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Preface

Extra dimensions – Extra benefit: Working in the local angle domain

This is the twelfth technical report of the “Modeling and Imaging Project” from the Modeling and Imaging Laboratory (MILAB), Center for Study of Imaging and Dynamics of the Earth (CSIDE), Institute of Geophysics and Planetary Physics (IGPP), University of California at Santa Cruz (UCSC).

This year’s technical report is devoted to the development and applications of the “local angle domain”. Comparing to the traditional scalar image field, the image in local angle domain has extra dimensions (local phase-space). The migrated image is no longer a scalar quantity at each point, but becomes a multi-dimensional matrix. Each matrix element represents an image from a local scattering experiment with an incident-scattering angle pair. This angle pair can be also transformed into a dip-reflection angle pair. For the 2D case, the matrix is two-dimensional, while in 3D, four-dimensional. Certainly there is an extra cost for the calculations with these extra dimensions, but we have shown that the extra benefits working in these extra dimensions are much more rewarding and are worth the extra cost. We show also that the conventional common image gathers (CIG) that use only half dimensions of the image matrix in local angle domain, are not enough to get the full benefits.

In the following, we list the developments in theory and applications related to the local angle domain:

- Local scattering matrix: It is defined as the scattering amplitude obtained by local scattering experiments. It is the intrinsic property of the local heterogeneity.
- Local image matrix: It is obtained during the imaging process in local angle domain and serves as the basis of amplitude correction, velocity updating and other operations in the local angle domain.
- Local aperture efficacy matrix (Local illumination matrix): It is the basis for illumination analysis for a given acquisition system.
- Local amplitude correction matrix: It is the basis of amplitude correction, including the acquisition aperture correction, for the purpose of true-reflection imaging.
- Local resolution matrix (Point spreading function): It is the basis of resolution analysis.

From these matrices, different gathers (vectors), such as the common dip-angle gather, common scattering-angle gather, common incident-angle gather, etc., can be obtained by summing up the matrix elements along the specified lines. These gathers can be used for different purposes. However, many benefits will be lost if only gathers are used. In order to get the full benefit, we have to start from the matrices, such as the local image matrix, aperture matrix, amplitude correction matrix, etc. These will be explained later in the papers included in this volume.

The volume is mainly composed of four parts:
Part I devotes to the true amplitude, true reflection imaging (six papers). The first three papers are on the amplitude corrections, especially the acquisition aperture correction. The last three papers are related to the true amplitude propagators used in the imaging process.

Part II is on the velocity updating in the local angle domain, including the method of residual migration, and the Gaussian beam method. We have also included our preliminary result of waveform tomography in this part. There are totally four papers in this part.

Part III deals with resolution analysis (three papers). The resolution matrix is defined based on the inverse theory, and includes both the effects of acquisition apertures (spatial and frequency apertures) and the influence of propagator inaccuracy.

Part IV reports some progress in beamlet migration (two papers).

We have some other work related to wave propagation in random media, complex crustal waveguides etc, included in Part V.

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When review this year’s research, we are extremely grateful to the above-mentioned funding agencies, and to our sponsoring and supporting companies of the WTOPI-Phase III: Anardaco, BHP, BP America, ChevronTexaco, ConocoPhillips, CNPC/GS, ExxonMobil, GDC, Geotrace, Norsk-Hydro, Paradigm Geophysical, Shell, SITI, SUN Microsystems, TotalFinaElf, Unocal and Veritas. We welcome also some new members of the Consortium (in the process).

Help and support from our collaborators at Los Alamos National Laboratory and other industrial partners are highly appreciated. The facility support from the W.M.,Keck Foundation is acknowledged.

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PART I

TRUE REFLECTION IMAGING
Comparison of different schemes of image amplitude correction in prestack depth migration

Ru-Shan Wu and Mingqiu Luo

Summary

We compare different schemes of amplitude correction from the viewpoint of amplitude gain control (AGC) factors with different approximations. Four kinds of amplitude corrections are considered: the traditional vertical AGC, space-domain correction based on total illumination, correction in local scattering angle-domains, and the correction in local dip-angle domain. We analyze the different approximations involved in these schemes and compare their results for amplitude correction for the migrated images of the SEG/EAGE 2D salt model and the Sigsbee2A model. The advantages of the correction in dip-angle domain can be seen clearly. The image quality of subsalt structures is greatly improved and the image amplitudes along subsalt faults are more balanced. In the meanwhile the noises and migration artifacts become smaller than other schemes.

Introduction

Amplitude correction in local angle-domain for wave equation based migration method is currently a focus of investigation (e.g., Mosher et al., 1997; Rickett and Sava, 2002). Wu et al. (2004) proposed an amplitude correction scheme using local image matrix defined in local angle-domain by beamlet decomposition of the Green’s functions. The aperture correction is done in the dip-angle domain. The new theory and method of amplitude correction include both the effects of acquisition system configuration and the propagation through complex overburden. In this paper we compare the new scheme of amplitude correction in local dip-angle domain with other correction schemes from the viewpoint of amplitude gain control (AGC) factors with different approximations. The 2D SEG/EAGE salt model and Sigsbee2A model datasets are used to demonstrate the effects of different corrections.

Local image matrix (LIM) and the final image amplitude

Traditionally the final migrated image is plotted as a map of image strength represented by a scalar quality at each point in the image space. In order to relate the image strength to the local scattering property of heterogeneity, the image amplitudes of migrated image need to be corrected to eliminate the influences of different factors, such as (1) geometric spreading in complex media, (2) path effects (absorption and scattering losses during propagation), (3) acquisition aperture effects. It turned out that the acquisition aperture effect is the most significant one among these factors (Wu et al., 2004b). Although the image amplitude is a scalar quality, it is shown that a scalar correction factor directly applied to the final image strength cannot remove the acquisition aperture influence effectively, and the correction must be done in the local angle domain using local image matrix (LIM) (Wu et al., 2004a, b). Some operations must be applied to the matrix before the summation over matrix elements for getting the final image strength.
A local image matrix \( L(\vec{\theta}_i, \vec{\theta}_g) \) can be obtained for each image point in the image space during the migration process, where \( \vec{\theta}_i, \vec{\theta}_g \) are the local incident and receiving angles respectively. If we define a reflector-normal direction as the direction that bisect the source direction \( \vec{\theta}_i = \vec{\theta}_s \) and the receiving direction, \( \vec{\theta}_g \), we can change \( (\vec{\theta}_i, \vec{\theta}_g) \) into \( (\vec{\theta}_n, \vec{\theta}_r) \) with \( \vec{\theta}_n = (\vec{\theta}_i + \vec{\theta}_g) / 2 \), \( \vec{\theta}_r = (\vec{\theta}_g - \vec{\theta}_i) / 2 \), where \( \vec{\theta}_n \) is reflector-normal angle and \( \vec{\theta}_r \) is the reflection angle with respect to the normal, as shown in Figure 1. Note that reflector-normal is opposite to the migration-dip in direction, but \( \vec{\theta}_n \) is equal to the dip-angle (the angle between x-direction and the dip direction).

![Source and receiver](image)

**Figure 1.** The definition of reflection and dip angles.

For shot profile migration, the local image matrix (LIM) is defines as

\[
L_u(\vec{\theta}_i, \vec{\theta}_g, x, z, \omega) = k_0^2 \exp\{ik_0[\sin(\vec{\theta}_i) - \sin(\vec{\theta}_g)]x\} \cdot \int_{\lambda_s} dx U(x, z; x, \omega) \int_{\lambda_g} dx U^*(x, z; x, \omega)
\]

(1)

where \( k_0 = \omega / v(x, z) \). \( \vec{\theta}_i \) and \( \vec{\theta}_g \) are defined in Figure 1, \( U(x, z; x, \omega) \) is the local incident plane wave (incident beamlet) from a source at \( x_s \), and \( U^*(x, z; x, \omega) \) is the local scattered plane wave (scattered beamlet) from a receiver at \( x_g \), excited by the source at \( x_s \). The decomposition window has a nominal width \( a \), and \( \lambda_s, \lambda_g \) are the source and receiver apertures respectively. The inner integral sums up the contributions from all the receivers for the same shot to a scattered beamlet at \( \vec{\theta}_g \); the outer integral transforms the spatial sources for a given receiving beamlet at \( \vec{\theta}_g \) to local beamlet sources (with different \( \vec{\theta}_s \)). The decomposition of a wavefield can be done using either the GDF (Gabor-Daubechies Frame) transform or a local slant stack (local Radon transform) using the decomposition vector (atom):

\[
g_m(x) = \exp\{ik_0 \sin(\vec{\theta}_m)x\}g_a(x - \bar{x}_m)
\]

(2)

where \( g_a(x - \bar{x}_m) \) is a Gaussian window function with window width \( a \) centered at \( \bar{x}_m \). In (1) we take each \( x \) as the window center for the local plane wave decomposition.
Stacking the image matrices of all the frequencies with incident angle $\overrightarrow{\theta}_i$ and receiving angle $\overrightarrow{\theta}_g$ (image condition) results in images in the local angle domain,

$$I(\overrightarrow{\theta}_i, \overrightarrow{\theta}_g, x, z) = \text{Re} \int d\omega L_{g, \omega} (\overrightarrow{\theta}_i, \overrightarrow{\theta}_g, x, z, \omega)$$

(3)

The summation over frequency implies that the local incident and scattered plane waves meet at the image point $(x, z)$ at $t = 0$. $I(\overrightarrow{\theta}_i, \overrightarrow{\theta}_g, x, z)$ is the final local angle-domain image matrix.

Figure 2 shows the image matrices (LIM’s) for two special types of scatterers: point scatterer (on the left), and planar reflector (on the right). The top panel gives the LIM’s for a shallow target, and the bottom panel, for a deep target. We see that the image matrix could be very different for different scatterers. For an isotropic point scatterer, the amplitude distribution is more uniform; while for a planar reflector with different dips, the energy distribution are mainly along different line segments.

![Image](image.png)

Figure 2. Local image matrices for a point scatterer (left panel) and a plane reflector (right panel) in a homogeneous medium (total 201 shots with 176 left-hand receivers).

Note that the LIM derived from (1) uses the cross-correlation imaging condition, and therefore the image amplitude has not been taken care of. For amplitude corrections in local angle domain, see Wu et al. (2004).

The final total image in the space domain can be obtained from the local image matrix by summing up the contributions of all scattering experiments with different angle pairs $(\overrightarrow{\theta}_i, \overrightarrow{\theta}_g)$:

$$I(x, z) = \Delta\overrightarrow{\theta}_i \Delta\overrightarrow{\theta}_g \sum_{\overrightarrow{\theta}_i} \sum_{\overrightarrow{\theta}_g} I(\overrightarrow{\theta}_i, \overrightarrow{\theta}_g, x, z)$$

(4)

The local angle coordinates in incident-receiving angle pairs $\overrightarrow{\theta}_i, \overrightarrow{\theta}_g$ can be transformed to normal-reflection angle pairs $\overrightarrow{\theta}_n, \overrightarrow{\theta}_r$ as shown in Figure 1.
Image amplitude correction

Figure 3. Angular distribution of scattered energy for reflectors with different dips: The grey lines are the responses with 360 degree view angles (no aperture effect); the bold lines represent the responses with 180 degrees (aperture effect of the surface scattering experiments).

Aperture correction using local image matrix (LIM)

In order to see the acquisition aperture effect, we assume that the local reflector is a planar reflector. In this case, the LSM degenerates into a line segment perpendicular to the matrix diagonal as illustrated in Figure 3. Without aperture limitation (360° experiments) the line segment of the LIM has the same length but situated at different locations along the diagonal for different dip-angles (grey lines in Figure 3). This can be understood from the conservation of the total scattered energy. However, in surface reflection measurements, data are only available on the surface (180° view angles), the line segments of the LIM will have different lengths for different dip-angles (bold lines). Horizontal reflector has the longest length and vertical reflector has zero length. For other limited apertures, the corresponding LIM will be segments with different lengths. When we sum up the contributions from all the matrix elements of a image matrix (LIM) the final image amplitudes will be very different for local reflectors with different dips. In heterogeneous media, the aperture effect is further complicated by the wave propagation and scattering along the paths. Because the acquisition configuration and the overburden (such as salt bodies) distortion, the aperture effects for different reflectors under a salt body could be very different. Figure 4 is the Aperture matrices (acquisition aperture efficacy matrices) for four image points of SEG/EAGE salt model. Figure 5 shows the resulted local image matrices at the four points if reflectors with different dips are situated there. We see that if we sum up the matrix elements for the final image strength, the image amplitude could vary drastically from point to point even though the true reflection coefficients are constant.
From above arguments, we can see that aperture correction must be done in dip-angle domain. Depending on the purpose of the final image, the amplitude correction can be implemented in different ways:

1) **CRA (Common Reflection-Angle) imaging:**
   In this case the amplitude dip-angle correction is done to the matrix element of LIM. True amplitude CRA image gathers can be used for local AVA (amplitude vs. angle) analysis.
(2) *Total Strength imaging:*  
In this case the amplitude correction can be done to CDA (common dip-angle) images (see Wu et al., 2004b):

\[
|I(x)|^2 = \sum_{-\theta_1 \leq \theta \leq \theta_2} \left[ \sum_{\theta} |L(x, \theta_n, \theta')|^2 \right] / \left[ |D(x, \theta_n)|^2 + \varepsilon \right]
\]

where \(\varepsilon\) is a damping factor for regularization, \(-\theta_1\) and \(\theta_2\) form the angle-span for the dip-angle summation, and \(D(x, \theta_n)\) is the dip correction factor for a local reflector with dip angle \(\theta_n\) at the image point \(x\), and is obtained from the amplitude correction factor matrix \(F_a(x, \theta_n, \theta_r)\) by summing up all the contributions along the constant \(\theta_n\) line (see Figure 6):

\[
|D(x, \theta_n)|^2 = \sum_{\theta_r} |F_a(x, \theta_n, \theta_r)|^2 .
\]

From (5) we see that the final image amplitude (a scalar) is the summation of the matrix elements of LIM after amplitude correction on common dip-angle gathers.

**Comparison of different approximations for image amplitude correction**

In this paper we concentrate on the total strength \(I(x)\) imaging. In this case amplitude corrections can be considered as applying amplitude gain (AG) factors to the migrated images in prestack depth migration. We will compare four different schemes with different degrees of approximation:

1) *Correction in local dip-angle domain*  
For amplitude correction in local dip-angle domain, the amplitude gain (AG) factor is a dip-angle and space dependent function \(A(x, \theta_n) = 1 / |D(x, \theta_n)|\) as can be seen from equation 5, 6 and Figure 6. The correction can be rewritten as

\[
|I(x)|^2 = \sum_{-\theta_1 \leq \theta \leq \theta_2} |I_m(x, \theta_n)|^2 A(x, \theta_n)^2,
\]

where \(I_m(x, \theta_n)\) is the raw migrated image field in local dip-angle domain (common dip-angle image gather),

\[
|I_m(x, \theta_n)|^2 = \sum_{\theta_r} |L(x, \theta_n, \theta_r)|^2,
\]

and the AG factors are applied in this dip angle gather. The summation procedures for the image (CDA) gather and for the AG factors are shown in Figure 6a and 6b respectively. The resulted common-dip image gather is shown in Figure 6c, and the corresponding amplitude correction factors are plotted in Figure 6d. After binning and thresholding the CDA gather, the AG factors (Figure 6d) can be applied to correct the aperture effect.
2) Correction in local scattering-angle domain

If we do not apply the correction in dip-angle domain, and instead work on common scattering-angle (or reflection-angle) image gathers, the correction then becomes

\[
| I(x) |^2 = \sum_{-\bar{\theta}_0 \leq \bar{\theta}_r \leq \bar{\theta}_0} | I_m(x, \bar{\theta}_r) |^2 A(x, \bar{\theta}_r)^2 \tag{9}
\]

where \( I_m(x, \bar{\theta}_r) \) is the CSA image gather and \( A(x, \bar{\theta}_r) \) is corresponding AG factors. The summation procedures for getting these gathers are shown in Figure 7. As shown in Figure 7a, the summation is along the common scattering angle lines in the image matrix. The resulted gather is shown in Figure 7c. However, in the traditional correction in scattering angle domain, the angle gather is obtained by using offset plane waves, and the dip information is not available. The correction factor, therefore, is derived based on the flat interface assumption, that corresponds the red (thick) line in Figure 7b. The correct AG factors should be along the green (grey) line. The resulted AG factors are shown in Figure 7d. We can see clearly the inaccuracy of

Figure 6. Correction in the local dip-angle domain.
Image amplitude correction

In the same way we can apply the correction to common receiving-angle image gathers or other gathers. However, these corrections cannot correctly handle the acquisition aperture effects, because the aperture effect is mainly dip-dependent.

3) Correction in space-domain alone

If we totally neglect the angle dependence of aperture correction, the AG factors are only space dependent (see Figure 8)

\[ |I(x)| = |I_m(x)| A(x), \]

where

\[ |I_m(x)|^2 = \sum_{\theta \in \mathcal{S}, \bar{\theta} \in \mathcal{I}} |I_m(x, \bar{\theta})|^2, \text{ and } A(x)^2 = \sum_{\theta \in \mathcal{S}, \bar{\theta} \in \mathcal{I}} A^2(x, \bar{\theta}) \]
The summation procedure is shown in Figure 7a and 7b. The total image strength is the integration over the image matrix, and the AG factor is the integration over the amplitude correction matrix. Since the reflectors with different dips have different aperture effects, while the AG factors will be all the same, the correction will not be effective.

4) Correction by vertical AGC

The conventional AGC is the simplest amplitude correction, which has an AG factor dependent only on $z$:

$$ |I(x, z)| = |I_m(x, z)| A(z) $$  \hspace{1cm} (12)

While the vertical AGC can compensate the weak image amplitude at depth, it increases the noise background simultaneously.

Application to the imaging of SEG/EAGE salt model

We apply various image amplitude gain factors defined in the previous section to prestack depth migration for the SEG/EAGE salt model. Local cosine beamlet (LCB) prestack migration method (Wu et al., 2000; Wang and Wu, 2002; Luo and Wu, 2003) is employed for the imaging. The velocity model and the raw image of prestack depth migration are shown in Figure 9.
Image amplitude correction

Figure 9, the 2D SEG/EAGE velocity model and its raw prestack depth migration image by LCB method

We start from the simplest vertical gain control AGC factor $A(z)$. Figure 10 gives the AG factor distribution and the image after the AGC correction. We see that although image amplitudes are increased for the deep targets, but the noise background is also amplified at depth. More important is the fact that the shadow zones still exist and the images for steep faults are still weak.

Figure 11 gives the corresponding results for the correction on total strength (equation (10)).
This correction corresponds to a spatially varying AGC, which extends the vertical AGC to include the laterally variation of acquisition and propagation effects. We see that the amplitude balance and image quality have been improved. However, the signal and noise are enhanced simultaneously in the weak illuminated areas.

![AG factor for total strength](image1)

![Image after the total strength correction](image2)

Figure 11, AG factor and corrected image for the total strength

Next we show the results of amplitude corrections in local angle-domain. To calculate the correct AG factors, the effects of acquisition aperture to local reflectors with different dips must be taken into account. Without this consideration, even corrections in angle domain, such as that for offset plane waves, or the correction in receiving angle-domain for common-shot migration, will not give the correct AG factors for the purpose of true-reflection imaging. Figure 12 shows the results for the amplitude corrections in local receiving-angle domain. Figure 12a,b and c show the AG factors for the corresponding -45°, 0°, and 45° common receiving angle-gathers. Figure 12d gives the amplitude corrected image. We have tested also the case of correction in common scattering-angle domain. The results are similar. We see that even though the image quality and amplitude balance are significantly improved, however, the signal-to-noise ratio in the subsalt region is still low.

![AG factors corresponding to θg=-45°](image3)
Figure 12, AG factors and corrected image for correction in receiving-angle domain

(a) AG factors corresponding to $\theta_n=-45^\circ$

(b) AG factors corresponding to $\theta_n=0^\circ$

(c) AG factors corresponding to $\theta_n=45^\circ$

(d) Image after receiving angle domain correction
Finally we show the results of corrections in local dip-angle domain in Figure 13. The AG factor for dip=-45°, 0°, and 45° are given in Figure 13a, b, and c. The image after correction is given in Figure 13d. We can see clearly the superior performance of this scheme. While the images of steep reflectors in the subsalt region are enhanced, the noises in the same region are depressed in the same time. The AG factors in Figure 13a for the dip 45°, which is the dip of target reflectors, are the opposite of the dip -45° (Figure 13c), which is the dip of coherent noises in this case. The image quality of subsalt structures is greatly improved by the amplitude correction in local dip-angle domain. The image amplitudes, especially along the steep faults and the baseline are much more uniformly distributed. Figure 14 summarizes the comparison of image qualities in a zoomed subsalt region for different amplitude corrections.
Application to the imaging of Sigsbee2A salt model

We apply various image amplitude gain factors defined in the previous section to prestack depth migration for Sigsbee2A model. Local cosine beamlet (LCB) prestack migration method (Wu et al., 2000; Wang and Wu, 2002; Luo and Wu, 2003, Luo et al., 2004, Luo and Wu, 2005) is employed for the imaging. The velocity model and the raw image of prestack depth migration are shown in Figure 15.

