AVO modeling using one-return approximation
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Summary
In this paper we apply elastic thin-slab propagator, which is based on one-return approximation and scattering theory, to AVO forward modeling. For medium perturbation of about 20%, such as shale/oil sand and shale/gas sand cases, the reflection coefficients are accurate up to 45° incident angle for P waves and 20° incident angle for S wave, which can meet the requirement in most AVO analyses. Numerical results show that thin-slab propagator can easily handle thin-bed effects as well as lateral variations in lithology.

Introduction
As early as in 1979s, seismic amplitude variation with offset (AVO) analysis (‘bright spot’) had become common in petroleum exploration based on the fact that high-intensity seismic reflections may be indicators of hydrocarbon accumulations, particularly gas. The theoretical basis for AVO analysis has been the Zoeppritz equations for the calculation of reflection coefficients by a single interface with different medium impedances (Ostrander, 1984; Shuey, 1985). Today, there are some attempts to use AVO analysis for quantitatively estimating lithologic characters such as Poisson’s ratio, porosity, saturation, etc., and fluid contents (Custagoni, 2001). However, we now known that conventional AVO modeling and analysis based on the Zoeppritz equations (primaries-only) only work for a single interface, or for thick layers compared with dominant wavelength. For a thin layer (thickness less than one quarter dominant wavelength) separating two infinite half spaces, conventional AVO modeling gives even opposite amplitude variation with offset (Simmons and Backus, 1994). Reflectivity method is the most common method used for modeling AVO response in layered media (Widess, 1973; Lin and Phair, 1993; Simmons and Backus, 1994; Martinez, 1993). It can generate the exact AVO response in arbitrarily layered medium. But it cannot be used for modeling the effects of lateral variations, i.e., inhomogeneities, on AVO response, which may be the key to detecting hydrocarbons. Adriansyah and McMechan (1998) used pseudospectral method to investigate the effects of lateral variations on AVO response. Youn, et. al. (1998) investigated the effects of salt (as overburden) and subsalt (gas sand) on AVO response using finite difference method. However, they are very time-consuming and memory-demanding. They are also difficult to handle thin layers where fine grids must be utilized. Thus, developing new efficient AVO modeling algorithm in complex reservoir structure becomes highly desirable.

Elastic wave propagator method based on one-return approximation has been developed by Wu (1994, 1996) and applied to model reflections (Xie and Wu, 1996; Wu and Wu, 1999). It is an efficient numerical simulation method in both computation speed and memory requirement. It neglects all reverberations but can handle common-type waves as well as single-leg and double-leg converted waves within a layer. Compared with reflectivity method, it can handle arbitrarily lateral inhomogeneities. In this paper, we propose to use one-return elastic wave propagator for modeling the effects of thin layers as well as lateral inhomogeneities on AVO responses.

Elastic thin-slab propagator
First, the formulas for calculating elastic wave propagation in heterogeneous media based on one-return approximation is briefly summarized. The formulas shown here are based on the derivation of Wu (1994, 1996) and the phase matching improvement (Wu and Wu, 1999). Assume that the heterogeneous slab is thin enough so that velocity and density can be approximated as vertical homogeneous within each thin slab. For 2-D case, the scattered fields can be expressed as

\[ U^P(k_x, z) = \frac{k_{\alpha}^0}{2\gamma_\alpha} e^{i\frac{\Delta z}{2}} \Delta z \hat{k}_\alpha \{ \hat{k}_\alpha \}
\]

\[ \int dx e^{-ik_x x} \frac{\rho(x)}{\rho} \left[ \eta^{PP} u^P_p(x) + \eta^{SP} u^S_p(x) \right]
\]

\[ - \int dx e^{-ik_x x} \frac{\delta x}{\lambda + 2\mu i k_\alpha} \left[ \eta^{PP} u^P_s(x) - (\hat{k}_\alpha \hat{k}_\beta) \right]
\]

