Chen et al., 2002). In this study, the applications of G-D beamlet prestack migration is furthered studied to directional illumination (DI) and acquisition-aperture efficacy (AAE) analysis and to target-oriented imaging for the SEG-EAGE 2D salt model. The obtained results demonstrate the great potential of G-D beamlet migration in directivity-involved analysis and imaging.

Directional wave fields and angle-domain image matrix

For a 2D model, G-D beamlet decomposition of wave field can be expressed as (Wu et al., 2000; Wu and Chen, 2001):

\[ u(x, z, \omega) = \sum_{\alpha} \sum_{x} u_{\alpha}(z, \xi_j, \omega) g_{\omega x} \]

(1)

where \( u_{\alpha}(z, \xi_j, \omega) \) are beamlet coefficients, \( \omega \) is the circular frequency, \( g_{\omega x}(x - \xi_j) \) are G-D frame atoms with \( \xi_j = \Delta z, \xi_j = j\Delta_z \), and \( \Delta_z \Delta_x < 2\pi, g(x) \) is a Gaussian window function. As having been introduced by Wu and Chen (2002), a local plane wave is obtained through partial reconstruction of wave field (mixed domain wave field: local phase – space):

\[ u(x, z, \bar{\xi}_j, \omega) = e^{i\bar{\xi}_j x} \sum_{\alpha} g(\omega - \omega_{\alpha}) u_{\alpha}(z, \bar{\xi}_j, \omega) \]

(2)

Based on the wavenumber-angle relation \( \bar{\xi}_j = \sin^{-1}(\nu(x, z) / \omega) \), the local plane wave can be expressed in terms of its propagating angle as \( u(x, z, \bar{\xi}_j, \omega) \). We call such a partially reconstructed field a directional wave field. Substituting the directional incident wave field and the directional scattered wave field into the image condition, the angle-domain image matrix for each point can be calculated:

\[ \hat{n}(\theta_j, \phi_j, x, z) = \sum_{\bar{\xi}_j} \sum_{\nu} W_{\bar{\xi}_j}(\theta_j, \phi_j, \omega) W^{*}_x(x, \bar{\xi}_j, \omega) \]

(3)

where \( W_{\bar{\xi}_j}(\theta_j, \phi_j, \omega) \) and \( W^{*}_x(x, \bar{\xi}_j, \omega) \) are directional incident and directional scattered wave fields, respectively. Directional wave field and angle-domain image matrix will play a key role in the following analysis and imaging.

DI (Directional Illumination) and AAE (Acquisition-Aperture Efficacy) analysis
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With the above-mentioned directional wave fields obtained in (2), DI map, which illustrates the directional distribution of illumination energies from limited source aperture, and AAE matrix or acquisition-dip response (ADR) function, which accounts for the influence of both limited source and receiver apertures to the prestack image quality as well as the overlaying structure to the target area, can be constructed as proposed by Wu and Chen (2002). In this paper, a smoothly tapered window function is employed to distribute the illumination energy from a local incident plane wave within adjacent discrete angle bins, and energy summation instead of amplitude summation is used for calculating DI maps in a frequency band from single-frequency DI maps or multi-source DI maps from single-source DI maps:

\[
D_1(x, z, \bar{\theta}_j; x_s, \omega_0) = \left| G(x, z, \bar{\theta}_j; x_s, \omega_0) \right|
\]

\[
D_2(x, z, \bar{\theta}_j; x_s, \omega_0 \pm \Delta \omega) = \left[ \sum_{\omega_0 - \Delta \omega}^{\omega_0 + \Delta \omega} G(x, z, \bar{\theta}_j; x_s, \omega) \right]^{1/2}
\]

\[
D(x, z, \bar{\theta}_j) = \sum_{S_{1-4}} D_I(x, z, \bar{\theta}_j; x_s)
\]

where \( G \) is the Green’s function for the source \( x_s \) on the surface, \( N_s \) is the number of sources considered. AAE matrices and ADR can also be obtained in a simple way:

\[
AAE(\bar{\theta}_j, \bar{\theta}_s, x, z) = \left[ \sum_{S_{1-4}} N_s \sum_{j=1}^{N_s} D_I(x, z, \bar{\theta}_j; x_s) \right]^{1/2}
\]

\[
ADR(\bar{\theta}_s, x, z) = \sum_{\bar{\theta}_j} AAE(\bar{\theta}_j, \bar{\theta}_s, x, z)
\]

where \( N_{Rs} \) is the receiver number for each source, and \( \bar{\theta}_s = (\bar{\theta}_j - \bar{\theta}_s)/2 \), \( \bar{\theta}_j = (\bar{\theta}_j + \bar{\theta}_s)/2 \) are the normal of the dipping reflector and the reflection angle with respect to the normal respectively.