We start again from the vertical AGC factor $A(z)$. Figure 16 gives the AG factor distribution and the image after the AGC correction. We see that although image amplitudes are increased for the
deep targets, but the noise background is also amplified at depth. The image in the shadow zones did not improve much.

Figure 15. 2D Sigsbee2A velocity model and its raw prestack depth migration image by LCB method
Figure 17 gives the corresponding results for the correction on total strength (equation (10)). This correction corresponds to a spatially varying AGC. We see that the amplitude balance and image quality have been improved. However, the signal and noise are enhanced simultaneously in the weak illuminated areas.
Figure 18 shows the results for the amplitude corrections in local receiving-angle domain. The results are similar. We see that even though the image quality and amplitude balance are improved, however, the noise background is also increased.

Finally we show the results of corrections in local dip-angle domain in Figure 19. The AG factor for dip=-40°, 0°, and 40° are given in Figure 19a, b, and c. The image after correction is given in Figure 19d. We can see the superior performance of this scheme. While the images of reflectors in the subsalt region are enhanced, the noises in the same region are depressed compared with other schemes. The AG factors in the image quality of subsalt structures, the amplitude balance, the continuity of the subsalt reflectors are all improved by the amplitude correction in local dip-angle domain. The improvement in signal to noise ratio is not as dramatic as in the case of SEG/EAGE salt model in the previous section, where the target structures and the noise structures beneath the salt body are orthogonal to each other, and the correction in local dip-angle domain is most suitable for the case. In any case, the correction in local dip-angle domain has the greatest potential in improving the image amplitude and quality, and increases the S/N in the subsalt region.
Figure 18. AG factors and corrected image for receiving-angle correction.
The last figure (Figure 20) summarizes the comparison of image qualities in a zoomed subsalt region for different amplitude corrections. The features of different correction schemes can be seen more clearly in the zoomed regions.
Image amplitude correction

(a) image of the subsalt part without correction

(b) image of the subsalt region after AGC

(c) image of the subsalt region after total strength correction
Figure 20, comparison of images of the subsalt region by the four types of amplitude correction.

**Conclusion**

We compared the four kinds of amplitude correction schemes from the viewpoint of amplitude gain factors with different approximations: traditional vertical AGC, space-domain correction based on total illumination, and correction in scattering angle domain and correction in dip-angle domains. Through the tests using the SEG/EAGE salt model and the Sigsbee2A salt model, we see clearly the superior performance of the dip-angle domain correction scheme.

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**References**


Recover scattering wave amplitudes from back propagated waves

*Mingqiu Luo and Xiao-Bi Xie*

**Summary**

Reconstructing correct scattering waves is essential for the true reflectivity imaging. We investigate the relations between the scattering and back propagated waves in both space and local angle domains. By omitting the couplings between far away scatters and different directions, the complex relation between two waves can be simplified to an amplitude compensation factor, which can be used to eliminate image distortions resulted from acquisition and propagation effects. Numerical examples are conducted for a four-layer model and the 2D Sigsbee2A model to demonstrate the application of compensation factors to improve the image qualities.

**Introduction**

The prestack depth migration can provide images for the subsurface structures. Although migration images give relatively accurate locations of the structures, their amplitudes often fail to provide correct reflectivity from the impedance contrasts. The true reflectivity image can be obtained from the ratio between the incident and scattering waves at the target reflectors. Therefore, the amplitudes of these waves are crucial for the true reflectivity image and AVO analysis. The migration technique provides tools that can extrapolate the waves to the target area. However, due to the limited apertures of the acquisition system and the complex velocity models along the propagation path, only part of the scattering waves can be recorded by the receiver array and then propagated back to the target area for reconstructing the original scattering waves. This makes the amplitude of the scattering wavefield often a problem.

There have been many efforts to regularize these losses. The matrix relation between the depth image and the reflectivity has been studied and serves as the foundation for the migration deconvolution, which aiming at suppressing the impact of the acquisition effects and to improve the resolution and illumination (Hu, et al., 2001; Yu, et al., 2003, 2004). The relationship between the local angle domain image and the corresponding illumination are also studied and used for the amplitude corrections. These results can partially eliminate the acquisition and propagation effects on the image amplitudes (Wu and Chen, 2002; Xie and Wu, 2002; Wu, et al., 2003; Wu et al., 2004; Luo et al., 2004).

In this paper, we propose a method for reconstructing correct scattering wave amplitudes. We use an instant partial modeling to predict the upper- and down-going waves at the receiver side. Then calculate the losses in this process and use it to guide the amplitude compensation in the migration process. This method is consistent to the illumination correction at the receiver side. To demonstrate the applications of this method, numerical examples are calculated for a four-layer model and the Sigsbee2A data set.
Recover scattering wave amplitudes

Relationship between scattering and back propagated waves

Consider the scattering and back propagation geometry shown in Figure 1. The local scattering wavefield \( u(\vec{x}_{sc}) \) generated at the target location \( \vec{x}_{sc} \) forward propagates to the surface receiver at \( r \). The signals recorded by the receivers are then downward extrapolated to the target region \( \vec{x} \) for imaging. The downward wavefield can be expressed as

\[
\sum_{K} u(\vec{x}_{sc}, \vec{x}) A_{sc} = \sum_{K} \sum_{\vec{x}_{sc}} u(\vec{x}_{sc}, \vec{x}) A(\vec{x}_{sc}, \vec{x}),
\]

where \( A(\vec{x}_{sc}, \vec{x}) \) is defined as the contribution factor and can be expressed as

\[
A(\vec{x}_{sc}, \vec{x}) = \sum_{r} G(\vec{x}_{sc}, r) \partial_{n} G(r, \vec{x}).
\]

Here \( G(\vec{x}_{sc}, r) \) is the forward Green’s function from the scatterer to the receiver, and \( \partial_{n} G(r, \vec{x}) \) is the backward Green’s function from the receiver to the image point. By reciprocity, the contribution factor can be rewritten as,

\[
A(\vec{x}_{sc}, \vec{x}) = \sum_{r} G^{*}(r, \vec{x}_{sc}) \partial_{n} G(r, \vec{x}).
\]

Equations (1) and (2) form the upward-downward modeling which serve as the basis for amplitude modification. At scattering point \( \vec{x}_{sc} \) and imaging point \( \vec{x} \), both \( u(\vec{x}_{sc}) \) and \( u'(\vec{x}) \) can be decomposed into beamlets with local directional information (Wu and Chen, 2002; Xie and Wu, 2002; Luo, et al., 2004)

\[
u(\vec{x}_{sc}) = \sum_{\theta_{sc}} u(\vec{x}_{sc}, \theta_{sc}),
\]

\[
u'(\vec{x}) = \sum_{\theta_{sc}} u'(\vec{x}, \theta_{sc}).
\]

With these transforms, we obtain the angle domain equivalent of equation (1)

\[
u'(\vec{x}, \theta'_{sc}) = \sum_{\vec{x}_{sc}} \sum_{\theta_{sc}} u(\vec{x}_{sc}, \theta_{sc}) A(\vec{x}_{sc}, \theta_{sc}, \vec{x}, \theta'_{sc})
\]

where the local angle domain contribution factor can be expressed as
\[
A(\bar{x}_{sc}, \theta_{sc}, \bar{x}, \theta'_{sc}) = \sum_r G(\bar{x}_{sc}, \theta_{sc}, r) \partial_n G(r, \bar{x}, \theta'_{sc}).
\] (7)

In equation (7) \(G(\bar{x}_{sc}, \theta_{sc}, r)\) and \(\partial_n G(r, \bar{x}, \theta'_{sc})\) are the forward and backward angle domain Green’s functions, which propagate the beamlets with local angle information to and from receivers. By the local angle domain reciprocity of the Green’s function, the local angle domain contribution factor becomes,

\[
A(\bar{x}_{sc}, \theta_{sc}, \bar{x}, \theta'_{sc}) = \sum_r G^*(r, \bar{x}_{sc}, \theta_{sc}) \partial_n G(r, \bar{x}, \theta'_{sc})
\] (8)

Equations (1) and (6) can be rewritten in the matrix form

\[
u'(\bar{x}) = A(\bar{x}_{sc}, \bar{x}) u_{sc}(\bar{x}_{sc}) ,
\] (9a)

\[
u'(\bar{x}, \theta'_{sc}) = A(\bar{x}_{sc}, \theta_{sc}, \bar{x}, \theta'_{sc}) u_{sc}(\bar{x}_{sc}, \theta_{sc}) .
\] (9b)

where \(u'\) and \(u_{sc}\) are back propagated and scattering wave vectors, and \(A\) is the contribution factor matrix.

In the prestack migration process, we expect to reconstruct correct scattering waves from the back propagated waves. Based on equation (9), the scattering wavefield \(u_{sc}\) can be obtained by solving an inverse problem. However, this is usually a very time consuming process. In practice, the couplings between different scatterers and between different directions are usually weak. If these couplings can be neglected, we can simplify the contribution factor matrix by keeping only the diagonal elements from these matrixes. We obtain

\[
A_d(\bar{x}_{sc}) = A(\bar{x}_{sc}, \bar{x}_{sc}) ,
\] (10a)

\[
A_d(\bar{x}_{sc}, \theta_{sc}) = A(\bar{x}_{sc}, \theta_{sc}, \bar{x}_{sc}, \theta_{sc}) .
\] (10b)

The matrix inversions are simplified to

\[
u(\bar{x}_{sc}) = \nu'(\bar{x}_{sc}) / A_d(\bar{x}_{sc}) ,
\] (11a)

\[
u(\bar{x}_{sc}, \theta_{sc}) = \nu'(\bar{x}_{sc}, \theta_{sc}) / A_d(\bar{x}_{sc}, \theta_{sc}) .
\] (11b)

From the definition of contribution factor matrixes (3) and (8), their diagonal values should be real nonnegative. The equations (11a)-(11b) provide only amplitude compensation. On the other hand, equations (11a)-(11b) are similar to the receiver side total- and directional- illuminations, respectively (Wu and Chen, 2002; Xie, et al., 2003, 2004).

**Numerical examples**

To demonstrate the application of this method, we calculate a group of shot profile prestack depth migrations. The traditional convolution image condition

\[
I(\bar{x}, \omega, s) = u_s(\bar{x}, \omega, s) \cdot u_s^*(\bar{x}, \omega, s) ,
\] (12)

and a weighting division image condition (Luo, 2005)

\[
I(\bar{x}, \omega, s) = f(\omega) \cdot u_s(\bar{x}, \omega, s) / u_s(\bar{x}, \omega, s)
\] (13)

are used for comparison. In (12) and (13), \(u_s(\bar{x}, \omega, s)\) and \(u_s(\bar{x}, \omega, s)\) are source (incident) and the back propagated scattering waves, \(S\) is the source index. A Ricker function \(f(\omega)\) gives the frequency dependence. With the amplitude compensation (10a), the convolution image condition (12) becomes

\[
I(\bar{x}, \omega, s) = u_s(\bar{x}, \omega, s) \cdot u_s^*(\bar{x}, \omega, s) / A_d(\bar{x}) .
\] (14)
Similarly, with the amplitude compensation (11b), the weighting division image condition becomes

\[
I(\bar{x},\omega,s) = \frac{f(\omega)}{u_r(\bar{x},\omega,s)} \sum_{g} u_g(\bar{x},\omega,s,\theta_g) / A_d(\bar{x},\theta_g).
\]  

(15)

Example 1: Four-layer constant velocity model

For the four-layer model (Baina et al., 2002) with a constant background velocity, the synthetic data are generated using a full wave FD method. The data set consists of 181 shots with 120 right-side receivers separated by 25m. The distance between shots is 50m. The velocity for the background is 4km/s and for the reflectors is 6km/s, as shown in Figure 2(a).

![Diagram of a four-layer constant velocity model](image)

Figure 2. Images of the four-layer constant background velocity model without the amplitude compensation.
Prestack depth migrations using the wave equation based shot profile method plus the image conditions (12) and (13) are conducted and the results are shown in Figure 2(b) and 2(c). The image amplitudes along the reflectors are picked and compared in Figure 3. We see strong amplitude variations along the curved reflectors T1 and T3 due to different acquisition apertures for different dips.

We applied the amplitude compensation using the convolution image condition (14). The results are shown in Figure 4, where the amplitudes for all four reflectors are almost corrected to the same level. The weighting division image condition (15) with local angle domain amplitude compensation is tested, and the results are shown in Figure 5. We see that the amplitudes for all the four reflectors are almost corrected to the same level.
Recover scattering wave amplitudes

Figure 4. Image and amplitudes of the four reflectors using the convolution image condition with space domain amplitude compensation.

Figure 5. Image and amplitudes using weighting division image condition with local angle domain amplitude compensation.

Example 2: 2D Sigsbee2A model

The benchmark data Sigsbee2A from the SMAART joint venture is used to test the angle domain amplitude compensation method. The velocity model has 3201 samples in distance and 1200 samples in depth, both with a sampling interval of 25ft (see Figure 6). There are 500 shots with right-side receivers and the maximum number of receivers for each shot is 348. The local cosine basis (LCB) method (Wu, et al., 2000; Wang and Wu, 2002; Wang et al., 2003; Luo and Wu, 2003; Luo, et al, 2004; Luo and Wu, 2005), together with convolution image conditions (12) and (14), are used in the depth migration. The results in Figure 7 show that: 1) the total image is more balanced with the amplitude compensation and 2) the steep boundary of the salt body is greatly improved.
Conclusion

Theoretically the true scattering waves can be reconstructed from back propagated waves through an inversion of the contribution factor matrix. If omitting the couplings between scatters and different directions, the contribution factor matrix is simplified to a diagonal matrix, and the scattering waves can be approximated as the back propagated waves with an amplitude compensation factor. Compared with the formal inversion, this method is more efficient. Numerical examples demonstrate that the amplitude compensation factors can greatly improve the image quality. The amplitude compensation in the space domain is an approximation for that in the angle domain.
Recover scattering wave amplitudes

(b) Image with space domain amplitude compensation

Figure 7. Comparison of the images of Sigsbee2A model.

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References


Image conditions for true reflection imaging

Mingqiu Luo

Summary

For prestack depth migration, the image amplitude and its relation to true reflectivity depend on the image condition applied. The true reflection image condition should provide image with amplitude representing accurately the true reflectivity. Images by approximate image conditions can only approximately represent the true reflectivity. In this paper, the true reflection image condition is proposed. With several approximations applied separately or combined, different approximate image conditions can be deduced and their relations to reflectivity are studied. The image amplitudes from the numerical results show their dependence on image conditions.

Introduction

Ideally, we expect the seismic migration can provide image that represents the true reflectivity of subsurface structures, at least approximately. The true reflectivity can be defined as the ratio of the incident wave and its scattered waves at target discontinuities. How close the actual image amplitudes approach to the true reflectivity depends on many factors, for example, the accuracy of propagators, the acquisition configuration and the image conditions applied.

The true amplitude propagator is the basis for true reflection imaging. The one-way wave equation based propagators usually provide more accurate wavefields compared to ray-based methods, but they still cannot be regard as true amplitude propagators unless corrections based on WKBJ method or transmission coefficient are considered (Zhang et al., 2004; Wu, et al., 2005; Luo, et al., 2005). The limited acquisition aperture also has essential impact on the true reflection imaging. The acquisition impact can be partly eliminated using local angle domain illumination correction (Wu et al., 2004). In other words, Due to the limited acquisition aperture, not all scattered waves can be recorded and back propagated to the image point. That is, local scattered wavefield usually cannot be fully recovered. In order to reconstruct correct scattered waves, proper compensation should be taken (Luo and Xie, 2005). This amplitude compensation can be formulated as part of the image condition.

The compensation should be processed in the local angle domain. However, there is no angle information in traditional wave-equation based image conditions. Local angle domain analysis (Wu and Chen, 2002; Xie and Wu 2002) makes it possible to define image conditions based on the angle information. In this paper, the theoretical image condition for true reflectivity will be formulated. Then, using different approximations and considering the trade-off between the accuracy and efficiency, several simplified versions are presented. Numerical examples based on 2D models are calculated to demonstrate the application of these image conditions.

Reflectivity and image condition

Generally, the reflectivity for subsurface reflector or scatterer can be described by the scattering
matrix (Wu and Chen, 2002)

\[
R(\vec{x}, \theta_i, \theta_g) = \sum_{\omega} R(\vec{x}, \omega, \theta_i, \theta_g).
\]

(1)

Where \(R(\vec{x}, \omega, \theta_i, \theta_g)\) denotes the frequency dependent scattering coefficient. \(\theta_i\) and \(\theta_g\) denote the incident and receiving angles respectively (see Figure 1). The total scattering coefficient for a reflector or scatterer is

\[
R(\vec{x}) = \sum_{\theta_i} \sum_{\theta_g} R(\vec{x}, \theta_i, \theta_g).
\]

(2)

The scattering coefficient in (1) and (2) can be used in the analysis of the media.

The ultimate goal of the true reflection imaging is providing the scattering coefficients for subsurface targets. Given an acquisition system, each shot \(s\) generate directional incident waves \(u_{in}(\vec{x}, \omega, s, \theta_i)\) and their directional scattered waves \(u_{sc}(\vec{x}, \omega, s, \theta_g)\) at the scattering point \(\vec{x}\). The true reflection image condition can be defined as

\[
I(\vec{x}) = \sum_{\theta_i} \sum_{\theta_g} I(\vec{x}, \theta_i, \theta_g) = \sum_{\theta_i} \sum_{\theta_g} I(\vec{x}, \omega, \theta_i, \theta_g)
\]

\[
= \sum_{\theta_i} \sum_{\theta_g} \sum_{\omega} \frac{1}{n_{stk}(\vec{x}, \omega, \theta_i, \theta_g)} \sum_{s=1} n_{stk}(\vec{x}, \omega, \theta_i, \theta_g) u_{sc}(\vec{x}, \omega, s, \theta_g)
\]

(3)

where \(n_{stk}(\vec{x}, \omega, \theta_i, \theta_g)\) denotes the multifold (or hit count) at each point \(\vec{x}\) with incident angle \(\theta_i\) and receiving angle \(\theta_g\).

![Figure 1. Definition of local incident, receiving, dip, and reflect angles.](image)

**Basic image condition for true reflection imaging**

In prestack depth migration, the angle decomposed source waves \(u_i(\vec{x}, \omega, s, \theta_i)\) can be regarded as a good approximation to the true incident waves when true amplitude propagators is used, i.e.

\[
u_{in}(\vec{x}, \omega, s, \theta_i) = u_i(\vec{x}, \omega, s, \theta_i).
\]

(4)

Due to the limited acquisition aperture, the back-propagated waves differ greatly from the true
scattered waves. Amplitude compensation should be applied to reconstruct the true scattered waves. An approximate formula between the true scattered wave \( u_{sc}(\tilde{x}, \omega, s, \theta_g) \) and its back-propagated wave \( u_g(\tilde{x}, \omega, s, \theta_g) \) is (Luo and Xie, 2005b)

\[
u_{sc}(\tilde{x}, \omega, s, \theta_g) = \frac{u_g(\tilde{x}, \omega, s, \theta_g)}{A(\tilde{x}, \omega, s, \theta_g)}, \quad A(\tilde{x}, \omega, s, \theta_g) = \sum r |G(\tilde{x}, r, \omega)\partial_s G(r, \tilde{x}, \omega, \theta_g)|,
\]

where \( A(\tilde{x}, \omega, s, \theta_g) \) is the amplitude compensation factor, \( G(\tilde{x}, r, \omega) \) is the Green’s function from scattering point \( \tilde{x} \) to receiver \( r \), and \( \partial_s G(r, \tilde{x}, \omega, \theta_g) \) stands for the angle component of the back-propagated Green’s function from receiver \( r \) to scattering point \( \tilde{x} \).

With the approximations (4) and (5), the true reflection image condition (3) can be simplified to,

\[
I(\tilde{x}) = \sum_{\theta_1} \sum_{\theta_2} \sum_{\omega} \sum_{n_{stk}} u_{sc}(\tilde{x}, \omega, s, \theta_g) \cdot u_g(\tilde{x}, \omega, s, \theta_g)^* \cdot |u_g(\tilde{x}, \omega, s, \theta_g)|^2 A(\tilde{x}, \omega, s, \theta_g).
\]

Since image condition (6) is the closest approximation to the true reflectivity, it will be referred as a basic image condition.

**Approximations**

It is impossible to apply the basic image condition (6) directly in prestack depth migration due to the huge computations. Further approximations should be taken to simplify it.

