Depth migration using the windowed generalized screen propagators

Shengwen Jin* and Ru-Shan Wu, University of California, Santa Cruz

Summary

Windowed generalized screen propagators are applied to poststack and prestack depth migrations. For the SEG/EAGE salt model, the velocity distributions are windowed into a few blocks with large overlapping areas. By combining the windowed Fourier transform and the interpolation technique, the method improves both the accuracy and computational efficiency. Comparisons with the phase-screen and the Kirchhoff migration methods demonstrate that the windowed prestack migration has the highest potential for subsalt imaging in the case of strong velocity contrasts.

Introduction

Depth migration is an important step in identifying the potential targets for oil and gas exploration. Strong-contrast heterogeneous media present a great challenge to the migration methods. Many standard methods whic h work well in weak-contrast media often fail in this case. The Kirchhoff approach, the most commonly used method for prestack depth migration, relies on the high-frequency approximation of ray-tracing. It may face the problems of caustics, multiple arrivals, shadow zones, and even chaotic rays (Fei et al., 1996; Audibert et al., 1997). More accurate ray-tracing that takes into account multi-pathing and computes the correct amplitudes of each arrival is computationally more expensive and more difficult to implement. Wave equation based methods can avoid these difficulties. Full (two-way) wave equation (Başal et al., 1985) are capable of solving wave solutions for arbitrary complex media but are time consuming. One-way methods based on the paraxial wave equation are more efficient but are difficult to improve the angular accuracy (Claerbout, 1985; Hale, 1991; Nautiyal et al., 1993). Higher-order approximations are more accurate, but are also more costly. Methods in frequency-wavenumber domain, such as phase-shift (Gazdag, 1978) and phase-shift plus interpolation approaches (Gazdag and Sguazzero, 1984), generally require smooth variation of the lateral velocity.

The screen propagators, including split-step Fourier method (Stoffa et al., 1990), phase-screen and pseudo-screen methods (Wu, 1994, 1996; Huang and Wu, 1996; de Hoop et al., 1998), work well for weak-contrast media. In the case of strong-contrast media, such as the saltdome related structures where velocity of the salt can be 3-4 times higher than the surrounding media, how ever the method of windowing is only used for accurate small angle incident waves and produces large errors for wide-angle waves. Multiple reference velocity methods (Kessinger, 1992; Huang and Fei, 1997) can improve significantly the performance in this case, but with increased computations due to the multiple global Fourier transforms involved. Windowed screen propagator (Wu and Jin, 1997) in which the windowed Fourier transform is used, can overcome the shortcoming of the global background velocity and still keep many advantages of the screen methods. For the salt model, the velocity distributions can be windowed into a few blocks with large overlapping areas. In those areas, w e modif i the window weighing in to an in terpolation summation.

In this work we will apply the method in poststack and common-shot prestack depth migration of the SEG/EAGE salt model and further explore the advantages of the method by comparing with those of the phase-screen and Kirchhoff methods.

The windowed screen propagator

Starting from the acoustic wave equation and taking the screen approximation, the dual-domain expression for the forward scattered wavefield $P_s(K_T, z_{i+1})$ at the extrapolated depth $z_{i+1}$ is

$$P_s(K_T, z) = \frac{i}{2\gamma} k^2 \Delta x e^{i\gamma z_0} FT_{K_T} [S(x_T, z_0)p(x_T, z_0)]$$

(1)

where $FT_{K_T} […]$ denotes the 2D Fourier transform over $x_T$ and $K_T = (K_x, K_y)$ is the transverse wavenumber. $k = \omega/\nu_0(z)$ is the wavenumber in background medium with the reference velocity of $\nu_0(z)$, $\gamma$ is the vertical wavenumber with $\gamma = \sqrt{k^2 - K_T^2}$ and $S(x_T, z_0)$ is the slowness perturbation within the screen. The total wavefield $P(K_T, z)$ at depth $z_{i+1}$ is then calculated by the sum of the scattered wavefield and the primary wavefield $P_0(K_T, z)$, which propagate in the background medium. Under the small angle approximation, we obtain the phase-screen propagator

$$P(K_T, z_{i+1}) = e^{i\gamma z_0} FT_{K_T} [e^{ikl(x_T, z_1)} \Delta x e^{i\gamma z_0} p(x_T, z_0)]$$

(2)

In the case of strong-contrast heterogeneous media, the global nature of the background velocity selection makes the method less accurate for steep dip reflectors. The windowed Fourier transform can be applied to the screen propagator to overcome these shortcomings.
The representation of windowed Fourier transform is (Kaiser, 1994)
\[
F(x, k_x) = \int g(x - \tilde{x}) f(\tilde{x}) e^{-i k_x \tilde{x}} d\tilde{x}
\] (3)

where, \( f(x) \) is the original function, \( F(x, k_x) \) is its windowed transform, \( \tilde{x} \) is the spatial variable of input function, \( k_x \) is the transformed wavenumber, and \( g(x) \) is the window (or weighting) function.

If \( g \) is chosen to be a Gaussian function
\[
g(x) = \left( \frac{\Omega}{\pi} \right)^{\frac{1}{4}} e^{-\frac{x^2}{2 \Omega}} \] (4)

then the uncertainty relationship between \( \Delta x \) and \( \Delta k_x \) is minimized. The value of \( \Omega \) gives the width of the Gaussian and determines the resolution of the transform. When \( \Omega = 1 \), it reduces to the Gabor transform.