\[ \int dx e^{-ik_x x} \frac{\delta x}{\lambda + 2\mu i k_\beta} \left[ \eta^{PS} u^P_s(x) + \eta^{PS} u^S_s(x) \right]
\]

\[ U^S(k_x, z) = \frac{k_{\alpha}^0}{2\gamma_\beta} e^{i\frac{\Delta z}{2}} \Delta z \{ I - \hat{k}_\beta \hat{k}_\alpha \}
\]

\[ \int dx e^{-ik_x x} \frac{\delta x}{\mu} \left[ \eta^{SP} u^S_p(x) + \eta^{SP} u^P_s(x) \right] - \hat{k}_\beta \cdot \hat{k}_\alpha
\]

\[ \int dx e^{-ik_x x} \frac{\delta x}{\mu} \left[ \eta^{PS} u^S_p(x) + \eta^{SP} u^S_s(x) \right]
\]

with \( k_\alpha^0 = +\gamma_\alpha \) and \( k_\beta^0 = +\gamma_\beta \) for forescatterings and \( k_\alpha^0 = -\gamma_\alpha \) and \( k_\beta^0 = -\gamma_\beta \) for backscatterings. In the above equations, \( I \) is the unit dyad, and \( u^P_p(x) \), \( u^S_p(x) \) (displacement), \( \Delta \cdot u^P_s(x) \), \( \varepsilon^P_s(x) \) and \( \varepsilon^S_s(x) \) (strain) can be calculated by

\[ u^P_p(x) = \frac{1}{2\pi} \int dk' e^{ik'_p x} u^P_p(k'_p) e^{i\gamma_\alpha \Delta z/2}
\]
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\begin{align*}
\nabla \cdot u^p_d(x) &= \frac{i k^0_d}{2 \pi} \int dk'' e^{i k''_d x} k'_d \cdot u^0_d(k'_d) e^{i \gamma_d \Delta z/2} \\
\varepsilon^p_d(x) &= \frac{i k^0_d}{2 \pi} \int dk'' e^{i k''_d x} k'_d \cdot u^0_d(k'_d) e^{i \gamma_d \Delta z/2} \\
u^p_d(x) &= \frac{1}{2 \pi} \int dk'' e^{i k''_d x} u^0_d(k'_d) e^{i \gamma_d \Delta z/2} \\
\varepsilon^p_d(x) &= \frac{i k^0_d}{2 \pi} \int dk'' e^{i k''_d x} \frac{1}{2} \left[ k''_d u^0_d(k'_d) + u^0_d(k'_d) k''_d \right] e^{i \gamma_d \Delta z/2}
\end{align*}

where \( \gamma_a, \gamma_b, \gamma'_a \) and \( \gamma'_b \) are the vertical components of scattered and incident wavenumbers. \( k_a \) and \( k'_a \) are corresponding transverse components. \( k^0_a = \omega/\alpha \) and \( k^0_b = \omega/\beta \) are P and S wavenumbers, respectively, and \( \alpha \) and \( \beta \) are P and S wave velocities in the background medium. \( \lambda_0, \mu_0 \) and \( \rho_0 \) are the Lamé constants and density for background medium, and \( \delta \lambda, \delta \mu \) are the corresponding perturbations. \( \tilde{k}_a \), \( \tilde{k}_b \), \( \tilde{k}'_a \) and \( \tilde{k}'_b \) are unit wavenumber vectors. \( u^0_d(k'_d) \) and \( u''_d(k''_d) \) are the incident P and S wave displacements at the entrance \( z = z^0 \), respectively. \( z = z_1 \) is the thin slab exit. The slab thickness is \( \Delta z = z_1 - z' \). The factors \( \eta^{PP}, \eta^{SS} = \eta^{PS} \) and \( \eta^{PS} \) are calculated plane wave reflection coefficients at a single interface. The background medium parameters are \( \lambda_p = 3.6 \text{ km/s}, \mu_p = 2.08 \text{ kPa}, \rho = 2 \text{ g/cm}^3 \). The interface is with contrasts of -20% and +20% for both P wave and S wave velocities. The model is defined on a 20x40 rectangular grid. The grid spacing in horizontal direction is 16m and that in vertical direction is 4m. A plane P wave (or S wave) at frequency 15Hz and certain angle is incident on the interface. Then we can calculate the reflected/converted waves using one-return approximation. For stability, we pick up 500 samples (displacement amplitude in space domain) for both incident wave and reflected waves in the middle of the model to avoid the edge effects. We take the averaged displacement amplitudes to calculate the reflection coefficients. Figure 1 shows the reflection coefficients for P wave incidence. The top panel corresponds to +20% perturbation and the low panel to -20% perturbation. The theoretical values (dotted) are given and used as references. We see good agreements between thin-slab propagator and the exact solution up to large scattering angles which approach the critical angles. For critical angles, there exists a singularity in the scattered formulae, which may be removed by other approximate approaches such as Pade expansion. We will further improve wide-angle capability of thin-slab propagator in future study. Figure 2 shows the reflection coefficients for S wave incidence. The top panel corresponds to +20% perturbation for both P and S wave velocities and the low panel to -20% perturbation. Similar results as for P wave incidence can be seen from Figure 2. For most cases where the formation is composed of shale, oil/gas sand or brine sand, velocity and density perturbations are about 20%. Under such perturbation, the reflection coefficients are accurate up to 45° incident angle for P wave.