Let’s see a simple mathematic approximation formula. For a series of complex numbers \( a_i, b_i, c_i \), there is an approximation

\[
\forall i, j \in [1, N], c_i = \frac{a_i}{b_i} \quad \Rightarrow \quad \sum_{i=1}^{N} c_i \approx N_{\text{nonzero}} \frac{\sum_{i=1}^{N} a_i}{\sum_{i=1}^{N} b_i}.
\]

This approximation will be used for deducting the simplified image conditions.

The following are some possibilities toward more efficient calculations:

**a)** No amplitude compensation. That is,

\[
A(\tilde{x}, \omega, s, \theta_g) \equiv 1.
\]

With this approximation, the acquisition and propagation impact is neglected, which may lead to incorrect image amplitudes for the steep reflectors and sub-salt area.

**b)** No multifold approximation. That is,

\[
n_{stk}(\tilde{x}, \omega, \theta, \theta_g) = 1.
\]

With this approximation, the multifold at each point is omitted.

**c)** Neglecting the local angle information for the source waves. This approximation can save the angle decomposition computation time in processing, but the image will lost its incident angle dependence. According to (7), this approximation can be formulated as,
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True reflection image condition

\[ \sum_{\theta_i} \approx \sum_{\theta_i} \sum_{\theta_i}. \] (8c)

Here, we omit the total number of incident waves angles for simplicity, because it is constant.

d) Neglecting the local angle information for the back propagated waves. This approximation can save the angle decomposition computation time in processing, but the image will lose its receiving angle dependence. According to (7), this approximation can be formulated as,

\[ \sum_{\theta_i} \approx \sum_{\theta_i}. \] (8d)

Here, we omit the total number of receiver waves angles for simplicity, because it is constant.

e) Assuming certain frequency independence, so the summation on frequency can be done before the division. That is, if omit the total number of frequency for simplicity, the approximation can be formulated as

\[ \sum_{\omega} \approx \sum_{\omega}. \] (8e)

f) Assuming contributions from different shots are either similar or can be neglected, so the summation on shot can be done before the division. This approximation can be formulated as

\[ \sum_{s} = n_{stk} \sum_{s}. \] (8f)

Here, the multifold can be different from point to point and cannot be omitted.

g) Assuming the contribution from dominant frequency similar to all frequencies’ contributions, so the summation on frequencies can be omitted. This approximation can be formulated as

\[ \sum_{\omega} = \delta(\omega - \omega_d). \] (8g)

Here, \( \omega_d \) stands for the dominant frequency.

h) Applying convolution image condition. That is,

\[ \left| u_s(\hat{x}, \omega, s, \theta) \right|^2 = 1. \] (8h)

In the numerical calculation, the amplitude of the source wave field can be very small, which may enlarge the useless noise. This approximation makes the image amplitude a little far from the true reflectivity, but can reduce the noise.

i) Applying weighting division image condition. The Ricker function \( f_{\text{ric}}(\omega) \) is used as the weighting factor for each frequency. That is,
$$\sum_\omega I(\bar{x}, \omega) \rightarrow \sum_\omega f_{\text{rec}}(\omega) I(\bar{x}, \omega).$$  \hspace{1cm} (8i)

It reduces the numerical noise while keeping the image amplitudes close to the true reflectivity.

j) Acquisition reciprocity approximation. Different acquisitions favor different structures. Left side acquisition favors reflectors with negative dip angles. Right side acquisition favors reflectors with positive dip angles (Liu, etc., 2004). Even for the same acquisition system, due to the source wave field comes from Greens function, but the receiver back propagated wave field comes from partial Greens function. The images from common shot gather dataset is different from its common receiver gather’s image. In order to eliminate the one-side acquisition’s angle favorites, a symmetry acquisition should be reconstructed. A simple way is combining the shot gather with its resorted receiver gather to get the approximate symmetry acquisition. A more balance images can be gotten from the combined dataset.

**Simplified image conditions**

Applying the above mentioned approximations separately or combined, the following are some simplified image conditions.

1. **Conducting division shot by shot**

Applying approximations (b) and (c) to the basic image condition (6), we get,

$$I(\bar{x}) = \sum_{\theta_s} \sum_\omega \sum_s u_s(\bar{x}, \omega, s, \theta_g) \cdot u_s(\bar{x}, \omega, s)^*, \hspace{1cm} (9)$$

where $u_s(\bar{x}, \omega, s, \theta_g) = \sum_{\theta_j} u_j(\bar{x}, \omega, s, \theta_j)$ stands for the total source wave field. Alternatively,

Further applying approximations (i) to the image condition (9), we get,

$$I(\bar{x}) = \sum_{\theta_s} \sum_\omega \sum_s f_{\text{rec}}(\omega) \cdot u_s(\bar{x}, \omega, s, \theta_g) \cdot u_s(\bar{x}, \omega, s)^*, \hspace{1cm} (10)$$

Alternatively, further applying approximations (h) to image condition (9), we have the convolution image condition

$$I(\bar{x}) = \sum_{\theta_s} \sum_\omega \sum_s u_s(\bar{x}, \omega, s, \theta_g) \cdot u_s(\bar{x}, \omega, s)^* \cdot A(\bar{x}, \omega, s, \theta_g). \hspace{1cm} (11)$$

We can further apply approximation (d) to image condition (10) and (11). The two image condition can be simplified to,

$$I(\bar{x}) = \sum_\omega \sum_s f_{\text{rec}}(\omega) \cdot u_s(\bar{x}, \omega, s) \cdot u_s(\bar{x}, \omega, s)^*, \hspace{1cm} (12)$$

$$I(\bar{x}) = \sum_\omega \sum_s u_s(\bar{x}, \omega, s) \cdot u_s(\bar{x}, \omega, s)^* \cdot A(\bar{x}, \omega, s). \hspace{1cm} (13)$$

Where $u_s(\bar{x}, \omega, s) = \sum_{\theta_j} u_j(\bar{x}, \omega, s, \theta_j)$ stands for the space domain back propagated waves, and
amplitude compensation factor is \( A(\vec{x},\omega,s) = \sum_r |G(\vec{x},r,\omega)\partial_n G(r,\vec{x},\omega)| \).

If no amplitude compensation is considered, i.e., applying approximation (a) to image condition (12) and (13), we get

\[
I(\vec{x}) = \sum_{\omega} \sum_{s} \frac{f_{\text{rec}}(\omega)u_g(\vec{x},\omega,s) \cdot u_s(\vec{x},\omega,s)^*}{|u_s(\vec{x},\omega,s)|^2}, \tag{14}
\]

\[
I(\vec{x}) = \sum_{\omega} \sum_{s} u_g(\vec{x},\omega,s) \cdot u_s(\vec{x},\omega,s)^*. \tag{15}
\]

Here (15) is the traditional convolution image condition. If cancel the approximation (i) for image condition (14), we get the traditional division image condition

\[
I(\vec{x}) = \sum_{\omega} \sum_{s} \frac{u_g(\vec{x},\omega,s) \cdot u_s(\vec{x},\omega,s)^*}{|u_s(\vec{x},\omega,s)|^2}, \tag{16}
\]

2. Conducting division after summation on shots in incident and receiving angles

Applying approximation (e), (f) and (g) to the basic image condition, we get

\[
I(\vec{x}) = \sum_{\theta_1} \sum_{\theta_2} \sum_{\omega} \sum_{s} \frac{\sum u_g(\vec{x},\omega,s,\theta_g) \cdot u_s(\vec{x},\omega,s,\theta_g)^*}{|u_s(\vec{x},\omega,s,\theta_g)|^2} \cdot A(\vec{x},\omega,s,\theta_g) \tag{17}
\]

Further applying approximations (b) to the image condition (17), we get

\[
I(\vec{x}) = \sum_{\theta_1} \sum_{\omega} \sum_{s} \frac{u_g(\vec{x},\omega,s,\theta_g) \cdot u_s(\vec{x},\omega,s)^*}{|u_s(\vec{x},\omega,s,\theta_g)|^2} \cdot A(\vec{x},\omega,s,\theta_g) \tag{18}
\]

It is also referred as amplitude correction in receiving angles.

Further applying approximation (d) to image condition (18), we get

\[
I(\vec{x}) = \sum_{\omega} \sum_{s} \frac{\sum u_g(\vec{x},\omega,s) \cdot u_s(\vec{x},\omega,s)^*}{|u_s(\vec{x},\omega,s)|^2} \cdot A(\vec{x},\omega,s,\theta_g) \tag{19}
\]

This image condition is similar to the image amplitude correction with total AAE illumination (Wu etc., 2004).

Further applying approximation (a) to image condition (19), here comes the image condition denotes the amplitude correction with total DI illumination (Wu etc., 2004)

\[
I(\vec{x}) = \sum_{\omega} \sum_{s} \frac{\sum u_g(\vec{x},\omega,s) \cdot u_s(\vec{x},\omega,s)^*}{\sum |u_s(\vec{x},\omega,s)|^2} \cdot A(\vec{x},\omega,s,\theta_g) \tag{20}
\]

3. Conducting division after summation on shots in dip and reflection angles

Actually, the local incident and receiving angles can be converted to dip and reflect angles using
\[ \theta_n = (\theta_i + \theta_s) / 2 \] and \[ \theta_r = (\theta_i - \theta_s) / 2 \] (see Figure 1). Similar to (17), we have the image condition for amplitude correction in dip and reflection angles domain,

\[
I(\bar{x}) = \sum_{\theta_i, \theta_r} \sum_{s} \sum_{\omega} u_s(\bar{x}, \omega, s, \theta_n - \theta_r) \cdot u_s(\bar{x}, \omega, s, \theta_n + \theta_r)^*.
\]

(21)

It is also regarded as the AAE correction (Wu et al., 2004).

Further neglect the image dependence on the reflection angle in image condition (21), we have the image condition for the dip angle domain amplitude correction,

\[
I(\bar{x}) = \sum_{\theta_i, \theta_r} \sum_{s} \sum_{\omega} u_s(\bar{x}, \omega, s, \theta_n - \theta_r) \cdot u_s(\bar{x}, \omega, s, \theta_n + \theta_r)^* \cdot |A(\bar{x}, \omega, s, \theta_n - \theta_r)|^2.
\]

(22)

Further neglect the image dependence on the dip angle in image condition (21), we have the image condition for the reflection angle domain amplitude correction,

\[
I(\bar{x}) = \sum_{\theta_i, \theta_r} \sum_{s} \sum_{\omega} u_s(\bar{x}, \omega, s, \theta_n - \theta_r) \cdot u_s(\bar{x}, \omega, s, \theta_n + \theta_r)^* \cdot |A(\bar{x}, \omega, s, \theta_n - \theta_r)|^2.
\]

(23)

4. Acquisition reciprocity approximation

For the acquisition with only one-side receivers, the acquisition reciprocity approximation can be applied. For image condition (15) and (13), considering the acquisition reciprocity approximation, we get

\[
I(\bar{x}) = \sum_{\omega} \sum_{s} u_s(\bar{x}, \omega, s) \cdot u_s(\bar{x}, \omega, s)^* + \sum_{\omega} \sum_{r} u_s(\bar{x}, \omega, r) \cdot u_s(\bar{x}, \omega, r)^*.
\]

(24)

\[
I(\bar{x}) = \sum_{\omega} \sum_{s} \frac{u_s(\bar{x}, \omega, s) \cdot u_s(\bar{x}, \omega, s)^*}{A(\bar{x}, \omega, s)} + \sum_{\omega} \sum_{r} \frac{u_s(\bar{x}, \omega, r) \cdot u_s(\bar{x}, \omega, r)^*}{A(\bar{x}, \omega, r)}.
\]

(25)

Numerical examples

In this section, using synthetic datasets, numerical examples are calculated to compare different image conditions.

1. The four-layer model with constant background velocity

For the four-layer model (Baina et al., 2002) with a constant background velocity (Figure 2), the synthetic data are generated through full wave FD method. The data set consists of 181 shots with 120 right-side receivers separated by 25m. The distance between shots is 50m. The sound velocity for the background is 4km/s and for the layer reflectors are 6km/s. Wave equation based shot-gather prestack migration is applied to get the image of the model.
The traditional convolution image condition (15), division image condition (16) and weighting division image condition (14) are applied to get the prestack images (see Figure 3), and their image amplitudes along the reflectors are picked and shown in Figure 4. We see that the convolution image condition provides the smoothest image but the amplitudes are farthest from the true reflectivity.
Figure 3. Images of the four-layer model with constant background velocity without amplitude compensation.

Figure 4. Amplitude curves without amplitude compensation.

To eliminate the acquisition effect, the space domain amplitude compensation, equation (13) and (12) are applied, see Figure 5. The amplitude curves along the four reflectors are shown in Figure
6. We can see that the image amplitudes for convolution and weighting division image conditions are almost on the same level. While for the steep reflectors, the image amplitudes still have small variants from the average level, especially for the weighting division image condition.

![Figure 5](image1.png)

**Figure 5.** Images of the four-layer model with constant background velocity with space domain amplitude compensation.

![Figure 6](image2.png)

**Figure 6.** Amplitude curves with space domain compensation.
Similarly, the local angle domain amplitude compensations, (11) and (10), can be considered in the image conditions. We apply the two image conditions respectively and sum up all the angle images to get the space domain images, see Figure 7. The amplitude curves along the four reflectors are shown in Figure 8. We see that the convolution condition’s image amplitudes with local angle domain amplitude compensation have better balance than without amplitude compensation, but are not as good as those of space domain amplitude compensations. The weighting division condition’s image amplitude curves have better balance than those of space domain amplitude compensation, but with a little more unexpected noises.

Figure 7. Images of the four-layer model with constant background velocity with local angle domain amplitude compensation.
2. The four-layer model with vertical variation background velocity

For the same four-layer model but with a vertical variation background velocity as showed in figure 9, synthetic data are generated through full wave Finite-Difference method. The same acquisition system as for the constant background velocity model is applied. The sound velocity for the background is \((4000+0.36\times\text{depth})\) m/s, and for the layer reflectors are 1.3 time the background velocity.

We applied the convolution image condition (15) to the shot gather dataset and the resorted receiver gather dataset, respectively. We also sum up shot and receiver gather’s results to follow the image condition (24). The images and their amplitudes curves are shown in Figure 10 and Figure 11, respectively. We see from the figures that the shot gather and receiver gather favorite different angle reflectors. After applying the reciprocity approximation, the angle reflectors have similar amplitudes.
Figure 10. Image of the four-layer model with $v(z)$ background velocity without amplitude compensation.
Figure 11. Amplitude curves along the images of the four layers without amplitude compensation.

We also applied the convolution image condition with space domain amplitude compensation (13) to the shot gather dataset and the resorted receiver gather dataset, respectively. The reciprocity image condition (25) is implemented by adding the shot and receiver gather’s results together. The image results are shown in Figure 12 and heir amplitudes curves along the four layers are shown in Figure 13.
Figure 12, image of the four-layer model with v(z) background velocity with space domain amplitude compensation.
We see from the figures that after the space domain amplitude compensation. The image amplitudes for the shot gather are balanced except for reflector with negative dip angle. While for the receiver gather, the image amplitudes are balanced except for reflectors with positive angles. The image amplitudes with acquisition reciprocity are almost balanced for reflectors with both negative and positive angles.

3. 2D SEG/EAGE salt model

We also apply the different simplified image conditions in the prestack depth migration for SEG/EAGE salt model, as shown in Figure 14. The synthetic data for the model has 361 shots with 176 left-side receivers separated by 80 feet. The distance between shots is 160 feet. Local cosine beamlet (LCB) propagator (Wu, et al., 2000; Wang and Wu, 2002; Wang, et al., 2003; Luo and Wu, 2003; Luo, et al., 2004; Luo and Wu, 2005) is employed to get the images.
We also apply the convolution image condition with amplitude compensation (13) to the shot gather dataset (Figure 16a) and the resorted receiver gather dataset (Figure 16b), respectively. We can see left-side gathers (shot gather) favor reflectors with negative dip angles and right-side gathers (receiver gather), the positive dip angles. While applying the image condition (25) (Figure 16c), the image amplitudes become more balanced for all reflectors.

Figure 15. Image result from convolution image condition (15).

Figure 16. Images from convolution image condition with space domain amplitude compensation.
We also applied simplified image condition (18), (23) and (22), as shown in Figure 17. We see that image of subsalt structures is greatly improved if the amplitude compensation is applied. The amplitudes along the steep faults and the baseline are much more uniformly distributed after the correction.

![Image result from image condition (18).](image1)

![Image result from image condition (23).](image2)

![Image result from image condition (22).](image3)

Figure 17. Images from local angle domain amplitude compensation.

4. Sigsbee2A model

We applied some the simplified image conditions to the Sigsbee2A model. The dataset contains 500 shots and each with right-side receivers (maximum receivers per shot is 348). The velocity model has 3201 samples in horizontal direction, and 1200 samples in depth (Figure 18).

The local cosine basis (LCB) method (Wu, et al., 2000; Wang and Wu, 2002; Wang, et al., 3003; Luo and Wu, 2003; Luo, et al., 2004; Luo and Wu, 2005) is employed to provide prestack depth
images. The results with image conditions (15), (13), (18), and (22) are shown in Figure 19. We see that imaging with the amplitude compensation show much better results than that of no compensation, especially for the lower salt boundary and in the subsalt area.
True reflection image condition

Figure 19. Image results for Sigsbee2A dataset with different image conditions

Conclusion

The amplitudes of the seismic migration image are greatly affected by the image conditions used. The image conditions that closely related to the true reflectivity definition generate better results. However, trade-off often need be considered to improve the efficiency. The amplitude compensations, space or local angle domain corrections play important roles in pursuing the true image amplitude.

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References


WKBJ Solution and Transparent Propagators

Ru-shan Wu and Jun Cao

Summary

In this paper, we derived the WKBJ solution from the principle of power flux conservation for acoustic media. The numerical results demonstrated that the one-way wave propagator with WKBJ correction provides amplitude that agrees very well with that from the full wave equation method. And we also found that the evanescent waves have a significant influence on the wave amplitude, especially for the near-field waves. Furthermore, we generalize the WKBJ solution for smooth $c(z)$ media to generally heterogeneous $c(z)$ media by introducing the concept of “transparent boundary condition” and “transparent propagators”, which can be useful for true-reflection imaging.

Introduction

Traditionally WKBJ solution is derived by asymptotic approximation for smoothly varying $c(z)$ media (vertically inhomogeneous media), where $c(z)$ is the wave speed at level $z$ (e.g. Morse and Feshbach, 1953; Aki and Richards, 1980; Clayton and Stolt, 1981; Stolt and Benson, 1986). It has been also obtained by introducing an extra amplitude term based on transport equation of high-frequency asymptotics to the traditional one-way wave equations that satisfy only the eikonal equations (Zhang, 1993; Zhang, et al., 2003). In this work, we will derive the WKBJ solution from the principle of energy conservations. In this way, we can generalize the WKBJ solution for smooth $c(z)$ media to a transparent propagator (or energy-conservative Green’s function) for general heterogeneous $c(z)$ media. For $c(z)$ media with discontinuities, we introduce the concept the transparent boundary condition, which implies the neglect of all the scattering and reflection loss during the propagation. Therefore the energy flow is continuous and conserved in both the slowly varying media or across sharp boundaries. Although the transparent boundary condition does not reflect the physical reality, it may be useful and preferred for some inversion procedure or true-reflection imaging (Wu et al., 2004). Further along this line, we can generalize the “transparent propagators” to 3D heterogeneous media.

In this paper, we will first derive the WKBJ solution from the principle of power flux conservation for acoustic media. Then, we demonstrate the correctness of wave amplitude by comparing our results with the results of full-wave equation or theoretical predictions for smooth $c(z)$ media.

WKBJ solution got from energy conservation

Here, we derive the WKBJ solution from power flux conservation. Power flux is defined as “The amount of energy transmitted for unit time across unit area normal to the direction of propagation” (Aki and Richards, 1980). In general elastic media, the strain energy density is...
\[ \frac{1}{2} \tau : \varepsilon = \frac{1}{2} \sum_{i,j} \tau_{ij} \varepsilon_{ij}, \]  

(1)

where \( \varepsilon, \tau \) is the stress and strain tensor. It can be shown that for plane waves the strain energy is equal to its kinematic energy. In the case of acoustic media, the kinematic energy density of plane waves is \( \frac{1}{2} \rho u^2 \), where \( \rho, u \) is media density and particle displacement and \( \dot{u} = \ddot{v} \) is the particle velocity (vector). This means, for plane waves, energy flux rate is \( \rho c \dot{u}^2 \), where \( c \) is the wave propagation speed. Now let us derive the particle velocity vector in terms of pressure field and the propagation direction.