Since the basis functions of this transform are non-orthogonal, the inverse windowed Fourier transform is non unique. Here we choose the form of
\[
f(x) = \frac{1}{2\pi} \int g^*(x - \tilde{x}) F(\tilde{x}, k_x) e^{ik_x \tilde{x}} d\tilde{x} d\theta \] (5)

where \( g^* \) is the complex conjugate of \( g \).

The dual domain implementation of the windowed screen propagator can then be expressed by the following formulation for each step in the propagation direction (here, along the \( z \)-axis)
\[
P(x_T, z_{i+1}) = \int dK_T g^*(x_T - \tilde{x}_T) \frac{1}{2\pi} \int dK_T \\text{e}^{i K_T \tilde{x}_T} g(\tilde{x}_T - x_T \text{e}^{i K_T x_T}) \frac{\text{e}^{-i K_T x_T}}{2\pi} \int d\tilde{x}_T \text{e}^{i K_T \tilde{x}_T} p(x_T, z_i) \] (6)

where \( g(x_T) \) is the window with its norm \( \|g\| = 1 \), \( \Delta \gamma \) is the wavenumber perturbation corresponding to the interaction of wavefields with the heterogeneities and can be approximated with difference schemes. For the phase-screen approximation, it is simply the slowness perturbation multiplied by the frequency.

**Applications to depth migration for SEG/EAEG salt model**

A 2D slice, profile A-A' of the 3D SEG/EAEG salt model (O'Brien and Gray, 1996), is used to test the accuracy of the windowed screen propagator migration and to compare with other methods.

First we do depth migration on the zero-offset synthetic data. As we discussed before (Wu and Jin, 1997), the high-velocity and irregular shape of the salt body cause some difficulties in obtaining good image quality. The traditional phase-screen method works well for weak contrast media. In this case, however, it's only accurate for nearly vertical incidence waves. Also the choice of reference velocity will have a significant impact on imaging accuracy. Figure 1(a) shows a phase-screen migration result with the minimum velocity as the reference velocity. We see that strong migration noises are present due to the erroneous phase corrections for large angle waves and the interferences of different wavefronts. It could be difficult to interpret the true steep steep dip structures. Figure 1(b) is the depth image with the same phase-screen method, but with an average reference velocity in each depth level. The noise seems to be attenuated, but some portion of energy is lost for the salt flank. The image is also distorted at the top of salt. Because of the choice of the average reference velocity wave modes with large wavenumbers that are associated with the steep dip events are suppressed.

The result using the windowed screen method is shown in Figure 1(c). Not only most of the subsalt structure is reconstructed clearly, but also the steep reflectors, including the steeply dipping interfaces and the faults, are correctly imaged. In contrast, for the Kirchhoff migration method shown in Fig.1(d), almost all steep subsalt structures are missing. The base of the salt is not continuous. The background noise from incorrectly migrated fields are also strong beneath the salt body. Furthermore, some false faults appear due to the multi-pathing arrivals and coherent noises.

The second example is the prestack depth migration on the SEG/EAEG salt model. The synthetic data set consists of 325 shot gathers. For each shot, there are 176 receivers at 80 ft spacing, with the spread from 14000 to 0 ft (negative \( v \) means the location of receivers left to the shotpoint). The source position ranges from 0 to 51840 ft with its increment of 100 ft. The sampling interval is 8 ms with 626 samples per trace.

The Kirchhoff wavelet is used in the migration as a source function with dominant frequency of 12 Hz. The offset range used here is 0-14000 ft. Figure 2(a) shows the phase-screen migration result with the reference velocity being the minimum at each depth level. The left salt flank and the subsalt structures are not well positioned. Furthermore, strong noises exist beneath the salt. When the average reference velocity is chosen, the subsalt structures are better imaged, however, the image at the top of salt is distorted (Fig. 2b). Figure 2(c) is the result by our windowed screen method. Most structure elements are reasonably clear and correctly positioned. It is evident that the image is much better than the top two panels of figure 2. No w let's see the result of the common-offset Kirchhoff migration done by Michael O'Brien and Samuel Gray. First arrival traveltime was used in the method. As shown in the bottom panel of figure 2, strong noises also exist beneath the salt, while the flat layer at the base of the model is
not appeared, since the ray-tracing fails to give either the maximum-energy arrivals or those of other multiple arrivals in complex structures.

Conclusions

A windowed screen propagator is applied to poststack and prestack depth migrations. By use of the windowed Fourier transform, the method overcomes the shortcoming of the global nature of the background velocity and still keeps many advantages of the screen method. It works well in poststack and prestack depth migration for the SEG/DAEG salt model data where strong-contrast velocities exist between the salt body and the surrounding medium. Comparisons with the conventional phase-screen and the Kirchhoff methods demonstrate that the windowed screen method has great potential for subsalt imaging and other strong-contrast structure imaging.

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References


Figure 1: Poststack depth migration on the SEG/DAEG salt model. (a) phase-screen method with the minimum reference velocity, (b) is same as (a) but with the average reference velocity, (c) windowed screen method, and the bottom panel is from the Kirchhoff migration method (Courtesy of Michael O’Brien).


Figure 2: Prestack depth migration on the SEG/EAGE salt model. (a) phase-screen method with the minimum reference velocity. (b) same as (a) but with the average reference velocity. (c) windowed screen method, and the bottom panel is from the Kirchhoff migration method (Courtesy of Michael O’Brien).

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