\[
\text{Fig.1 DI maps. (a) from a single shot at (271,0); (b) from a shot array of 40 shots at every other point from (431,0) to (509,0).}
\]

\[
\text{Fig.2 AAE album from 325 shots with 176 left-side receivers for each shot. (a) horizontal dip; (b) +30° (down from horizontal); (c) +45° (down from horizontal); (d) -30° (up from horizontal); (e) -45° (up from horizontal); (f) total response intensity.}
\]

\[
\text{Fig.3 AAE matrices for one shallow point (A) and four deep points (B, C, D, E) from all the 325 shots with 176 left-side receivers (upper panels) and 351 two-side receivers (lower panels) for each shot, respectively.}
\]

Target-oriented G-D beamlet prestack migration
As for the SEG-EAGE model, the subsalt structures, especially the steep faults, are mainly concerned for various purposes. Taking the three subsalt steep faults as the specific target, three types of sources are used: natural point sources (individual shots), G-D frame-based beam sources, and plane sources. The principle and procedures of synthesis and migration of G-D beam sources and plane sources for wave-equation based imaging methods have been introduced and tested previously (Wu et al., 2002). Target-oriented prestack migration is conducted by the selection of illuminating sources or/and the local angle image matrix elements before stacking.

**Target illumination-based partial source migration** Since usually only a few specific structures are of great interest in migration, and the target area is relatively small compared with the whole model, it is expected that the target can be well illuminated and imaged with higher computation efficiency using only part of the sources and the data. As for the subsalt steep faults of the SEG-EAGE salt model, it is obvious that only the shots or beam sources located at the right side of the surface, or the left-propagated beam sources or plane sources can provide effective illumination to the target. With the target-illuminating sources, prestack migrations are performed and the images are shown in Fig.4a, 4c and 4e, respectively. Among them Fig.4a is obtained by common-shot migration with the space-limited shots, while for beam source migration, G-D beam sources are controlled in both space and direction to generate the final image (Fig.4c). Plane sources, on the other hand, occupy full aperture in space but can provide various directional illuminations to the image space. Therefore the target image with good quality is obtained by direction-controlled plane sources as shown in Fig.4e. For comparison, we give the subsalt residual images in Fig.4b (image using other shots), Fig.4d (image using other beam sources), and Fig.4f (image using other plane sources), respectively. From Fig.4, it can be seen obviously that the subsalt steep faults can be fully imaged by the selected shots, beam sources or plane sources with reduced background noises, while other sources almost have no contribution to the target imaging.

**Superposition of contributing local angle images** As mentioned in the first section, imaging of prestack beamlet migration can be performed in the mixed domain to construct the angle-domain image matrix \( I(\theta_1, \theta_2, x, z) \). As given in (3), \( I(\theta_1, \theta_2, x, z) \) is different from that defined in the previous study (Chen et al., 2002) where the image matrix is expressed in terms of local wavenumber pairs which is a direction-blurred version of \( I(\theta_1, \theta_2, x, z) \). The local angle image matrix can be obtained either by full prestack migration or by target-illuminating partial source migration. It measures the contributions from different directional incident – scattered field pairs to the final image.

It can be outputted in terms of different incident – scattering angle pairs to construct an image album. Fig.5 shows the subsalt image album for some angle pairs \((\theta_1, \theta_2)\) using the target-illuminating beam sources. Besides each image gives the propagating directions of the corresponding incident-scattered field pair. From Fig.5, we can see that the images with different \((\theta_1, \theta_2)\) pairs reveal different directivity features. The three subsalt steep faults can be seen more clearly in Fig.5b, 5c and 5d, while can hardly be recognized in the bottom four panels (5e – 5h). It is obvious that only vertical or left-propagated incident and scattered fields have contributions to the image of the subsalt structures, especially the target of steep faults. Based on this observation, we superpose the local angle images of only the contributing angle pairs, i.e. incident angles \( \theta_1 \leq 0 \) and scattering angles \( \theta_2 \leq 0 \), resulting in noticeable improvement on the image of the steep faults as shown in Eq.6c. For comparison, Fig.6a and Fig.6b give the subsalt images by full beam-source migration and target-illuminating beam-source migration, respectively. From the three images, we can see that both target-illuminating beam source migration and structure-based superposition of local angle images can provide improved image qualities for the target by enhancing the effective signals and suppressing background noises. Similar results can be obtained using individual shots or plane sources instead of beam sources.
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Conclusions

Directional illumination and acquisition-aperture efficacy analysis as well as target-oriented prestack imaging are conducted based on beamlet wave field decomposition and propagation using Gabor-Daubechies frames for the SEG-EAGE 2D salt model. Beamlet decomposition provides localizations in both space and direction of the wave field, and is more flexible and accurate compared with the traditional illumination analysis methods. The analytic results of DI and AAE distributions can be further used to do aperture correction for the purpose of improving the image quality, and the prestack migration in a target-oriented way may result in both better image quality and high computation efficiency.

References


Acknowledgements

The supports from the WTOPI (Wavelet Transform On Propagation and Imaging for seismic exploration) Project and the DOUBES Project at University of California, Santa Cruz are acknowledged.