For acoustic media, Newton’s Law in frequency domain is

\[ -\nabla P = -i \omega \rho \dot{v}. \]  

(2)

From equation (2), we get

\[ \frac{\partial P}{\partial z} = i \omega \rho v_z. \]  

(3)

\[ v_z = \frac{1}{i \omega \rho} \frac{\partial P}{\partial z}. \]  

(4)

Assume the plane wave pressure field

\[ P = P_0 \exp[i(k_z x + k_z z - \omega t)]. \]  

(5)

We have

\[ \frac{\partial P}{\partial z} = i k_z P = i k_0 \cos \theta P, \]  

(6)

where, \( k_0 = \omega/c \); \( \theta \) is the angle of wave propagating direction with respect to vertical direction (see Figure 1). Substituting equation (6) into (4), we get

\[ P = \frac{\rho cv_z}{\cos \theta} = \rho cv. \]  

(7)

With equation (7), we can get the power flux along z-direction

\[ \rho cv^2 \cos \theta = \frac{P^2 \cos \theta}{\rho c}. \]  

In case of vertically heterogeneous media \( c(z) \) and \( \rho(z) \), the power flux conservation leads to (see Figure 1).

\[ \frac{P_z^2 \cos \theta_1}{\rho_1 c_1} = \frac{P_2^2 \cos \theta_2}{\rho_2 c_2}. \]  

(8)

We can recognize that \( \rho_1 c_1 / \cos \theta_1 = Z_1 \) is the local impedance of media to the plane wave with propagation angle \( \theta_1 \), and the power flux can also be expressed as \( P^2(z)/Z(z) \). From (8) we get the pressure field change along the z-direction

\[ \frac{P_z}{P_1} = \sqrt{\frac{\cos \theta_1 \rho_2 c_2}{\cos \theta_2 \rho_1 c_1}} = \sqrt{\frac{\rho_2}{\rho_1}} \sqrt{\frac{c_2 / \cos \theta_2}{c_1 / \cos \theta_1}} = \sqrt{\frac{\rho_2}{\rho_1}} \sqrt{\frac{k_z(c_2)}{c_1}.} \]  

(9)

Equation (9) agrees with the WKBJ solution for smoothly varying media (e.g. Stolt and Benson, 1986). However, we can generalize the formulation to heterogeneous media with discontinuities. At each discontinuity, the solution (9) corresponds to a “transparent boundary condition” and the
propagator becomes a “transparent propagator” which neglects all the scattering (reflection) loss across the boundary. The concept of transparent propagator of wave field can be further generalized to 3D heterogeneous media, which can be used in wave field inversion and the true-reflection imaging (Wu et al., 2004).

**Numerical tests**

In this part, we will test the correctness of wave amplitude by comparing the one-way wave propagators with WKBJ correction with the results of full-wave equation (FD) method for smooth $c(z)$ media. The media velocity $c(z)=3.0+0.36z$ (km/s). The source time function is Ricker wavelet with dominant frequency 15Hz. To compare the amplitude clearly, we draw the curve of amplitude vs. distance from the source along the radial direction for different angles $\theta$ (see Figure 2). Figure 3 shows the curves of amplitude vs. distance $r$ along the radial direction for angle $\theta = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$. The upper panel of Figure 3 is for the one-way propagator without WKBJ correction and the lower one is for propagator with WKBJ correction. Comparing with the results from full wave FD method (solid lines in Figure 3), we can see that the one-way propagator without WKBJ correction cannot give the correct amplitude, but with WKBJ correction, the one-way propagator can give almost the same amplitude as the full wave FD method does; even for the larger angles (e.g. $\theta = 75^\circ$) their difference is very small. Figure 4 shows the seismograms got from above two methods along radial direction $\theta = 60^\circ$. The waveforms got from the one-way propagator with WKBJ correction agree very well with those got from FD method. It demonstrates that the one-way propagator with WKBJ correction can give correct travel time either.

![Figure 1](image1.png)

**Figure 1.**

![Figure 2](image2.png)

**Figure 2:** Diagram for extracting the amplitude along the radial direction for different angle $\theta$. Here, the dots represent the receivers.
Figure 3: Curves of amplitude vs. distance from the source for $\theta = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$. The solid lines are the results from FD. The dashed lines are results from phase shift method. The upper panel is for the conventional propagator without WKBJ correction and the lower panel is for the new propagator with WKBJ correction.
Seismograms in v(z) media ($\theta=60^\circ$)

Figure 4: seismograms got from one-way propagator with WKBJ correction (dashed lines) and full-wave FD (solid lines) method along radial direction $\theta = 60^\circ$ in smooth $c(z)$ media.

**Discussion**

We know that the vertical wavenumber $k_z = \sqrt{k_0^2 - k^2}$. In the literature, usually only the propagating waves are considered, i.e. those waves corresponding to $k^2 < k_0^2$, and the evanescent waves corresponding to $k^2 > k_0^2$ are discarded. All of our above implementations include the contribution from evanescent waves. Figure 5 shows the results from the same method as the lower panel of Figure 3 except that Figure 5 did not consider the contribution of the evanescent waves in the one-way propagator. Comparison of Figure 3 and 5 demonstrates that the evanescent waves have a significant influence on the amplitude, especially the near field waves. With the increase of the depth, the contribution from the evanescent waves decrease because the evanescent waves attenuate quickly with the increase of depth.

Above we discussed the theory and numerical implementation of WKBJ correction in smooth $c(z)$ media. Next we will briefly discuss the preliminary application of WKBJ correction in smooth $c(x, z)$ media ($c(x, z) = 3.0 + 0.18x + 0.36z$ (km/s)). The model is 10.24km x 5.12km. The source location is ($x_s=5.12km, z_s=0km$). The one-way propagator we use here is GSP (Generalized-Screen Propagators or Fourier FD) method (Xie and Wu, 1998). Figure 6 shows the comparison of seismograms along radial direction $\theta = 45^\circ$ from GSP method with WKBJ correction (dashed lines) and from the full wave FD (solid lines) method. Here the WKBJ correction uses the minimum velocity of each depth as the reference velocity. The results demonstrate that with WKBJ correction GSP method still can get the correct travel time, but the
amplitudes have obvious difference from those got from FD method. Above discussion demonstrates that the wide-angle correction in current implementation of GSP method may not be able to give correct amplitude of the wave field as a transparent propagator. Further study for wide-angle amplitude correction is on the way.

Figure 5: Same as the lower panel of figure 3 except that here we did not consider the contribution of the evanescent waves in one-way phase shift method.

Figure 6: Seismograms along radial direction \( \theta = 45^\circ \) in smooth \( c(x, z) \) media from GSP (dashed lines) and FD (solid lines) method
Conclusion

We derived the WKBJ solution for one-way wave propagators from the principle of power flux conservation and the solution is generalized to generally heterogeneous media by introducing the concept of transparent propagator. Numerical results demonstrated that the one-way wave propagators with WKBJ correction can get the correct amplitude wavefield in smooth $c(z)$ media. We also found that the evanescent waves have a significant influence on the wave amplitude, especially for the near-field waves. The wide-angle correction in current implementation of GSP or Fourier FD method may not be able to give correct amplitude of the wave field for larger-angle waves.

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References


Influence of Propagator and Acquisition Aperture on Image Amplitude

Jun Cao and Ru-shan Wu

Summary

We apply the local angle domain migration method using one-way wave equations to study the influence of different propagator (with/without WKBJ correction) and different acquisition aperture on the image amplitude. An example of a point scatterer is used to demonstrate the concept and methods in a $c(z)$ media. The results indicate that the propagator and acquisition aperture both influence the imaging amplitude, however aperture correction has much stronger effect on image amplitude than the WKBJ correction for migration with limited aperture acquisition.

Introduction

Traditional wave equation based migration can provide a reflector map consistent with the real velocity model, but provides unreliable amplitude information of the reflectors. True-amplitude imaging tries to give both the correct position and amplitude of the reflectors. The factors influencing the amplitude of the imaging include focusing and defocusing by heterogeneity, geometrical spreading, path absorption and path scattering loss, numerical dispersion and numerical anisotropy, propagator errors and acquisition aperture effect. Among these factors, the amplitude errors caused by one-way wave propagators have been studied extensively in recent years (e.g. Zhang, et al., 2003; Zhang, et al., 2004). In this paper we will compare this effect with the acquisition aperture effect to understand the nature and magnitudes of these two effects.

The one-way wave equation based propagators provide powerful and fast tools for forward modeling and migration, but the original one-way wave equations cannot provide accurate amplitude (Zhang, et al., 2003). With the true-amplitude one-way wave equations, better image amplitude is got in common-shot migration (Zhang, et al., 2003) and common-angle gathers (Zhang, et al., 2004). Most true-amplitude propagators are formulated and implemented in the space-domain. Wu & Cao (2005) proposed a method of amplitude correction based on WKBJ solution in local angle-domain. They demonstrated the agreement of the new one-way wave equations with the full-wave equation by comparing their amplitude and waveforms in a smooth $c(z)$ media.

Due to the limited data acquisition aperture in reality, the inverse-propagated waves cannot completely recover the scattered wave field, which will influence the amplitude of the image. Wu et al. (2004) proposed an amplitude correction method in angle domain with acquisition aperture correction. Their numerical examples showed significant improvement in both the total strength images and the angle-dependent reflection amplitudes, which demonstrated the significance of aperture correction in true-amplitude imaging.

The true reflectivity (or scattering coefficient) depends on the angle, but usually there is no local angle information in the wave equation based migration methods. The recently developed methods, local plane wave analysis based on window Fourier Frame theory (Wu & Chen, 2002)
or local slant stack (Xie & Wu, 2002), can decompose the wave field into localized beamlets carrying angle information, which make it possible to get the image in local angle domain.

In this paper, we will first briefly discuss the WKBJ correction in local angle domain. Then we apply different propagator to get the migration image and study the influence of the propagator and acquisition aperture on the image amplitude in scattering angle domain for a point scatterer in a smooth \( c(z) \) media.

**True amplitude one-way propagator**

**(1) WKBJ solution**

Traditionally WKBJ solution is derived by asymptotic approximation for smoothly varying \( c(z) \) media, where \( c(z) \) is the wave speed at level \( z \) (e.g. Morse and Feshbach, 1953; Aki and Richards, 1980; Clayton and Stolt, 1981; Stolt and Benson, 1986). It has been also obtained by introducing an extra amplitude term based on transport equation of high-frequency asymptotics to the traditional one-way wave equations that satisfy only the eikonal equations (Zhang, 1993; Zhang, et al., 2003). In a recent paper by Wu & Cao (2005), WKBJ solution is also derived from the conservation of power flux in \( z \)-direction,

\[
\frac{P_2}{P_1} = \frac{\cos \theta_1, \rho_2, c_2}{\cos \theta_1, \rho_1, c_1} = \sqrt{\frac{P_2}{P_1} \frac{k_z(c_1)}{k_z(c_2)}},
\]

where \( P, \rho, c, \theta, k \) is pressure, density, velocity, propagation angle and vertical wavenumber, respectively (see Figure 1). In this way, we can generalize the WKBJ solution for smooth \( c(z) \) media to a transparent propagator (or energy-conservative Green’s function) for general heterogeneous \( c(x, z) \) media. For \( c(z) \) media with discontinuities, we introduce the concept the transparent boundary condition, which implies the neglect of all the scattering and reflection loss during the propagation. Therefore the energy flow is continuous in \( z \)-direction and conserved in both the slowly varying media or across sharp boundaries. Although the transparent boundary condition does not reflect the physical reality, it may be useful and preferred for some inversion procedure or true-reflection imaging (Wu et al., 2004). Further along this line, we can generalize the “transparent propagators” to generally heterogeneous media, \( c = c(x, z) \), however the correction then should be done in the local angle domain.

![Figure 1: Diagram for WKBJ correction.](image)
(2) The method of wavenumber integration
In the numerical implementation of the one-way wave equation, we use the frequency-wavenumber domain phase-shift method plus WKBJ correction; so discrete wavenumber method (Bouchon and Aki, 1977; Bouchon, 2003) is actually applied. We calculate the wavenumber integral with discrete Fourier transform. To minimize the influence of the periodic boundary condition and remove the singularities of integrand in wavenumber domain, a small constant imaginary part $\omega_i$ is given to the frequency (Bouchon and Aki, 1977; Bouchon, 2003). The final time-domain single-source solution $f(t)$ can be obtained from the integration of the frequency-domain solution $G(\omega)$ by

\[
f(t) = e^{\omega_i T} \int_{\omega_0}^{\omega_f} G(\omega) e^{-i\omega t} d\omega \ ,
\]

where the integral is computed by using FFT.

(3) Numerical examples about true-amplitude one-way propagator
To compare the amplitude from one-way and full-wave equations clearly, we draw the curve of amplitude vs. distance from the source along the radial direction for different emergency angles $\theta$ (see Figure 2). The media velocity $c(z)=3.0+0.36z$ (km/s). The source time function is Ricker wavelet with dominant frequency 15Hz. Figure 3 shows the curves of amplitude vs. distance $r$ along the radial direction for angle $\theta=0\degree, 15\degree, 30\degree, 45\degree, 60\degree, 75\degree$. Figure 3(a) is for the one-way propagator without WKBJ correction and Figure 3(b) is for propagator with WKBJ correction. Comparison with the results from full wave finite difference (FD) method (solid lines in Figure 3) shows that the one-way propagator without WKBJ correction cannot give the correct amplitudes. The larger the emergency angle is, the larger the error of the amplitude is. For the emergency angle $\theta=75\degree$, the amplitude got from one-way propagator without WKBJ correction is only about half of the true amplitude. However, with WKBJ correction, the one-way propagator can give almost the same amplitude as the full wave FD method does. Even for large angle (e.g. $\theta=75\degree$), their difference is very small. It should be noted that all of our above implementations include the contribution from evanescent waves, which are usually discarded in the literature. In our previous research (Wu & Cao, 2005), we found that the evanescent waves have a significant influence on the wave amplitude, especially for the near-field waves.

Figure 2: Diagram for extracting the amplitude along the radial direction for different emergency angle $\theta$. The dots represent the receivers.
Influence of propagator and acquisition aperture on image amplitude

Figure 3: Curves of amplitude vs. distance from the source for $\theta = 0', 15', 30', 45', 60', 75'$. The solid lines are the results from FD. The dashed lines are results from phase shift method. (a) is for the propagator without WKBJ correction and (b) is for the true-amplitude propagator with WKBJ correction.

Imaging with one-way propagator

In previous part, we have demonstrated that the one-way propagator with WKBJ correction can give almost the same amplitude as the full wave FD method does even for large angles. In this part, we will use the one-way wave propagator to study the influence of the WKBJ correction and aperture correction on image amplitude. Because the true reflectivity (or scattering coefficient) depends on the angle, we use the imaging condition in local angle domain (Wu and Chen, 2002, 2004) to get the local image matrix (LIM) $L(\vec{x}, \vec{\theta}, \vec{\phi})$, which is distorted from the
local scattering matrix (LSM) due to the acquisition aperture limitation and the propagation path effects. In $L(\bar{x}, \bar{\theta}, \bar{\psi})$, $\bar{x} = (x, z)$ is the window position at depth $z$, $\bar{\theta}$ and $\bar{\psi}$ are the source and receiving angles, respectively (Wu, et al., 2004). For a single shot (point source) the imaging condition to obtain the scattering strength (for a single frequency) in local angle domain can be written as,

$$L(\bar{x}, \bar{\theta}, \bar{\psi}) = 2 \frac{G_i(\bar{x}, \bar{\theta}; x, \theta)}{|G_i(\bar{x}, \bar{\theta}; x, \theta)|} \int_{A(x, \theta)} dx \frac{\partial G_i(\bar{x}, \bar{\psi}; x, \psi)}{\partial z} u_i(x, \psi),$$  \hspace{2cm} (3)

where $G_i$ is Green’s function used in the imaging process, which could be different from the Green’s function of forward modeling; “$*$” stands for complex conjugate; $G_i(\bar{x}, \bar{\theta}; x, \theta)$ is the incident field in the local angle domain at the image point $\bar{x}$; and the integral is a back propagation Rayleigh integral, $A(x, \theta)$ is the spatial receiver aperture and $u_i(x, \psi)$ is the recorded scattered waves at receiver $x$, from the source at $x$, on the surface.

To illustrate the influence of WKBJ and aperture correction on image amplitude, we design a simplified problem. A point scatterer is put in a smooth $c(z)$ media and the data is recorded on surface, which theoretically should have the same scattering coefficient in all direction. To avoid the numerical errors during generating the dataset by directly putting the source on surface, which may cause the anisotropy of the scattering coefficient, we generate the dataset by firstly generating a data by a point source at the position of the point scatterer with full wave FD method and then give the data a time delay by which the waves travel from the real source to the imaginary source (see Figure 4). Because there is only one incident angle $\theta = 0^\circ$ for the example here, we can investigate the image at the scattering point $x_0$ in receiving angle $\theta$ domain, $L(\bar{x}_0, 0^\circ, \theta)$, to compare it with the theoretical prediction.

Here, we take the velocity $c(z)$=1.5+0.36$z$ (km/s) and the shot is in the center of the section and the receivers cover the surface in an aperture of 5000m on both sides with a 25m interval. The point scatterer is 2km below the source. The source time function is Ricker wavelet with dominant frequency 30Hz. The final input data for the migration is shown in Figure 5.

First, we will study the influence of WKBJ correction on the image amplitude. Figure 6 shows the image amplitude at the scattering point in the local receiving angle domain for the peak frequency. For the migration with true-amplitude propagator and with full 10km-aperture data (solid line in Figure 6a), the amplitude curve within $\pm 30^\circ$ is almost flat, which agrees well with the theoretical prediction. With the same aperture data but without WKBJ correction (dashed line in Figure 6a), the image is smaller than that from true-amplitude propagator. With smaller 6km aperture data, the results are similar (Figure 6b).

Figure 4: Diagram for the point scattering problem.
Influence of propagator and acquisition aperture on image amplitude

Figure 5: Input data for migration.

Figure 6: Comparison of image amplitude for true-amplitude and original one-way propagator. The solid lines are results with true-amplitude propagator, and the dashed lines are results without WKBJ correction. (a) Full 10km-aperture with receivers on both sides; (b) 6km-aperture with receivers on both sides (The thick solid line is the result for the full 10km-aperture with true-amplitude propagator as the reference curve). The amplitudes are normalized with the maximum among all images.

From Figure 6b, we can see that the acquisition aperture also has significant influence on the image amplitude. Here we will use the amplitude correction method in local angle domain with acquisition aperture correction proposed by Wu et al. (2004) to eliminate the aperture effect. We can get the correspondent amplitude correction factor $F_a$ for above imaging condition (3) according to Wu et al. (2004),
If we use the true-amplitude propagator in migration, the amplitude correction factor can be simplified as

\[
|F_a(\overline{x}, \overline{\vartheta}, \overline{\vartheta_g})| = \left| \frac{G_F(\overline{x}, \overline{\vartheta}_g; x)}{G_F(\overline{x}, \overline{\vartheta}_g; x)} \right| \left\{ \int_{\mathcal{A}(\overline{x})} dx |G_F(\overline{x}, \overline{\vartheta}_g; x)|^2 \right\}^{1/2}, \tag{4}
\]

and

\[
|F_a(\overline{x}, \overline{\vartheta}, \overline{\vartheta_g})| = \left( \int_{\mathcal{A}(\overline{x})} dx |G_F(\overline{x}, \overline{\vartheta}_g; x)|^2 \right)^{1/2}. \tag{5}
\]

Since we only have the image for one incident angle \( \theta_i = 0^\circ \) here, we can do the aperture correction in the local receiving angle \( \theta_r \) domain with above amplitude correction factor. So, the final image in the local receiving angle domain can be written as

\[
\Phi(\overline{x}, 0, \overline{\vartheta}_g) = L(\overline{x}, 0, \overline{\vartheta}_g) [F_a(\overline{x}, 0, \overline{\vartheta}_g) + \varepsilon], \tag{6}
\]

where \( \varepsilon \) is a damping factor for regularization.

We will see first the effect of aperture correction for the original one-way wave propagator without WKBJ correction. Figure 7 shows the image amplitude at the scattering point in receiving angle domain before and after aperture correction for the 6km-aperture data with receivers on both sides. The amplitude after aperture correction (dotted line) is greatly improved for the large scattering angle compared with that before aperture correction (dash-dot line). Figure 8 shows the similar results for the case with 3km-aperture on the right side.

Figure 7: Comparison of image amplitude for one-way propagator before and after aperture correction for the 6km-aperture data with receivers on both sides. The dash-dot line is the result before aperture correction; the dotted line is result after aperture correction; and the thick solid line is the result for the full-10km aperture with true-amplitude one-way propagator as reference curve. The dashed line is the result with WKBJ correction only; and the solid line is with both WKBJ and aperture correction. The amplitude after aperture correction is normalized with its maximum. All the other amplitudes are normalized with their maximum.
Influence of propagator and acquisition aperture on image amplitude

Figure 8: Same as Figure 7 except that the data is 3km-aperture on the right side.