Numerical examples and discussion

Reflection coefficients at a single interface

For investigating accuracy and wide-angle capacity of thin-slab propagator applied to AVO modeling, we first calculate plane wave reflection coefficients at a single interface. The background medium parameters are \( \lambda_p = 3.6 \text{ km/s}, \mu_p = 2.08 \text{ kPa}, \rho = 2 \text{ g/cm}^3 \). The interface is with contrasts of -20% and +20% for both P wave and S wave velocities. The model is defined on a 20x40 rectangular grid. The grid spacing in horizontal direction is 16m and that in vertical direction is 4m. A plane P wave (or S wave) at frequency 15Hz and certain angle is incident on the interface. Then we can calculate the reflected/converted waves using one-return approximation. For stability, we pick up 500 samples (displacement amplitude in space domain) for both incident wave and reflected waves in the middle of the model to avoid the edge effects. We take the averaged displacement amplitudes to calculate the reflection coefficients. Figure 1 shows the reflection coefficients for P wave incidence. The top panel corresponds to +20% perturbation and the low panel to -20% perturbation. The theoretical values (dotted) are given and used as references. We see good agreements between thin-slab propagator and the exact solution up to large scattering angles which approach the critical angles. For critical angles, there exists a singularity in the scattered formulae, which may be removed by other approximate approaches such as Pade expansion. We will further improve wide-angle capability of thin-slab propagator in future study. Figure 2 shows the reflection coefficients for S wave incidence. The top panel corresponds to +20% perturbation for both P and S wave velocities and the low panel to -20% perturbation. Similar results as for P wave incidence can be seen from Figure 2. For most cases where the formation is composed of shale, oil/gas sand or brine sand, velocity and density perturbations are about 20%. Under such perturbation, the reflection coefficients are accurate up to 45° incident angle for P wave.
Figure 2: Reflection coefficients for S wave incidence. The top panel corresponds to +20% velocity perturbation for both P and S waves and the low panel to -20% velocity perturbation for both P and S waves.

Table 1. Oil sand

<table>
<thead>
<tr>
<th>Material</th>
<th>V_p (m/s)</th>
<th>V_s (m/s)</th>
<th>Density (g/cm^3)</th>
<th>Thickness (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>shale</td>
<td>3170</td>
<td>1668</td>
<td>2.36</td>
<td>1500</td>
</tr>
<tr>
<td>oil sand</td>
<td>3734</td>
<td>2280</td>
<td>2.27</td>
<td>5</td>
</tr>
<tr>
<td>shale</td>
<td>3170</td>
<td>1668</td>
<td>2.36</td>
<td>—</td>
</tr>
</tbody>
</table>

and 20° incident angle for S wave, which can meet the requirement in most AVO analyses.