Finally, we compare the influence of WKBJ correction and aperture correction. The dashed line and thin solid line in Figure 7 are the same as the dash-dot line and dotted line respectively except that they use the true-amplitude one-way propagator with WKBJ correction. Figure 7 shows that the result only with aperture correction improves the amplitude more than that only with WKBJ correction, and the result with both WKBJ and aperture correction gives the best amplitude distribution with scattering angle. These results demonstrate that aperture correction has stronger effect on image amplitude than the WKBJ correction for the example shown here. The results are similar for the one-side receiver aperture case (see Figure 8).

Conclusion

We apply the local angle domain migration method using one-way wave equations to study the influence of different propagator (with/without WKBJ correction) and different acquisition aperture in migration on the image amplitude in the local scattering angle domain for a point scattering problem in a c(z) media. The results show that the propagator and acquisition aperture both influence the imaging amplitude, however the result only with aperture correction improves the amplitude more than that only with WKBJ correction, and the result with both WKBJ and aperture correction gives the best amplitude distribution with scattering angle. These results demonstrate that aperture correction has much stronger effect on image amplitude than the WKBJ correction for migration with limited acquisition aperture.

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Reference


True amplitude one-way propagators implemented with localized corrections on beamlets

Mingqiu Luo, Ru-Shan Wu and Xiao-Bi Xie

Summary

The transmission coefficient and WKBJ approximation can be adopted for constructing the true amplitude propagators. For vertically inhomogeneous media, the WKBJ approximation or transmission coefficients can be applied on the plane waves at each depth to get the true amplitudes. While for the media with lateral variation, the correction coefficients vary with horizontal locations, and cannot be applied on the global plane waves. However, the beamlet propagators decompose the wavefield into localized beamlets, and the laterally varying WKBJ or transmission correction coefficients can be conducted on a localized basis at each depth. In this paper, the theory for the localized WKBJ and transmission corrections are proposed and implemented with the local cosine basis (LCB) beamlet propagator. Numerical examples of the impulse responses are calculated to demonstrate the feasibility of the local WKBJ and local transmission corrections.

Introduction

Theoretically, the WKBJ solution is valid only for vertically smoothly varying media, in which velocity \(v(z)\) is a function of depth (e.g. Morse and Feshbach, 1953; Aki and Richards, 1981; Clayton and Stolt, 1981; Stolt and Benson, 1986). It has been also obtained and generalized to complex media by introducing an extra amplitude term based on transport equation of high-frequency asymptotics to the traditional one-way wave equation that satisfy only the eikonal equations (Zhang, 1993; Zhang, et al., 2003). Based on the principle of energy conservations, it also can be derived as transparent propagator, by introducing the concept of transparent boundary condition (Wu and Cao, 2005).

While for the media with rapid velocity variation, the WKBJ approximation could be invalid. For one-way methods, the reflected waves are omitted and the transmitted waves may not be their true values. But for most cases, especially for the media with limited sharp velocity interfaces, the transmission coefficient for one-way wave provides a good approximation to the true transmitted waves.

For the vertical heterogeneous media, the wavenumber dependent WKBJ approximation and transmission coefficient can be implemented on each global wavenumber (plane wave) at each depth. While for the heterogeneous media, the correction coefficients vary with location and can’t be applied on the global wavenumber directly. On the other hand, the beamlet propagation method decomposes the wavefield into beamlets and each beamlet has a location and local wavenumber (Wu et al., 2000; Wang and Wu, 2002; Luo and Wu, 2003; Luo, et al, 2004; Luo and Wu, 2005), the location and wavenumber dependent WKBJ and transmission corrections can be applied on localized beamlets at each depth.
Global WKBJ and transmission correction

For heterogeneous media $v(x, z)$, the wave equation in Cartesian coordinates is

$$[\partial_{zz} + \partial_{xx} - v^2(x, z)\partial_t^2]u(x, z, t) = 0,$$  \hspace{1cm} (1)

where we are considering the 2-D wave propagation problem and $u(x, z, t)$ is the space domain wavefield. In frequency-space ($f$ - $x$) domain, the wave equation can be written as

$$[\partial_{zz} + \partial_{xx} + \omega^2/v^2(x, z)]u(x, z, \omega) = 0.$$  \hspace{1cm} (2)

We first consider a simple case where velocity is a function of depth $z$ alone, i.e., $v = v(z)$. The wavefield at depth $z$ can be decomposed to a superposition of plane waves

$$u(x, z, \omega) = \sum_{k_x} u(k_x, z, \omega) \exp(ik_x \cdot x),$$  \hspace{1cm} (3)

where $k_x$ is the horizontal wavenumber. Each plane wave satisfies

$$(\partial_{zz} + k_z^2)u(k_x, z, \omega) = 0, \quad k_z^2 = \omega^2/v^2(z) - k_x^2$$  \hspace{1cm} (4)

where $k_z$ is the vertical wavenumber and it is velocity dependent.

For each plane wave, the phase and the amplitude should change after one step of propagation. Fig.1 shows two plane waves propagating in the $v(z)$ model. The phase and amplitude relation between the incident plane waves and transmission plane waves depend on the change of velocities in the vertical direction.

If the velocities vary continually with the depth and the scattering (reflection) loss can be omitted, the WKBJ approximation can be applied, and the downward continuation from depth 0 to $z$ can be written as

$$u(k_x, z, \omega) = u(k_x, 0, \omega) \frac{k_z(z)}{k_z(0)} \exp\{i\int_0^z k_z(z')dz'\}. \hspace{1cm} (5)$$

Equation (5) agrees with the WKBJ solution for smoothly varying media (e.g. Stolt and Benson, 1986).

![Fig. 1. Plane waves propagation in v(z) model.](image)

If the velocity varies rapidly with depth and the transmission loss exists, the downward continuation from depth 0 to $z$ can be written as

$$u(k_x, z, \omega) = u(k_x, 0, \omega) \frac{2k_z(0)}{k_z(0) + k_z(z)} \exp\{i\int_0^z k_z(z')dz'\} \hspace{1cm} (6)$$

Equation (6) is a simplified transmission coefficient correction for plane waves, which omits the
contribution from $k_x$.

**Local WKBJ approximation and transmission correction**

For laterally varying media $v(x, z)$, the relations (5) and (6) are no longer valid. However, instead of the global plane wave decomposition of the wavefield, the beamlet decomposition is applied to decompose the wavefield into beamlets (Wu et al., 2000; Wang and Wu, 2002; Wang, et al., 2003; Luo and Wu, 2003)

$$u(x, z, \omega) = \sum_n \sum_m u(\bar{x}_n, \bar{\xi}_m) b_{mn}(\bar{x}_n, \bar{\xi}_m, z, \omega),$$  \hspace{1cm} (7)

where $b_{mn}$ is the decomposition vector (beamlet), $u(\bar{x}_n, \bar{\xi}_m)$ are the coefficients of the decomposition beamlets located at $\bar{x}_n$ (space locus) and $\bar{\xi}_m$ (wavenumber locus), with

$$\bar{\xi}_m = m\Delta_\xi, \quad \bar{x}_n = n\Delta_x.$$  \hspace{1cm} (8)

For each beamlet with $\bar{x}_n$ and $\bar{\xi}_m$, it satisfies approximately,

$$\left[ \partial_{zz} + k_z^2(z, \bar{x}_n)b_{mn}(\bar{x}_n, \bar{\xi}_m, z, \omega) = 0, \quad k_z^2(z, \bar{x}_n) = \omega^2 / v^2(\bar{x}_n, z) - \bar{\xi}_m^2 \right]$$  \hspace{1cm} (9)

where $k_z(z, \bar{x}_n)$ is the vertical wavenumber for the beamlet and depends on the velocity in the window.

Each beamlet is localized to its window, and in most cases, the horizontal velocity variation in a window can be very small. As an approximation, instead of the original heterogeneous media $v(x, z)$, a window-constant velocity model $v(\bar{x}, z)$ is used to calculate the phase and amplitude change during the propagation (see Fig.2). The phase and amplitude between the incident beamlet and transmission beamlet depend on the velocity vertical variation in the window. Similar to the global plane wave propagation in $v(z)$ media, the WKBJ approximation and transmission coefficient can be applied to the beamlet propagation in windows.

![Fig. 2. Beamlet waves propagation in heterogeneous media](image)

If the velocities vary continually with depth within the window and the scattering (reflection) loss can be omitted, the downward continuation beamlet with WKBJ approximation at depth 0 and $z$ can be written as

$$b_{mn}(\bar{x}_n, \bar{\xi}_m, z, \omega) = b_{mn}(\bar{x}_n, \bar{\xi}_m, 0, \omega) \frac{k_z(0, \bar{x}_n)}{k_z(z, \bar{x}_n)} \exp(i\int_{0}^{z} k_z(z', \bar{x}_n) dz').$$  \hspace{1cm} (10)

Here vertical wavenumber $k_z$ also depends on the beamlet window location. As shown in Fig.2, two incident beamlet have different window locations will result to different transmission corrections.
If the velocity varies rapidly with depth and the transmission loss cannot be neglected, the downward continuation beamlet with transmission approximation between depth 0 and \( z \) can be written as

\[
b_{mn}(\vec{x}_n, \vec{x}_m, z, \omega) = b_{mn}(\vec{x}_n, \vec{x}_m, 0, \omega) \frac{2k_z(0, \vec{x}_n)}{k_z(0, \vec{x}_n) + k_z(z, \vec{x}_n)} \exp\{i\int_{0}^{z} k_z(z', \vec{x}_n)dz'\} \tag{11}\]

**Numerical examples**

Here, the full wave FD method, one-way phase shift method and LCB method are applied to generate impulse responses for comparison. The full wave FD method is supposed to provide the true amplitude impulse responses in all cases. The phase shift method is convenient for applying the global transmission and WKBJ correction according to (5) and (6). For the LCB beamlet method, at each depth, the frequency space domain wave field is decomposed into local beamlets, and each beamlet has its location and wavenumber. The location and wavenumber related transmission or WKBJ correction coefficient, equation (10) and (11), can be applied to each beamlet separately. For the following calculations, a same Ricker wavelet with dominant frequency 15Hz is used for all the models and methods.

1. **Vertically heterogeneous media**

We first calculate the impulse responses in a two-layer model and a vertically linearly varying model to see the effects of the transmission and WKBJ approximation. The LCB method is also applied to these models to see the difference between global and local transmission or WKBJ approximation.

The two-layer model has an upper velocity of 2km/s and a lower velocity of 4km/s, with the interface located at depth 1.5km. The full wave FD method, one-way phase shift method and LCB method are applied to generate impulse responses. As shown in Fig.3, the wavefields at 0.5s, 1.0s and 1.5s are added together for comparison. We also pick the wave fields at distance 5.11km and 4.39km for comparison. As shown in Fig.4, the first trace is the wave field by full wave FD methods. The second trace is the difference between the phase shift and FD method. The third trace is with transmission correction, and the fourth trace is with WKBJ correction.
We can see from Figs. 3 and 4 that: 1) The general amplitude difference between the one-way phase shift method and full wave FD method does exist, but not very large in the small angles, even after the sharp velocity interface; 2) The transmission or WKBJ correction can improve the wave field amplitude after crossing the velocity interface, especially for wide angles; 3) The WKBJ correction seems over corrected the wide angle amplitudes slightly. It is reasonable since...
it omits the transmission loss; 4) The LCB method with local transmission or WKBJ correction can provide the same results as the phase shift method with global correction; 5) the transmission and WKBJ methods provide nearly the same amplitude improvement in the near vertical angles. It is reasonable because the formula (5) and (6) are similar for small wavenumbers.

The vertically linearly varying model has a minimum velocity of 2km/s, and a linear varying parameter $\frac{dv}{dz}$ of 0.4/s. The wave fields at 0.5s, 1.0s and 1.5s are picked and shown in Fig.5 for comparison. We also pick the wavefield curves at distance 5.11km and 4.39km for comparison, which is shown in Fig.6.

Similarly we see from the Figs.5 and 6 that: 1) The amplitude errors by the one way phase shift method are generally small but increase with the angle and the distance to the source; 2) Global and local transmissions or WKBJ corrections can improve the amplitudes to their true values; 3) The transmission and WKBJ correction make nearly same amplitude improvements, especially for waves with small incident angles, but the WKBJ correction seems provide better amplitude.
improvements for wide-angle waves.

2. Heterogeneous media

We also applied the full wave FD method and LCB method to get the impulse response for two heterogeneous media, one is the model which has a high velocity lens, the other is depth and distance related linearly varying model. In these models, the one-way phase shift method and global transmission or WKBJ correction are not valid.

The lens model has a back ground velocity 2km/s and high lens velocity 4km/s. The wavefields at 0.5s, 1.0s and 1.5s are added together for comparison, as shown in Fig.7.
We can see from Fig. 7 that the amplitude error for LCB method exists but is small, except for the wide angle waves. With the local transmission or WKBJ corrections, the amplitudes of wide angle waves are improved. However, using the WKBJ method, they are slightly over corrected.

The model in which the velocity varying with depth and distance linearly have a minimum velocity of 2km/s, and linear varying parameters $\frac{dv}{dz}$ of 0.25s$^{-1}$ and $\frac{dv}{dx}$ of 0.125s$^{-1}$. The wave fields at 0.5s, 1.0s and 1.5s are picked and added together for comparison, as shown in Fig. 8.

We can see from Fig. 8 that both local transmission and WKBJ correction can improve the amplitudes, especially for wide angle waves.
Conclusion

The transmission coefficient and WKBJ approximation can be adopted for constructing the true amplitude propagators. The transmission correction is suitable for rapid velocity changes while the WKBJ approximation favors the smoothly varying media. For vertically heterogeneous media, the transmission and WKBJ corrections can be applied to global plane waves. Similarly, for laterally varying media, the localized transmission and WKBJ correction can be conducted using localized beamlets.

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PART II

VELOCITY UPDATING IN LOCAL ANGLE DOMAIN
Residual migration velocity analysis in local angle domain

Hui Yang, Mingqiu Luo, Xiao-Bi Xie and Ru-Shan Wu

Summary

Common angle image gathers generated from local image matrix in local angle domain can extract more angle related information, such as the dip angle of the reflector. This new information can be used in residual migration velocity analysis for curved interfaces in locally inhomogeneous media. We demonstrate how the relationship between the depth and reflection angle, dip angle of the reflector for a given event on a local angle domain can be expressed as a function of the migration interval velocities and true interval velocities of overlying layers. Two simple numerical examples are given to show the features of the new approach and its feasibility.

Introduction

Based on the assumptions of small dip, small offset and/or constant-velocity layered model, many velocity analysis methods are derived to update the migration velocity model by investigating the migration residual moveout in various types of common image gathers (e.g., Yahya, 1989; Liu and Bleistein, 1992; Meng et al., 1999; Jiao, et al, 2002; Mosher and Foster, 2001; Sava, et al., 2003). However, for most of these velocity analysis methods, the structure dipping angle is usually not considered.

In this paper, we investigate the velocity updating using information in the local imaging matrix (Wu and Chen, 2002; Xie and Wu., 2002; Wu, etal., 2004). The local imaging matrix is composed of angle domain partial images from all possible local scattering events. The information on reflector dipping angle and angle domain common image gather can be extracted from the imaging matrix. Using this information, the common image gathers for dipping structures can be obtained and velocity analysis based on correct migration moveout can be conducted.

(a)                                           (b)
Fig1. Local angle domain plane wave decomposition.
Review of the local image matrix

The local image matrix (LIM) can be seen as the expansion of the conventional image condition. It is a function of the incidence-scattering angle pairs at the target reflector. The local image matrix can be defined as:

\[
L_a(\theta_s, \theta_g, x, z, \omega) = \int \int W_a(\theta_s, x, z; x_s, \omega) U_a^*(\theta_g, x, z; x_g, \omega) dx_g dx_s
\]

where \(\omega\) is the frequency, \(\theta_s\) is the local incident angle, \(\theta_g\) is the local receiving angle, \(W_a(\theta_s, x, z; x_s, \omega)\) is the local incident plane wave (incident beamlet) from a source at \(x_s\), and \(U_a^*(\theta_g, x, z; x_g, \omega)\) is the local scattering plane wave (scattered beamlet) from a receiver at \(x_g\). The subscript \(a\) denotes the width of the decomposition window, \(A_s\) and \(A_g\) are the apertures of source and receiver arrays, respectively. The inner integral sums up the contributions from all the receivers for the same shot to a scattered beamlet at \(\theta_g\); the outer integral sums contributions from different sources. The decomposition of wavefields into angle domain can be conducted using the GDF (Gabor-Daubechies Frame) transform (Wu and Chen, 2002), using a windowed Fourier transform or using a local slant stacking (local Radon transform) (Xie and Wu, 2002).

Applying the image condition to local incident wave with angle \(\theta_s\) and local scattering wave with angle \(\theta_g\), and stacking them for all frequencies result in a partial image in local angle domain \((\theta_s, \theta_g)\). The local image matrix is composed of all the partial images. The final total image in the space domain comes from the local image matrix by summing up the contributions of all scattering events \((\theta_s, \theta_g)\).

The local angle coordinates in source-receiving angle pairs \((\theta_s, \theta_g)\) can be transformed to normal reflection angle pairs \((\theta_n, \theta_r)\). For the case of acoustic waves (P-P scattering), we have the angle coordinate transform from \((\theta_s, \theta_g)\) to \((\theta_n, \theta_r)\):

\[
\begin{align*}
\theta_g + \theta_s &= 2\theta_n \\
\theta_g - \theta_s &= 2\theta_r
\end{align*}
\]

where \(\theta_n\) is the normal direction of the dipping reflector and \(\theta_r\) is the reflection angle with respect to the normal vector of the dipping reflector (Fig 1). Because of the mirror reflection of plane interfaces, we can sum up all the responses for different reflection-angles for a common normal-direction \(\theta_n\), i.e.,

\[
I(\theta_n, x, z) = \sum_{\theta_g} I(\theta_n, \theta_g, x, z)
\]

This reflector-dip angle image gather is one type of the CAI (common-angle image) gather, and can be called CDAI (common dip-angle image) gathers. For local planar reflectors, the CDAI gathers have peaks at the corresponding reflector dip-angles (Xie and Wu, 2002).

For the purposes of local AVA (amplitude versus angle) analysis or residual migration velocity analysis, we can sum up all elements of different dip-angles for a common reflection-angle
image gather,

\[ I(\theta_s, x, z) = \sum_{\theta_r} I(\theta_r, \theta_s, x, z) \]  

(4)

Other image gathers can also be extracted from the image matrix (Wu and Chen, 2002; Xie and Wu, 2002; Wu et al., 2004).

**Residual moveout in z – \( \theta \) domain**

Considering a plane wave incident into the layer at angle \( \theta_s \) to the vertical direction, and the reflected wave passing through the layer at \( \theta_r \), we have \( \theta_s = \theta_r = \theta \) at a horizontal reflector in the locally lateral homogeneous medium (see Fig 2). For \( \Delta t \) which is dual interval vertical delay time across the layer, we have,

\[
\Delta t = \frac{2\Delta z \cos \theta}{\nu'} \tag{5}
\]

where \( \Delta z \) is the vertical thickness of the model, \( \nu' \) is the interval velocity of the layered medium, which is locally lateral homogeneous. \( \theta_r \) is the reflection angle of the plane wave in the layer. We can obtain

\[
\Delta z = \frac{\Delta t \nu'}{2 \cos \theta_r} \tag{6}
\]

For migration, we use a trial interval velocity \( \nu'^m \) for the layer, similarly we can get

\[
\Delta z'^m = \frac{\nu'^m \Delta t}{2 \cos \theta'^m} \tag{7}
\]

The difference between the depth \( \Delta z \) from the true interval velocity pre-stack depth migration and the depth \( \Delta z'^m \) from the trial interval velocity pre-stack depth migration will leads to a residual moveout \( \Delta z^d \),

\[
\Delta z^d = \Delta z - \Delta z'^m = \frac{\nu' \Delta t}{2 \cos \theta_r} - \frac{\nu'^m \Delta t}{2 \cos \theta'^m}. \tag{8}
\]

Considering equation (6), the equation (7) can be rewritten as,
\[
\Delta z^d = \Delta z^m \left( \frac{v' \cos \theta^m - 1}{v'' \cos \theta_r} \right). \tag{9}
\]

According to equation (8), we obtain the true depth,
\[
\Delta z = \Delta z^d + \Delta z^m \tag{10}
\]

For a local dipping reflector with dip angle \(\theta_n\) (as shown in Fig 3), considering a plane wave incidents into the layer at angle \(\theta_s\), and the reflected wave passing through the layer at angle \(\theta_g\), the two-way intercept time can be expressed,
\[
\Delta t = \Delta z \left( \frac{\cos \theta_s + \cos \theta_g}{v'} \right) \tag{11}
\]

Incident wave reflec ted wave

![Dipping reflector and related wave propagation direction.](image)

where \(\Delta z\) is the vertical interval, \(v'\) is the true interval velocity of the \(i\)th layer around the reflecting point, which is locally lateral homogeneous. The thickness can be written as,
\[
\Delta z = \frac{v' \Delta t}{\cos \theta_s + \cos \theta_g} \tag{12}
\]

Similarly, after migration with a trial interval velocity \(v''\), we have
\[
\Delta z^m = \frac{v'' \Delta t}{\cos \theta_s^m + \cos \theta_g^m} \tag{13}
\]

where \(\Delta z^m\) is the apparent depth for the layer.

The difference between the true depth \(\Delta z\) from the true interval velocity prestack depth migration and the depth \(\Delta z^m\) from the trial interval velocity prestack depth migration leads to a residual moveout \(\Delta z^d\),
\[
\Delta z^d = \Delta z - \Delta z^m = \Delta z^m \left( \frac{v' \cos \theta^m + \cos \theta_g^m}{v'' \left( \cos \theta_s^m + \cos \theta_g^m \right)} - 1 \right). \tag{14}
\]
Using equation (2), we have

\[ \Delta z^d = \Delta z^m \left( \frac{v' \cos \theta_i \cos \theta_e - v \cos \theta_i \cos \theta_e}{\cos \theta_i \cos \theta_e - 1} \right). \]  

(15)

After a trial velocity shot migration, and with the application of local angle domain imaging conditions, we obtain the CDAI and CRAI. Then the local reflector dip can be estimated from the CDAI. With this estimation, dip residual moveout can be calculated on the CRAI. We will demonstrate the method through two simple numerical models which will be the initial procedure of the residual migration velocity analysis.

**Horizontal layer model**

We first use a constant velocity model with a horizontal reflector at depth 1 km, and the velocity is 5 km/s (as shown in Fig 4). A 15 Hz wavelet is used for calculating synthetic data set. The geometry is dual-side 480 receivers. The interval of receivers and shots are both 25 m. The first shot is at distance 8 km and there are total 80 shots are calculated for this model. The constant velocity model used for prestack migration is 741 points in x-direction and 401 points in z-direction. The interval of x-direction and z-direction are 25 m and 10 m, respectively.

We use two trial velocities, one is 10 percent higher and another is 10 percent lower, and as a comparison we also calculate the true velocity.

On the horizontal reflector, we only demonstrate the result at 9 km in x-direction. After applying the reflector-dip imaging condition and the reflector-angle imaging condition during the prestack depth migration, we obtain the CDAI, as shown in Figs 5a, 5b and 5c; and CRAI, as shown in Figs 6a, 6b and 6c, corresponding to velocity 4.5 km/s, 5.0 km/s and 5.5 km/s, respectively. The horizontal coordinate is degree for dip angle or reflector angle, and the vertical coordinate is km. For the horizontal reflector in constant velocity, the CDAIs have peak energy near the zero dip angle even for erroneous migration velocity. Apparently, the CRAIs have curve-up smile for lower velocity and curve-down cry for higher velocity. Our residual migration velocity analysis will be based on these two types of local angle domain common image gathers.

To compare with the traditional residual migration velocity analysis, we calculate the shot index common image gathers at the same point and show them in Figs 6d, 6e and 6f, corresponding to velocity 4.5 km/s, 5.0 km/s and 5.5 km/s, respectively.
Migration velocity analysis in angle domain

The final images of the prestack migration are shown in Figs 7a, 7b and 7c for velocities 4.5 km/s, 5.0 km/s and 5.5 km/s, respectively.

Fig 5. The CDAIs for constant velocity horizontal reflector model. (a) velocity 4.5 km/s, (b) 5.0 km/s, and (c) 5.5 km/s.

Fig 6. The shot index CIGs and CRAIs after residual migration moveout correction for constant velocity horizontal reflector model. (a), (b) and (c) are CRAIs for 4.5 km/s, 5.0 km/s and 5.5 km/s, respectively. The horizontal coordinate is degree for reflector angle. (d), (e) and (f) are the shot index CIGs. The horizontal coordinate is km. All vertical coordinates are km.

The final images of the prestack migration are shown in Figs 7a, 7b and 7c for velocities 4.5 km/s, 5.0 km/s and 5.5 km/s, respectively.
Normally, we don’t know the true velocity. To obtain the true velocity, we can use velocity scan. We decide to adopt the layer-stripping and residual migration velocity scan strategy. We choose 50 m/s as the velocity scan interval and take the trail migration velocity as the middle value, then proceed 101 steps. Shown in Fig 8a is the angle domain common image gather, overlapped with the theoretical migration moveouts. Numbers on the left side of the curves are different velocities. Note that the flat line near 5.0 km corresponds to the migration moveout for the true velocity. Shown in Fig 8b are residual moveout after subtracted the moveout at zero reflection angle. In the cross-corelation profile (Fig 8c), the peak amplitude focuses near the true velocity 5.0 km/s.

For a higher trail velocity (5.5 km/s), the similar results are shown in Fig 9.

Fig7. The final image of horizontal reflector model, with (a) velocity 4.5 km/s, (b) velocity 5.0 km/s, and (c) velocity 5.5 km/s, respectively.
Fig 8. (a) CRAI gather overlapped with theoretical moveouts curves for different velocities. The numbers denote different velocities in km/s. The horizontal coordinate is angle; (b) Similar to 8a, except the moveout at zero angle have been subtracted from theoretical curves. (c) Result of velocity scan. The horizontal coordinate is velocity in m/s. The vertical coordinates for all panels are km.

Fig 9. Similar to the results shown in Fig 8, except a higher trial velocity of 5.5 km/s is used.
Dipping reflector model

For a constant velocity model with a 20 degree dipping reflector and a 5 km/s velocity is shown in Fig10. A 15 Hz wavelet is used to calculate the synthetic data set. The geometry is dual side with 480 receivers. The interval of receivers and shots are both 25 m. The first shot is at 8 km and a total of 80 shots is used. The constant velocity model used for pre-stack migration is 741 points in x-direction and 901 points in z-direction. The interval of x-direction and z-direction are 25 m and 10 m respectively. Two trial velocities, one is 10 percent higher and the other is 10 percent lower, are used in the calculation. As a comparison, we also calculated the true velocity.

On the dipping reflector, we only demonstrate the result at point at 7.85 km in x-direction. After applying the reflector-dip imaging condition and the reflector-angle imaging condition during the pre-stack depth migration, we obtain the CDAI, as shown in Figures 11a, 11b and 11c, and CRAI, as shown in Figs 12a, 12b and 12c, corresponding to velocities 4.5 km/s, 5.0 km/s and 5.5 km/s, respectively. The horizontal coordinate is degree for dip angle or reflector angle and the vertical coordinate is km. For the dipping reflector in a constant velocity model, the CDAIs have peak amplitude near the 20 degree dip angle. Nevertheless, for erroneous migration velocity, there is an obviously departure to the theoretical dip angle. The CRAIs have some curve-up for lower velocity and curve-down for higher velocity. Our residual migration velocity analysis will be based on these two kinds of local angle domain common image gathers.

As a comparison with the traditional residual migration velocity analysis based on the shot index common image gathers, we also calculate them at the same point and show the results in Figures 12d, 12e and 12f, corresponding to velocities 4.5km/s, 5.0km/s and 5.5 km/s, respectively.
Fig 11. The CDAIs for constant velocity horizontal reflector model. (a) CDAI for velocity 4.5km/s, (b) CDAI for 5.0km/s, (c) CDAI for 5.5km/s. The horizontal coordinate is degree for dip angle or reflector angle and the vertical coordinate is km.

Fig 12. The CRAGs and shot index CIGs for constant velocity horizontal reflector model. (a), (b) and (c) are CRAGs for 4.5km/s, 5.0km/s and 5.5km/s respectively. The horizontal coordinate is degree for reflector angle; (d), (e) and (f) are shot index CIGs for 4.5km/s, 5.0km/s and 5.5km/s, respectively. The horizontal coordinate is km; All vertical coordinates are km.
The final images of the pre-stack migration are shown in Figures 13a, 13b and 13c, corresponding to velocity 4.5km/s, 5.0km/s and 5.5 km/s respectively.

For the velocity scan, we choose 50m/s as the velocity interval and take the trail migration velocity as the middle value, then proceeding 101 steps. As shown in Fig 14a, we compose the theoretical migration moveouts for different velocity on the CRAI of 4.5km/s. The numbers in the left side of the figure denote the different velocity used to scan, where we show every tenth theoretical move-out for corresponding velocity. Pay attention to the flat line near 5.0, it corresponds to the migration move-out of true velocity. In order to produce residual migration velocity scan profile as shown in Fig 14c, we only concern about the residual move-outs of different scan velocity (as shown in Fig 12b). In the cross-corelation profile (Fig 14c), the peak amplitude focuses near the true velocity 5.0 km/s. However, comparing with the velocity scan profile of horizontal reflector, there is a big dispersion. Nevertheless, we can also pick velocity from the profile interactically or automatically.

For higher trail velocity 5.5km/s migration, as in Fig15, we have the similar results to lower trail velocity 4.5km/s, as shown in Fig14. But the difference between Fig14c and Fig15c is the later has a more concentrated peak amplitude. At the same time, the peak amplitude focus at lower velocity to the true velocity.

**Conclusion and Discussion**

Common angle image gathers generated from local image matrix in local angle domain can extract more angle related information, such as the dip angle of the reflector, and further be used in residual migration velocity analysis for curved interfaces in laterally heterogeneous media. We demonstrate how the relationship between the depth and reflection angle, dip angle of the reflector for a given event on a local angle domain can be expressed as a function of the migration interval velocities and true interval velocities of overlying layers resulting from two numerical examples verified the validity of our approach.
**Migration velocity analysis in angle domain**

Fig 14. The theoretical residual moveouts for velocity scan on 4.5 km/s constant velocity horizontal reflector model. (a) Theoretical moveouts on CRAI for different velocity. The numbers denote the different velocity in km/s. The horizontal coordinate is reflective angle; (b) Theoretical residual move-outs on CRAI for different velocity. The horizontal coordinate is degree for reflective angle. (c) The profile of cross-corelated on CRAIs before stack for different velocity. The horizontal coordinate is velocity in m/s. For all these figures the vertical coordinates are km.

Fig 15. The theoretical residual moveouts for velocity scan on 5.5 km/s constant velocity horizontal reflector model. (a) Theoretical moveouts on CRAI for different velocity. The numbers denote the different velocity in km/s. The horizontal coordinate is reflective angle; (b) Theoretical residual move-outs on CRAI for different velocity. The horizontal coordinate is degree for reflective angle. (c) The profile of cross-corelated on CRAIs before stack for different velocity. The horizontal coordinate is velocity in m/s. For all these figures the vertical coordinates are km.
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References


Velocity analysis in local angle domain
Common image gathers in the plane-wave domain: A prestack Gaussian beam migration algorithm

Qiyu Han and Ru-Shan Wu

Summary

Prestack depth migration is a popular tool for velocity updating. A good migration method usually improves the performance of a velocity update. Gaussian beam migration is a desirable alternative to Kirchhoff migration due to its ability to image complicated geologic structures without loss of either accuracy or efficiency. Gaussian beam migration is often used in depth imaging, and is a potential tool for model building. In this paper we present a prestack plane-wave Gaussian beam depth migration algorithm which provides common image gathers for velocity analysis in the plane-wave domain. The algorithm may be performed with either common-offset gathers or common-shot gathers. Applying the imaging condition in the time-domain is more efficient than in the frequency domain. Common-offset migration is more efficient and accurate than common-shot migration with a time-domain imaging condition. Computational efficiency is an important factor in depth-migration based model building since velocity updating requires iterative migration processing. To speed up the algorithm with common-shot data, we ignore the amplitude of the shot field to efficiently apply the imaging condition in the time domain. With synthetic data sets, we test the proposed algorithm and illustrate its potential use for velocity updating with complex media. Our results are consistent with the expectation that in the plane-wave domain, the events in a common-image gather are sensitive to model velocity and interface inclination, while the imaged event contours are flat when the true velocity is used.

Introduction

Prestack depth migration is a powerful tool for migration velocity building. Image gathers from prestack depth migration methods provide imaging information for use with velocity updating approaches. The migration methods used for velocity building include common-shot migration and common-receiver migration (Al-Yahya, 1989), common-offset migration (Deregowski, 1990), plane-wave migration and constant-angle migration (Whitmore and Garing, 1993), and common focusing point migration (Berkhout, 1997; Kabir and Verschuur, 2000). Prestack Kirchhoff migration can be efficiently implemented in either the common-shot domain, the common-receiver domain, the common-offset domain, or the plane-wave domain (Akbar et al., 1996).

Kirchhoff depth migration is popular for use in velocity analysis. An alternative to Kirchhoff depth migration is Gaussian beam migration, which avoids many of the problems associated with complex migration models. Gaussian Beam depth migration is an accurate and efficient method for imaging in complex media (Hill, 1990, 2001; Hale, 1992; Gray, 2004). Hill (2001) presented an efficient and accurate prestack common-offset algorithm for Gaussian beam migration. This algorithm may be applied to velocity analysis in the offset domain. Gray (2004) performed a common-shot Gaussian beam depth migration and discussed the performance of these prestack migration algorithms.
A spherical wave from a point source can be decomposed into a series of plane waves. Stoffa et al. (1981) decomposed seismic reflection data into plane waves by slant stacking (the $\tau - p$ transform). This slant stacking decomposes the seismic data recorded at the surface into plane waves, with each reflection slope corresponding to a different plane wave component. A plane wave can be decomposed into Gaussian beams; the sum of the beams is approximately equal to the recorded plane wave (Cerveny, 1982; Hill, 1990). Hill (1990) showed how to decompose a general wave field into Gaussian beams, which are a series of local plane waves. Hale (1992) showed that the Gaussian beam migration is a local slant stack migration. It requires that the recorded seismic data be decomposed with slant stacking. As in slant stack migration, each local plane wave component of each subset contributes to the migrated image independently, and all contributions are summed to obtain the complete subsurface image.

Gaussian Beam migration has great potential in depth migration velocity analysis because of its accuracy, efficiency and ability of imaging the complex media. In this paper, we present prestack local plane wave Gaussian Beam algorithm for depth migration model building in the local plane-wave domain. It can be implemented with common-shot gathers or common offset gathers. Testing results with common-shot gathers shows that this algorithm can play an important role in migration model building. Han and Wu (2005) have successfully applied this algorithm to 2-D isotropic migration model building.

**Method**

The Gaussian beam approximation of a local plane wave can be obtained in both the common-offset domain and common-shot domain. In the common-offset domain (Hill, 2001), the local plane wave migration in a 3-D medium is expressed as,

$$ I_p(r, p_x^d, p_y^d) = \int dh_x \int dh_y I_h(r, p_x^d, p_y^d), $$

where $I_h(r, p_x^d, p_y^d)$ is a local plane-wave image with ray parameters $p_x^d, p_y^d$. The local plane-wave image is represented as

$$ I_h(r, p_x^d, p_y^d) = \frac{2i\omega}{\pi} C_0 \sum L \int d\omega \int dp_x^d dp_y^d \frac{u_{GB}^*(r; L + h, p_x^d, \omega)}{p_x^d} \times u_{GB}^*(r; L - h, p_y^d, \omega) $$

where $u_{GB}^*$ is the complex conjugate of $u_{GB}$ which is a normalized beam (Hill, 2001). Function $D_h$ is the wavefield with offset $h$. $C_0$ is a constant, $C_0 = \frac{3}{4\pi} \left( \frac{\omega l}{w_l} \right)^2$, where $w_l$ is the initial beam width at angular frequency $\omega_l$. The final image is a sum of the local plane-wave images,

$$ I(r) = \int dp_x^d dp_y^d I_p(r, p_x^d, p_y^d). $$

In the common-shot domain, the depth image is expressed as,
where $L_b$ is the beam center index. The full image is

$$I(\mathbf{r}) = \int dp_x dp_y I(\mathbf{r}; p_x, p_y).$$  \hspace{1cm} (5)$$

The imaging condition is a key factor affecting the efficiency and accuracy of the Gaussian beam migration. In equations (2) and (4), the imaging propagator of a source beam $u^*_{GB}(\mathbf{r}; \mathbf{r}, dp_x, dp_y; \omega)$ and a data beam $u^*_{GB}(\mathbf{r}; \mathbf{r}, dp_x, dp_y; \omega)$ is expressed as $A(\mathbf{r}; \mathbf{p}^s, \mathbf{p}^d) \exp[i \omega T(\mathbf{r}; \mathbf{p}^s, \mathbf{p}^d)]$. Function $A(\mathbf{r}; \mathbf{p}^s, \mathbf{p}^d)$ is the product of amplitude of the two beams, $A(\mathbf{r}; \mathbf{p}^s, \mathbf{p}^d) = A_s(\mathbf{r}; \mathbf{p}^s)A_d(\mathbf{r}; \mathbf{p}^d)$. $T(\mathbf{r}; \mathbf{p}^s, \mathbf{p}^d)$ is the complex traveltime, and $T(\mathbf{r}; \mathbf{p}^s, \mathbf{p}^d) = T_s(\mathbf{r}; \mathbf{p}^s) + T_d(\mathbf{r}; \mathbf{p}^d)$.

Gray (2004) discussed the advantages and disadvantages of the Gaussian beam migration in the common-offset domain and the common-shot domain. In the common-offset domain, the implementation of prestack Gaussian beam migration is accurate and efficient because of the appropriate application of the steepest descent method (Hill, 2001). It might be more practical to perform migration in the common-shot domain because of acquisition geometries such as orthogonal land and marine bottom-cable geometries (Gray, 2004). However, migration in the common-shot domain does not have the ability to handle multi-arrivals for the shot field.

To improve its efficiency in the common-shot domain, we ignore the amplitude information of the source field and set the amplitude equal to a value of one instead of to a frequency dependent complex number. By doing this, we do not need to use the procedure of steepest descents evaluation. The disadvantage is the loss of amplitude information of the source field.

Figure 1 illustrates that the prestack local plane-wave Gaussian beam migration is different from the prestack local plane-wave Kirchhoff migration (Akbar et al., 1996) in that the prestack local plane-wave Gaussian beam migration propagates local beams while the Kirchhoff migration propagates the whole plane wave during the migration. The advantage of the local plane-wave Gaussian beam migration over Kirchhoff migration is flexible sorting of the data in the beam-domain. In the beam-domain, the beam data of the shot-gathers can be sorted into either common-shot local plane-wave gathers or common-offset local plane-wave gathers. This leads to different model updating methods for use in a single domain or in allied domains.
Examples

We present two examples in this section. One shows the slope-dependent energy distribution in the local plane-wave domain. The other details the application of the algorithm for use with a complex medium.

The CIGs from the Gaussian beam migration in the local plane-wave domain provide inclination information of the reflectors: the distribution of the focused energy is slope-dependent. For the traditional CIGs in traditional offset domain, we know that the energy from all the subsurface reflectors is evenly spread across all traces, making it difficult to apply a dip-dependent correction to the data (Reshef and Roth, 2003). However, by using the Gaussian beam method in
the local plane-wave domain, the energy is shifted on the CIGs according to the reflector inclination. Figure 2 illustrates the phenomenon.

We now test the local plane-wave Gaussian beam migration with a complex model. Figure 3 shows a 2-D model with dipping reflectors. The dimensions of the model are 288 gridlines in the $x$ direction and 163 in the $z$ direction, with $dx = 20m$, $dz = 10m$. We simulated 84 shot gathers, each having 120 traces, each trace having 701 samples, and $dt = 4ms$. The shot spacing is 40$m$ and the receiver spacing is 20$m$. The first shot is located at 0.0$m$ and each shot gather has an offset of 0.0$m$.

![Figure 3: Velocity model.](image)

The common-shot migration experiment in the local plane wave domain was performed with two models, one is a homogeneous model and another is the true model. After migration, the local plane-wave image data were sorted into common-image gathers along the dimension of the ray-parameter. The common-image gathers at the positions of 520$m$, 1040$m$, and 2400$m$ are shown in Figure 4, with Figure 4 (a), (c) and (e) migrated with the constant velocity of 2600$m/s$, which is a little higher than the velocity of the first layer (2500$m/s$). Figure 4 (b), (d), and (f) were migrated with the true model. Note that the incorrect migration model results in the events curving upward or curving downward (Figure 4 (a), (c) and (e)). If the event contour curves downward (e.g. the first event in Figure 4 (e)), this indicates that the migration velocity was higher than the true velocity. If the event contour curves upward, this indicates that the overburden velocity is lower than the true velocity of the medium for those depths (the lower 3 events in Figure 4 (a), (c) and (e)). The events in the common-image gathers look flat when the true velocity model was applied (Figure 4 (b), (d) and (f)). Figure 5 shows the final, stacked depth images of the prestack local plane-wave Gaussian beam migration. Figure 5(a) is migrated with the true model and correctly illustrates the model structure. Figure 5(b) is migrated with a constant velocity of 2600$m/s$ and shows an incorrect map of the structure.
Figure 4: Common imaging gathers with common incident ray parameter located at 520m ((a), (b)), 1040m ((c), (d)) and 2400m ((e), (f)). The events in the common-imaging gathers are curved up or down if the migration velocity \(2600\, m/s\) is incorrect ((a), (c) and (e)). The events are aligned when the true model is applied ((b), (d) and (f)).
Discussion and Conclusions

We present an algorithm for computing common-image gathers in the local plane-wave domain based on Gaussian beam migration. Its imaging efficiency for complex media is comparable with Kirchhoff migration while its imaging quality approaches that of wave-equation migration. For velocity updating, this algorithm has an advantage of properly characterizing complex propagation paths of the wave energy before the beam summation. With its good computation efficiency and accuracy, the anticipated primary application of this algorithm is for 3-d isotropic migration velocity updating and anisotropic migration model building and reservoir detection.