Thin layer AVO response

A single thin-bed AVO response has been simulated by many authors. An efficient numerical simulation algorithm for AVO modeling must easily handle thin-bed effects. To test our algorithm for AVO modeling, we perform the same simulation and compare the results with reflectivity method. Table 1 shows the rock properties of an oil sand model (Simmons and Bacus, 1994). The thin-bed is 5m thick and located at the depth of 1500m. Source and receivers are on the surface. The Ricker wavelet used has the dominant frequency of 30Hz. All seismograms shown in this paper are the vertical component of displacement and NMO corrected to the top of the sand. To use thin-slab propagator, the spacing grid in horizontal direction is 10m and that in vertical direction is 10m for zero perturbation area and 0.5m for wave propagating through the thin-bed.

Figure 3 shows seismograms calculated by reflectivity method. (A) corresponds to reflections by a single shale/sand interface, (B) to reflections by a thin-bed but no converted shear waves included, (C) to all waves included. Comparing (A) and (C), we see big difference in amplitude variation with offset by a single impedance interface and by nearly close two impedance interfaces. Comparing (B) and (C), we see the important effect of locally converted shear waves on AVO response.

Figure 4 shows similar results as given in Figure 3 but calculated by thin-slab propagator. Comparing Figure 4 and Figure 3, AVO responses agree fairly well. Large angle waves corresponding to offset of 3km in Figure 4 are also predicted correctly. Figure 3C is an exact solution, while Figure 4C is obtained based on one-return approximation where all reverberations are neglected. This example shows that one-return thin-slab propagator can handle thin-bed effect in AVO modeling.

AVO response in laterally varying media

Compared with reflectivity method, one of the advantages of thin-slab propagator is that it can easily handle wave propagation in laterally varying media. Figure 5 shows a gas sand model with lateral heterogeneities (salt). Gas sand has parameters: \( V_p = 3.56 \text{km/s}, \ V_s = \ldots \)
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![Diagram](attachment:image.png)

**Figure 5:** A gas sand model with lateral heterogeneity.

![Diagram](attachment:image.png)

**Figure 6:** Seismograms calculated by thin-slab method for a gas sand model with different thickness salt overburden. (A) without salt, (B) salt thickness 5m, (C) 20m, (D) 100m and (E) 250m.

2.374km/s and $\rho = 2.1g/cm^3$ and salt has parameters: $V_p = 4.48km/s$, $V_s = 2.394km/s$ and $\rho = 2.1g/cm^3$. The spacing grids taken are the same as used in Figure 4 except for the spacing grid in salt body in vertical direction where it is 0.1m because of high velocity perturbation of salt body. Figure 6 shows seismograms calculated by thin-slab propagator. (A) corresponds to without salt, (B) to (E) correspond to salt thickness 5m, 20m, 100m and 250m, respectively. Comparing Figure 6A and Figure 4A, amplitude variation with offset for an oil sand is much different from that for a gas sand. From Figure 6B, for thin salt layer far away from target layer, thin salt layer doesn’t affect AVO response of target layer. An event following the gas sand response is the converted shear wave produced at the top interface of gas sand, and then converted back to P wave through the salt. When salt is close to the gas sand, this event will affect AVO response. So does primary reflection by salt. As salt thickness increases, the AVO response of target layer becomes more complex. The effects of more complicated medium model on AVO response will be investigated in the future.

**Conclusions**

Elastic thin-slab propagator based on one-return approximation and scattering theory, can be applied to AVO forward modeling for handling the effects of thin-bed and lateral inhomogeneities on AVO response. For medium perturbations of about 20%, such as shale/oil sand and shale/gas sand cases, the reflection coefficients are accurate up to 45° incident angle for P waves and 20° incident angle for S waves, which can meet the requirement in most AVO analyses.

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**References**


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