Acknowledgment

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A Velocity analysis algorithm based on prestack Gaussian beam migration

Qiyu Han and Ru-Shan Wu

Summary

A velocity model is necessary for prestack depth migration, while depth migration is itself a powerful tool for estimating interval velocities by analyzing the depth migration residuals on common image gathers (CIGs). The performance of a velocity building technique depends on the depth migration, and Gaussian beam migration is an efficient, accurate migration method for complex media. In this paper, we apply Gaussian beam migration to local plane-wave velocity analysis. CIGs in the local plane-wave domain are computed with a prestack local plane-wave domain Gaussian beam migration algorithm. Interval velocity updating is implemented with depth residuals of each selected CIG in the local plane-wave domain. The depth residuals in a CIG are picked from the events along chosen reflectors. The velocity is then estimated using the depth residuals, local plane-wave parameters, and traveltime information, all obtained from the prestack local plane-wave Gaussian beam migration. Residual correction is implemented with a top-down, layer-stripping procedure. Results show that this algorithm is robust and efficient for updating complex velocities in the local plane-wave domain. Like the Kichhoff method, the Gaussian beam migration velocity analysis is limited to use with a smoothed velocity model.

Introduction

Velocity analysis based on prestack depth migration is a popular technique. The performance of a velocity analysis technique depends primarily on the depth migration method. A practical velocity analysis technique requires the use of an efficient prestack migration method, since the depth migration is repeatedly performed to obtain the CIGs. In addition, a migration method that is robust in dealing with complex media is needed.

Existing work on velocity analysis is largely based on Kirchhoff migration. Gaussian beam migration is an alternative to the Kirchhoff method. Hill (1990) first introduced the Gaussian beam method for poststack depth migration. He then extended it to prestack common-offset depth migration (Hill, 2001). The research shows that Gaussian beam migration is an accurate and efficient depth migration method and is robust in dealing with complex media. The Gaussian beam migration should have potential for use in velocity analysis.

Most velocity analysis methods are performed with the CIGs in the offset domain (e.g., Al-Yahya, 1989; Liu, 1997). Velocity analysis can also be performed in the local plane-wave domain or the constant angle domain (e.g., Whitmore and Garing, 1993; Jiao et al., 2002).

In this paper, we study the feasibility of applying Gaussian beam migration to velocity analysis. We present a velocity analysis algorithm in the local plane-wave domain. The CIGs are computed with a prestack local plane-wave Gaussian-beam migration algorithm (Han and Wu, 2005).
Method

The CIGs in the local plane-wave domain

The migration velocity may be updated according to the alignment of an event in a common imaging gather. The common imaging gathers in the local plane wave image domain ($z - p$ domain) can be calculated with the Gaussian beam migration (Han and Wu, 2005). The depth image of the scattered local plane-wave is expressed as,

$$
I_s(r; p^d) = \frac{2i|\omega|}{\pi} C_0 \int d\omega \frac{dp^r}{p^r_c} u_{GB}^*(r; r^d; p^d; \omega) \times u_{GB}^*(r; r^r; p^r; \omega) D_z(L_b; p^r; \omega),
$$

where $p^r$ and $p^d$ are ray parameters for the source field and data field, $\omega$ is angular frequency, $L_b$ is the beam center index, $D_z$ is the wavefield of a common shot gather, $u_{GB}$ is the complex conjugate of $u_{GB}$ which is a normalized beam (Hill, 2001), $C_0$ is a constant, $C_0 = \sqrt{3}(\frac{\omega I_0}{w_i})^2$, with $w_i$, the initial beam width at angular frequency $\omega_i$, and $r$, $r^r$ and $r^d$ are positions of the image point, the source point and data. The final depth image is a sum of the local plane-wave images,

$$
I(r) = \int \int dp^d I_s(r, p^d).
$$

Many of the advantages of the Gaussian beam migration are well known, for example, low computation cost, high imaging accuracy for complex media, good stability at caustic regions. In addition, CIGs from the Gaussian beam migration in the local plane-wave domain also provide inclination information of the reflectors: the distribution of the focused energy is dip-dependent. For the traditional CIG in traditional offset domain, the energy from all the subsurface reflectors is evenly spread across all traces in which case it is difficult to apply a dip-dependent correction to the data (Reshef and Roth, 2003). The focused energy can be separated in the offset domain by transforming the image into a special output domain (Reshef, 2001). In the local plane-wave domain, however, the energy is directly separated on the CIGs by the Gaussian beam method. Figure 1 illustrates the phenomenon.

Velocity updating

After local plane-wave migration, the image data are sorted into CIGs before the final stacking. Extending the perturbation approach (Liu, 1997) from offset domain to local plane-wave domain, the imaging depth residual $\delta z$ of a reflection in a CIG is represented as

$$
\delta z(x_i, p^r) = \sum_{i=1}^n g_i(x_i, p^r) \delta \lambda_i,
$$

with CMP location $x_i$, ray parameter $p^r_i$, model perturbation $\delta \lambda_i$, and

$$
g_i(x_i, p^r) = \frac{1}{p^r_c + p^r} \left( \frac{\partial t_i(x_i, p^r)}{\partial \lambda} + \frac{\partial t_i(x_i, p^r)}{\partial \lambda_i} \right),
$$

where $t_i$ is the traveltime of the $i$-th reflection, and $\lambda$ is the velocity.
where $p^s_z$ and $p^r_z$ are vertical slowness of the source data and the receiver data, $t_s$ and $t_r$ are traveltimes of the source field and data to the image point. For a local plane-wave image, the ray-parameter $p^s_z$ of the source field is determined with vertical slowness $p^r_z$ of the data field, while $p^r_z$ is calculated with the ray at the image point. Figure 2 shows the relationship of these parameters. $p^s_z$ is calculated with the local velocity at the reflection point and the source ray angle $\alpha$, and $\alpha = \beta - 2\gamma$. $\gamma$ is the dipping angle of the reflector which is determined by reflector picking. Angle $\beta$ is the ray angle of the scattered data.

Figure 1. The distribution of the focused events is dip-dependent: (a) three-reflector model with different inclinations, (b) a CIG in the local plane-wave domain.

Figure 2. The relationship between the vertical slowness of the source data and the receiver data.

In the simplest case, the parameter updating is
Plane wave Gaussian beam velocity analysis

\[
\delta \lambda = - \frac{\sum_{x_i, p'_x} \left[ g(x_i, p'_x) - \bar{g}(x_i, p'_x) \right] \left[ z(x_i, p'_x) - \bar{z}(x_i, p'_x) \right]}{\sum_{x_i, p'_x} \left[ g(x_i, p'_x) - \bar{g}(x_i, p'_x) \right]^2},
\]

(5)

where \( z \) is the imaging depth after updating at the imaging point in the local plane-wave domain,

\[
z(x_i, p'_x) = z_0(x_i, p'_x) + \delta z(x_i, p'_x),
\]

(6)

where \( z_0 \) is the imaging depth of vertically scattered wave field. The overline denotes the mean value.

Some computational aspects

This algorithm is implemented with a top-down, layer-stripping procedure. Velocity updating is iteratively applied in the same layer (between two reflectors or the surface and the first reflector) based on the velocity corrections. Only selected CIGs are analyzed, and linear interpolation is applied to update the velocity in the other areas within the current layer. If all of the events in the selected CIGs are aligned, evaluation of the current layer is complete. Then velocity analysis for the next layer begins. This work continues until the last reflector is reached and the last layer is updated.

Figure 3. Picking of the reflector and its inclination.

Information on the interfaces and reflection events is required during the updating. For the interfaces, reflector depth and orientation of the reflector normal may be obtained by the energy distribution (see Figure 1). We currently get this information by picking the events from the stacked image (Figure 3). For the events, the residue of imaging depth can be obtained by scanning. To speed up the updating procedure, events are picked in the common image gathers to get their depth residuals. Note that, only the focused energy is picked. Computation cost is
significantly reduced for the local plane-wave images with focused energy instead of all images in the local plane-wave domain. Figure 4 shows how to pick an event.

![Figure 4](image1.png)

Figure 4. Only the well focused energy is picked for velocity updating.

![Figure 5](image2.png)

Figure 5. Reconstructed wavefield and corresponding images at different depth: 750m, 1500m, and 2250m. The upper are reconstructed wave fields and the lower are partially migrated image fields.

**Improvement of velocity updating in local angle domain**
In the Gaussian Beam migration, the beam width increases with the imaging depth. For complex media, conventional Gaussian Beam migration may lose its accuracy in the deep part of the model space because of its large beam width. In this case, information of the local plane-wave image gathers may not good enough for the velocity updating. To solve this problem, we improved the Gaussian Beam migration algorithm by downward continuation of the local Gaussian beams. The Gaussian beams decomposed at the surface first propagate to a depth level in the model. The wavefield is then reconstructed at this subsurface. The new constructed wavefield is decomposed into local Gaussian beams. The new decomposed Gaussian beams then propagate downward to a new depth level where the new wavefield will be reconstructed. Repeat the reconstruction and decomposition until the imaging in the whole model space is finished. The downward continuation effectively avoids rapidly increasing of the beam width.

We tested this algorithm with the popular poststack data of the SEG/EAEG salt model. Figure 5 shows wavefield downward continuation and reconstruction at different imaging depths: 750m, 1500m, and 2250m. Figure 6 shows final images migrated with the conventional Gaussian beam migration algorithm (the upper one) and our downward continuation Gaussian Beam algorithm (the lower one). The imaging quality within the salt body and at deep part of the model is obviously improved by Gaussian beam downward continuation.

Example

With some simple reflector models, we demonstrate the convergence ability of the velocity updating technique.
The inclinations of the reflectors, in a background velocity of 1800 m/s, are illustrated in Figure 7. The grid dimensions of the model are 120 gridlines in depth and 96 gridlines in the horizontal with $dx = 25m$ and $dz = 10m$. 15 shot gathers were simulated for the test and each shot gather includes 60 traces. Receiver increment is 25m (Figure 3(a)). The data was first migrated with an initial model of 2000 m/s. Then after two iterations of updating, the events of all the CIGs for different reflectors with close to horizontal alignment are compared with the ones migrated with the true model. Figure 8 shows the updating history of the test. For the imaging of each reflector, the common image gathers for different iterations are compared. This algorithm is efficient for both horizontal and inclined reflectors. The updating for the inclined reflector model is more efficient than for the horizontal reflector model. For event picking, it is easier to pick the imaging depth on a horizontal reflector than on a steeply inclined reflector, because of the difference in imaging aperture.

![Figure 7. Reflector models with different reflector inclinations: (a) 0°, (b) 10°, and (c) 20°.](image)

![Figure 8. velocity updating history for the reflectors with different inclinations: (a) 0°, (b) 10°, and (c) 20°.](image)
We note that the horizontal distribution of the focused reflection energy is depth-dependent. Figure 9 illustrates the phenomenon.

Figure 9. the distribution of the focused energy along $p_x$ is depth-dependent: 500m (left), 1000m (center), and 1700m (right). The reflector slope is about 10°.

**Discussion and conclusions**

CIGs computed with prestack local plane-wave Gaussian beam migration are convenient for depth migration velocity analysis. Computation of these CIGs is feasible for complex media because of the character of the Gaussian beam migration method. Therefore, the proposed velocity analysis algorithm based on the Gaussian beam migration is practical for complex model building. This algorithm makes it possible to do velocity analysis in areas where the structure is so complicated that the conventional Kirchhoff migration based velocity building techniques do not work well. Implementation in the local plane-wave domain enables us to directly use the local plane-wave parameter to update the velocity model.

In this paper, we demonstrate the velocity updating algorithm for isotropic media; extending this algorithm to deal with anisotropic media should not involve any theoretical or technical difficulties. In addition, model building in the local plane-wave domain is very suitable for anisotropic media because wave fields in this media are naturally dependent on direction. For complex anisotropic media, since the analytical perturbation approach does not behave well, significant improvements to the current updating procedures would be necessary.

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Preliminary results on velocity updating using waveform tomography

Jun Cao and Ru-shan Wu

Abstract

Waveform tomography has many successful and impressive applications since it was developed in 1980’s. We briefly introduce the theory of waveform tomography based on finite-difference (FD) method in frequency domain, including the forward modeling and inversion methods, especially the fast calculation of the gradient of misfit function which makes it possible to implement waveform tomography in reality. Then an application of waveform tomography method for an ultrasonic experiment data is used to demonstrate the strength of waveform tomography. Finally, we apply the waveform tomography method to a synthetic data set generated in the Marmousi velocity model, using a small offset acquisition geometry commonly used in the oil industry. Our preliminary results show that waveform tomography method works well even for the small offset dataset.

Introduction

Since the mid 1970’s geophysicists have attempted to apply tomographic reconstructions of the interior structure of the earth using earthquake arrivals, and since the 1980’s exploration seismologists have been using tomography to build velocity models for seismic imaging. The earliest tomographic methods used travel times, and a reconstruction of the velocity structure was built up by back-projecting these measurements along ray paths.

Waveform tomography represents an advanced approach, in which the whole waveform is used in the reconstructions. It first emerged in the 1980’s with the work of Lailly (1983), Tarantola (1984), Mora (1987) and Woodward (1992). Later authors continued this work (Zhou et al., 1993; Shipp, 1994), and others re-formulated the approach in the frequency domain (Pratt and Worthington, 1990; Song et al., 1995). In general the approach of waveform tomography represents an attempt to fully account for the complex interaction of the propagating wave and the target. (Pratt, 2005)

It is clear that in order to carry out waveform tomography, the key requirement is to be able to calculate the Green's functions. The more accurate the Green's function calculations, the more powerful the inversion method will be. The choices are many: ray theory, FD method, the pseudo-spectral method, the one-way wave propagator etc are all possibilities. Some of them are fast, but very limited (perhaps they can only handle 1-D earth models, or perhaps they don't handle multiple arrivals, etc). Some of them are very complete, but prohibitively expensive to run - an example would be a finite-difference simulation of the 3-D, anisotropic, anelastic wave equation. (Pratt, 2005)

Here we will focus on the waveform tomography method based on frequency domain finite-difference method developed by Pratt and his colleagues since 1990’s (Pratt and Worthington, 1990; Song et al., 1995; Pratt et al., 1998; Hicks and Pratt, 2001; Sircue and Pratt, 2004). Implementation in frequency domain has six major advantages:
(1). We only need to calculate some of the frequencies to do waveform tomography if we have a frequency domain method, instead of calculating all the frequencies at once in the time domain (Sirgue and Pratt, 2004).

(2). We can mitigate the non-linearity of the seismic inverse problem by progressing from low-frequency components in the data to high-frequency components.

(3). In the frequency domain there are significant efficiency gains for multi-source problems (Marfurt, 1984; Pratt, 1990). In contrast, calculating responses for many sources scales linearly for time domain methods. If you need 100 source responses, it will cost 100 times as much as one source response.

(4). In many cases the wavefront takes a long time to arrive at the receivers, and we are only interested in a short time window of data. To model this in the time domain requires many time steps to bring the wavefront to the receivers. In the frequency domain we only need to model a small number of frequencies required to represent the shorter time window.

(5). In the frequency domain, anelastic attenuation is easy to model. All we need to do is to make the velocity model complex-valued. Any frequency dependence of $Q(\omega)$ is possible.

(6). Any frequency dependence for dispersion of velocities $c(\omega)$ is possible.

Since the original development of frequency domain acoustic and elastic modeling and inversion methods by Pratt (1990) and Pratt and Worthington (1990), many progresses have been made in the past fifteen years. Song and Williamson (1995) developed the 2.5D method. Song et al. (1995) inverted for attenuation and first inverted real data. Brittan et al. (1997) first applied it to real surface refraction data. Pratt et al. (1998) fully described the theory using matrix formalism and developed it with Newton methods. Pratt & Shipp (1999) and Pratt (1999) applied it to real crosshole data in attenuating, anisotropic media, with verification on scale model data. Hicks and Pratt (2001) applied it to reflection data with time parameterization and Newton methods. Recently Sirgue and Pratt (2004) developed a strategy for selecting inversion frequencies for surface data, which makes the waveform tomography method more powerful in reality.

In this report, we will briefly introduce the theory of waveform tomography based on FD method in frequency domain firstly (include forward modeling and inversion methods). Then we will show an example for an ultrasonic experiment data. Finally, we will give our preliminary results got by the waveform tomography method for the Marmousi model which has only small offsets dataset, and compare the results with that from data with large offsets.

**Forward modeling in the space-frequency domain**

Pratt et al. (1998) fully described the waveform tomography theory using matrix formalism. In the next two parts we will introduce it briefly. The discretized equations for the acoustic or elastic wave equations with viscous damping using a FD method in time domain can be written as

$$M\ddot{u}(t) + C\dot{u}(t) + K\ddot{u}(t) = \hat{f}(t),$$

where $\dot{u}(t)$ is the discretized wavefield arranged as a column vector, $M$ is the mass matrix, $C$ is the damping matrix, $K$ is the stiffness matrix and $\hat{f}(t)$ are the source terms. The hat “$\hat{}$” and “$\ddot{}$” stand for the first and second derivative with respect to $t$. Taking the temporal Fourier transform of eq. (1) yields
\( \mathbf{Ku}(\omega) + i\omega \mathbf{C} \mathbf{u}(\omega) - \omega^2 \mathbf{M} \mathbf{u}(\omega) = \mathbf{f}(\omega), \) \hspace{1cm} (2)

where \( \mathbf{u}(\omega) \) and \( \mathbf{f}(\omega) \) is the Fourier transform of \( \tilde{\mathbf{u}}(t) \) and \( \tilde{\mathbf{f}}(t) \) respectively. For simplicity, we can rewrite eq. (2) as

\[ \mathbf{Su} = \mathbf{f}, \] \hspace{1cm} (3)

where the complex matrix, \( \mathbf{S} = \mathbf{K} - \omega^2 \mathbf{M} + i\omega \mathbf{C} \), is the differencing matrix.

Eq. (3) is often solved using direct matrix factorization methods, such as LU decomposition (Pratt 1990). If LU decomposition is used to solve eq. (3), the matrix factors can be re-used to solve rapidly the forward problem for any new source vector, \( \mathbf{f} \) (for typical problem sizes, the additional number of floating point operations required to generate the solution for a new source term is less than 1 per cent of the number of operations required to generate the LU matrix factors—see Stekl & Pratt (1998) for details). This point is especially important in the inversion for multiple sources.

The differencing matrix \( \mathbf{S} \) is very large; in general it’s of the order of \( N \times N \) where \( N \) is the number of grid points. And it’s also very sparse; it has a tridiagonal with outliers structure. To make the FD method more affordable and practical, some improvements on differencing operators and matrix solvers are needed. On the differencing operators, Stekl & Pratt (1998) combined the differencing operator in a rotated coordinate frame and a lumped mass term with the ordinary second order, finite-difference operators in an optimal manner to minimize numerical errors without increasing the size of the numerical operator. The new scheme is far more accurate than the original one: to maintain 99\% group velocity accuracy, we need approximately four grid points per wavelength; to maintain the same accuracy the original scheme would require more than ten grid points per wavelength. As a result the grid for a given model can be much smaller (or, we can choose to model much larger models). It is also very critical to use ordering schemes that allow taking maximum advantage of the sparsity of both \( \mathbf{S} \) and its LU factorization; nested dissection (George & Liu, 1981) is such a method. The nested dissection reduces the storage requirements from \( O(N^{3/2}) \) to \( O(N \log \sqrt{N}) \). There is an equally dramatic saving in the CPU time. Together these two modifications lead to a dramatic improvement in the computation times for acoustic wave equation modeling and inversion.

**Inversion in the space-frequency domain**

As is common in many inverse problems, we seek to minimize the \( l_2 \) norm of the data residuals. Thus we seek to minimize the misfit function (or objective function)

\[ E(\mathbf{m}) = \frac{1}{2} \hat{\mathbf{d}}^T \mathbf{H}^T \mathbf{H} \hat{\mathbf{d}} \]

where \( \mathbf{m} \) is the model parameters and \( \hat{\mathbf{d}} \) is the data residual; the superscript \( T \) represents the Hermitian transpose, introduced to ensure the misfit function is a real-valued norm for complex-valued data.

Due to the high computational cost of seismic forward modeling problem, we cannot search globally for the optimal model (e.g., Monte Carlo method, genetic algorithm, and simulated annealing algorithm). The waveform inverse problem is solved using local methods from a starting model. The methods employed to determine the macro-starting model mainly utilize the
travel time information of the data. They mainly include stacking velocity analysis that is most
frequently used (Yilmaz, 1987), travel time tomography (Bishop et al., 1985; Farra and
Madariaga, 1988; Zelt and Barton, 1998), stereotomography (Billette and Lambaré, 1998) and
migration velocity analysis (Al-Yahya, 1989; Symes and Carazzone, 1991; Docherty et al., 1997;
Chauris et al., 2002). Next we will discuss the local inversion methods.

(1) Newton methods of inversion
We expand the misfit function $E(m)$ in the neighborhood of the starting model $m$ with Taylor
series (up to quadratic order),

$$E(m + \delta m) = E(m) + \delta m \nabla_m E(m) + \frac{1}{2} \delta m^T H \delta m + O(\|\delta m\|^3),$$

where two important terms have been introduced. $\nabla_m E$ is the gradient of the misfit function
(with respect to model parameters). It points in the direction of steepest rate of change of the
misfit function. $H = \partial^2 E/\partial m \partial m^T$ is the Hessian of the misfit function. We seek a vector $\delta m$
that minimizes the misfit within the quadratic approximation. Minimizing with respect to all
components of $\delta m$, we find the solution is characterized by

$$H \delta m = -\nabla_m E(m).$$

The gradient looks like the true perturbation, except that the true perturbation is operated on first
by pre-multiplication by $H$. We can think of this operation as a kind of a filter on the true
perturbation: the gradient is a filtered version of the answer we are looking for.

Newton method directly inverts the Hessian which leads to

$$\delta m = -H^{-1} \nabla_m E(m).$$

But the Hessian matrix is usually very huge ($N \times N$, $N$ is the number of grid points). Even for
small problems the Hessian has problems: it is almost always a singular matrix, hence it cannot
be mathematically inverted in any case. Instead of using the Newton method, a method that is
very common and very practical is the gradient algorithm. We will discuss it in the next part.

(2) Gradient methods of inversion
Since the gradient $\nabla_m E$ tells us about the direction in which the misfit is changing most rapidly,
we can look for a smaller misfit function based on the steepest descent direction $\nabla_m E$ of the
misfit function. So we can update the model using the equation

$$m^{(k)} = m^{(k-1)} - \alpha^{(k-1)} \nabla E^{(k-1)},$$

where $k$ is the iteration number and $\alpha$ is a step length chosen to minimize the $l_2$ norm in the
direction given by the gradient of $E(m)$.

We may calculate the gradient by directly taking partial derivatives of eq. (4) with respect to the
model parameters $m$

$$\nabla_m E = \frac{\partial E}{\partial m} = \text{Re} \left\{ \left( \frac{\partial u}{\partial m_1}, \frac{\partial u}{\partial m_2}, \ldots, \frac{\partial u}{\partial m_n} \right)^T \delta \hat{a} \right\} = \text{Re} \{ J \delta \hat{a} \},$$

where matrix $J$ is the Fréchet derivative matrix, or the sensitivity matrix. The individual elements
of $J$ are

$$J_{ij} = \frac{\partial u_i}{\partial m_j}.$$
Each column in $\mathbf{J}$ is a partial derivative wavefield: the column vector contains the change in the wavefield $u$ due to the infinitesimal perturbation to a single model parameter. The best way to think of this is as a diffracted wavefield.

From eq. (9) we can see that the $j$th element of the gradient vector contains: The partial derivative wavefield, $\partial u / \partial m_j$, sampled at the receiver points, Hermitian transposed, and multiplied by the data residuals, and then finally summed over all receivers.

Although we now know how to calculate the gradient, we can see that the inversion is going to be very expensive unless we use some tricks. If we calculate the Fréchet matrix directly, it seems we would have to perturb each model parameter in turn and then calculate the wavefield, which requires $N$ forward modeling runs for one iteration of the inversion! Fortunately this isn’t necessary after all. Lailly (1983) and Tarantola (1984) found a fast calculation of the gradient and we don’t need to calculate the partial derivative matrix explicitly at all. They found that the calculation of the gradient of the misfit function is of the same computational order as the forward modeling task, which makes it possible to implement waveform tomography in reality, and that the gradient calculation is closely related to seismic prestack depth migration.

**Fast calculation of the gradient**

We start by differentiating equation (3) with respect to the $i$th model parameter $m_i$,

$$ S \frac{\partial u}{\partial m_i} + \frac{\partial S}{\partial m_i} u = 0, \quad (11) $$

or

$$ \frac{\partial u}{\partial m_i} = S^{-1} f^{(i)}, \quad (12) $$

which is the partial derivative wavefield; it can be got by a new forward modeling problem in which

$$ f^{(i)} = - \frac{\partial S}{\partial m_i} u $$

is the $i$th virtual source. The virtual source is the result of interaction of the original wavefield $u$ with the changes caused by the perturbation in the model parameter $m_i$. We know that the differencing matrix only have very few elements (usually only one element) that depends on a given model cell. So $\partial S / \partial m_i$ is a very sparse matrix indeed; for velocity perturbations there will only be one non-zero value in the whole matrix. Thus the whole operation $\partial u / \partial m_i$ “selects” a single value out of the forward wavefield $u$ and scatters the waves from it as if it were a new source.

From eq. (9) and (12), we have

$$ \mathbf{J} = \left[ \frac{\partial u}{\partial m_1} \frac{\partial u}{\partial m_2} \cdots \frac{\partial u}{\partial m_n} \right] = S^{-1} \left[ f^{(1)} f^{(2)} \cdots f^{(n)} \right] = S^{-1} \mathbf{F}, \quad (14) $$

where $\mathbf{F}$ is a matrix with all the virtual sources in its columns. Substitute eq. (14) into eq. (9), we can get the gradient

$$ \nabla_m E = \text{Re} \{ \mathbf{J}^\tau \mathbf{\partial d} \} = \text{Re} \{ \mathbf{F}^\tau \left[ S^{-1} \right]^\tau \mathbf{\partial d} \}, \quad (15) $$

or
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\[ \nabla_m E = \text{Re}\{F^T v\} \]  \hspace{1cm} (16)

where

\[ v = [S^{-1}]^T \delta l = [S^{-1}]^T \delta l \]  \hspace{1cm} (17)

is the backpropagated wavefield. Therefore, the gradient can be computed in two steps:

1. The back-propagated wavefield is computed, in which the source terms are replaced by the data residuals. The differencing matrix itself is replaced by its complex conjugate, which means it corresponds to the wave equation with time running backwards. The data residuals are thereby fed back into the model.

2. To form an image, the back-propagated wavefield is then multiplied by the virtual sources at each grid point (These are generated from the original, forward propagated wavefield).

Since the convergence rate of the gradient method is generally quite slow; convergence can be improved by adopting a conjugate gradient approach (e.g. Mora 1987), which does not require any significant additional computations.

Application of waveform tomography for ultrasonic experiment data

Waveform tomography has many successful applications since it was developed. Here we will display a physical scale model example for ultrasonic experiment data (Pratt, 1999) to see the strength of waveform tomography.

The physical scale model experiment was conducted in an ultrasonic modeling tank. As shown in Figure 1a, the model contains a number of horizontal layers of different velocities and thicknesses, a single channel feature, and a single dipping layer with a small fault. The model is sufficiently wide in the out-of-plane direction to allow us to assume the geometry is two-dimensional. The source and receiver piezoelectric transducers were operated at frequencies between approximately 200 and 800 kHz. The full survey consisted of a total of 51 source positions and 51 receiver positions, each at 2.5mm intervals and generating 2601 records. The target region was 55mm wide by 125mm long (i.e., 125mm deep).

All distances, times, and frequencies may be scaled by a factor of 1000 to produce cross-borehole dimensions that are realistic. The scaled survey is 55 m across and 120 m deep, and scaled seismic frequencies lie between 200 and 800 Hz.

The result for the travel time tomography is shown in Figure 1(b). The tomogram exhibits the expected velocity variations, but at a very poor resolution. The large, strong, horizontal low velocity layers are correctly imaged, but the central, semicircular channel feature is only very faintly present on the image. The dipping interface is recovered, but there is no trace of the small fault on this image.

Using the result by travel time tomography as the starting model, the final result by waveform tomography is shown in Figure 1(c). The image represents a significant gain in resolution when compared with the starting model. It shows a reasonable representation of the true model geometry and velocities. It has clearly resolved all the geometric features in the true model, including each of the individual layers, the thin, low-velocity layer containing the simulated channel feature and the small fault on the dipping layer in the bottom half of the model.
Preliminary results from small offset Marmousi data using waveform tomography

The example shown before is the crosshole acquisition, so the main energy in the data therein is the transmitted waves, which is regarded as the main information to resolve the velocity structure in the waveform tomography method. The seismic experiment with the sources and receivers on the surface is also very important in exploration seismology. For this kind of acquisition geometry, most of the applications of waveform tomography are focused on the large offsets data (e.g. Forgues, 1998; Sirgue and Pratt, 2004). Here we use the waveform tomography method to invert small offsets synthetic surface reflection data for the Marmousi model and compare the result with that from large offsets data (Forgues, 1998) to see how well the waveform tomography can work for such data.

Synthetic data

We recalculated a new acoustic dataset containing the same offset as that used in the original synthetic dataset computed by the Institut Francais du Pétrole, which is from –2575m to –200m in the left of the source. The model used is the original Marmousi model, re-sampled to a grid spacing of 25m (see Figure 2a). The first and last shots are located at 3000m and 8925m, and the shot interval is 75m (extracted from one of every 3 shots in the original data). The synthesis of the data used the FD method in frequency domain which is included in the package “FULLWV” by Pratt. The following parameters were used in the modeling: record length = 2900 ms, dominant frequency=5Hz, maximum frequency = 14.8Hz, frequency interval = 0.2 Hz, number of frequency steps = 74.

Inversion

Here we just invert some of the frequencies according to the strategy for selecting the frequencies by Sirgue & Pratt (2004). If we start from 3Hz and to about 10Hz, we can get the frequencies we need to invert are 3.0, 3.2, 3.4, 3.6, 3.8, 4.0, 4.3, 4.6, 4.9, 5.2, 5.6, 6.0, 6.4, 6.8, 7.3, 7.8, 8.3, 8.9, 9.5, 10.2Hz.

No pre-processing has been carried out on the dataset. The frequency components at the selected frequencies are directly extracted. Our starting model is shown in Figure 2b. This model is obtained by smoothing the exact model. Figure 2c is the model obtained by consecutively inverting the 3.0, 3.2, 3.4, 3.6, 3.8, 4.0, 4.3, 4.6 and 4.9Hz data. At this stage the velocity structure of the model in the shallower part is already visible. In particular, the low velocity zone at the top of the reservoir (at about x = 6000m, z =1700m) has been imaged. The edges of the model especially the left part are poorly resolved due to the very limited ray coverage there. Figure 2d displays our final result, after inverting the 5.2, 5.6, 6.0, 6.4, 6.8, 7.3, 7.8, 8.3, 8.9, 9.5 and 10.2Hz data components, using the model in Figure 2c as an intermediate starting model. The use of higher frequency data components yields a sharper velocity structure and the fine sedimentary layer structure is better resolved.

Forgues (1998) applied waveform tomography to a similar synthetic data for Marmousi model but with a very large offset record. There are 384 fixed receivers at intervals of 24m along the surface. The first and last receivers are located respectively at x = 0m and x = 9192m. The shot interval is 96 m. The first and last shots are located respectively at x = 48m and x = 9168m. The following parameters were used in the modeling: record length = 8192 ms, maximum frequency = 25 Hz, frequency interval = 0.122 Hz, number of frequency steps = 204. Figure 3 displays his
results with the large offsets data. He chose the frequencies 3.0, 4.0, 5.0, 8.0, 9.0, 10.0Hz to invert. Due to the use of large offset refracted waves, the velocity structure is better resolved than that with the small offset data.

Conclusions

Here we briefly introduced the basic theory of waveform tomography based on finite-difference method in frequency domain, including the forward modeling and inversion methods, especially the fast calculation of the gradient of misfit function which makes it possible to implement waveform tomography in reality. The successful application of waveform tomography method for the ultrasonic experiment data in a physical scale model has demonstrated the strength of waveform tomography. Finally, we apply waveform tomography method in the Marmousi model with a small offset reflection data that is usually used in oil industry. Our preliminary results show that waveform tomography method can roughly resolve the velocity structure for the small offset data. It works well for the small offset data although the large offset can yield better-resolved velocity structure.
Figure 1. Shaded-relief displays showing (a) the true velocity model, (b) the traveltime tomography result, and (c) the final wavefield inversion result. (FIG. 11 in Pratt (1999))
Figure 2: Results from a small offset Marmousi data. (a) is the true model, (b) is the starting model, obtained by strongly smoothing the exact model (c) is the inversion result with frequencies 3.0, 3.2, 3.4, 3.6, 3.8, 4.0, 4.3, 4.6, 4.9Hz, (d) is the inversion result with frequencies 3.0, 3.2, 3.4, 3.6, 3.8, 4.0, 4.3, 4.6, 4.9, 5.2, 5.6, 6.0, 6.4, 6.8, 7.3, 7.8, 8.3, 8.9, 9.5, 10.2Hz.
Figure 3: Results from a large offset Marmousi data. a) The exact Marmousi model (to a depth of 2 km); b) The starting model for the inversion, obtained by strongly smoothing the exact model; c) The result after inverting for 3 Hz, 4 Hz and 5 Hz data d) The result after inverting for 3 Hz, 4 Hz, 5 Hz and 8 Hz, 9 Hz, 10 Hz data. (Fig. 3 in Forgues, 1998)
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PART III

SEISMIC RESOLUTION ANALYSIS
Spatial resolution of seismic imaging

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Summary

We define and formulate the resolution of an imaging system based on the inverse theory and local angle domain decomposition of Green’s functions. The resolution defined this way includes both the effects of acquisition system and imaging (migration) process. The theory and method are based on wave theory and no asymptotic approximation is made in the calculation. It represents and quantifies the actual resolution we observed in the migrated images. The resolution taking into account only the influence of the acquisition system (frequency band and spatial aperture) and neglecting the factors such as the errors in backpropagation (migration) can be considered as the resolution limit of the system, the best resolution an acquisition system can get. Theoretical analysis and numerical examples are given to show the importance of propagator accuracy in the evaluation of resolution in complex media.

Introduction

Spatial resolution has been studied by Beylkin et al. (1986) based on generalized Radon transform and the mapping of domain integration (frequency band and spatial aperture) into a spectral coverage in spatial frequency domain (wavenumber domain). The mapping is done by simple ray-tracing which does not take into account of the frequency dependence and other wave phenomena. Since then, the topic has been investigated by many authors from different points of view (Chen and Schuster, 1999, 2001; Gelius et al., 2002; Gibson, R.L. Jr. and C. Tzimeas, 2002). While these analyses are useful tools for the resolution problem, most analyses are formulated to calculate the influences of acquisition system to resolution, and did not look at other factors in the imaging/migration process, such as the accuracy of the propagators, errors in velocity model, etc. Some authors called the resolution this defined as the resolution limit, the best resolution an acquisition system can get. In this research, we will formulate the resolution of imaging system, which includes the acquisition system and imaging (migration) process. The analysis is based on the inverse theory and the local angle domain decomposition of the Green’s functions. This resolution of imaging system represents and quantifies the actual resolution we observed in the migrated images and provides a theoretical basis for the estimation of different factors influencing the resolution and image quality in complex media. The other important feature of our formulation is that the resolution problem is treated totally based on wave theory and no high-frequency asymptotic approximation is made. This paper is the first part of a collaborated research and presents the basic theory and physical explanation. A few numerical examples are shown to demonstrate the special features of the new theory. The second part (Xie et al., 2005) is devoted to the theoretical resolution of an acquisition system (the resolution limit) treated by wave equation based method. The problem is studied from the illumination in spectral domain using the decomposition of Green’s function in local angle-domain. In the third part (Fehler et al., 2005) Numerical experiments and analyses are given to show the influence of propagator accuracy in heterogeneous media to the resolution of imaging system. The comparison between the Kirchhoff ray propagator and the wave-equation based propagators are

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Formulation of Spatial Resolution Based on Inverse Theory

Spatial resolution can be studied under the general frame of inversion theory. Resolution operator or the discrete form, resolution matrix has been defined to quantify the resolution of parameter inversion by a particular inversion scheme (Bucks and Gilbert, 1970; Aki and Richards, 1980, section 12.3; Tarantola, 1987, section 4.33, 7.22). The resolution matrix has dependence on both the acquisition system and the inversion scheme. Assume the acquisition process can be modeled by

\[ Fm = d \]  

Where \( F \) is the forward modeling operator and \( d \) is the data. If we adopt a specific inversion operator \( B \) to apply to the data we can get a set of model parameters \( m_I \), which is different from the real \( m \),

\[ m_I = Bd \]  

Substituting (2) into (1) we get the relation between the inverted model and the real model

\[ m_I = BFm \]  

and the resolution matrix (or operator) is defined as

\[ R = BF \]  

For an exact inversion of a well-posed problem, we should have

\[ R = I \]  

where \( I \) is the identity matrix. For most the cases, the resolution matrix is not an identity matrix and the spreading of the matrix elements along the diagonal give some quantitative measure of the parameter resolution of the inversion. For the sake of simplicity, here we will use the noiseless formulation. For the stochastic approach (Tarantola, 1987) a similar derivation can be obtained.

The imaging problem can be formulated as a specific inverse problem. We assume the media can be decomposed into a smooth variation of velocity and a sharp jumps of impedance (discontinuities). The velocity distribution of the background media can be derived with different approaches and is assumed known in the “imaging problem”. The unknowns in the imaging problem are the parameter strengths and their locations (distribution). For the problem of spatial resolution, we assume the scattering coefficients everywhere are unity and therefore are known. Then the resolution matrix is totally defined by the spatial resolution.

In the following, we will use the migration operator (simply backpropagation integral) as the inversion operator (here the imaging operator) to show the effects of different factors to the
final resolution. We can write the space-domain formulations for modeling (acquisition process) and imaging (migration) as:

\[
u_s(\omega, x_g; x_s) = -k^2 \times \left( \int_{\mathbb{R}^3} d^3x' G_M(\omega, x_g'; x_s) s(x') G_M(\omega, x_s'; x_g') dx' \right) \tag{6}
\]

\[
I(x) = \int d\omega \int_{A_g} dx_s W_s(\omega, x_s) G_I^*(\omega, x_s; x) \\
\times 2 \int_{A_g} dx_s W_s(\omega, x_s) \frac{\partial G_I^*(\omega, x_s; x)}{\partial z} u_s(\omega, x_s', x) \tag{7}
\]

where \( u_s \) is the scattered wave field observed on the surface at \( x_g \) excited by a source on the surface at \( x_s \). \( G_M \) is the Green’s function of the modeling (acquisition) process, which may include all the factors (geometric spreading, intrinsic and scattering attenuation, boundary scattering, etc.) for the real heterogeneous media; \( G_I \) is the Green’s function of inversion process, which could be quite different from \( G_M \) (see Figure 1). The integrations on the receiver aperture and source aperture are for the prestack migration process, \( W \)'s are the weighting functions for integrations. The integration on the receiver aperture \( A_g \) is the Rayleigh integral which simulates the backpropagation process. The integration on frequency is from the imaging condition which states that the downward extrapolated source field and the scattered field will meet at zero time at the scattering points. In (7) we used the cross-correlation imaging condition. Other imaging conditions for correcting the imaging amplitude can be also applied, but the general conclusion of resolution analysis will not be influenced.

Equation (6) is formulated for volume perturbations (heterogeneities), such as in the case of tomographic inversion or other parameter inversion. In the formulation, \( s(x) \) is assumed as a scalar quantity, and the scattering pattern due to this point scatterer is isotropic. However, in many cases the local scattering pattern is not isotropic, such in the case of elastic wave scattering, or boundary scattering. For now let us consider the case of scalar potential (point scatterer with isotropic scattering pattern). Theory for other scattering problems can formulated in a similar way.

Write modeling and imaging process (6) and (7) into operator form, resulting in

\[
U(\omega, x_g, x_s) = F(\omega, x_g, x_s | x_0) S(x_0) \tag{8}
\]

\[
I(x) = B(x | \omega, x_g, x_s) U(\omega, x_g, x_s) \tag{9}
\]

where \( F \) is the acquisition (modeling) operator and \( B \) is the imaging operator which invert the data \( U \) into the image \( I \). Therefore the resolution operator is obtained as

\[
R(x, x_0) = B(x | \omega, x_g, x_s) F(\omega, x_g, x_s | x_0) \tag{10}
\]

The kernels for the resolution operator can be obtained as
Spatial resolution of seismic imaging

\[
R(x, x_0) = -\int d\omega k^2 \int dx W_x(\omega, x, x) G_i^*(\omega, x, x_0) G_m(\omega, x_0, x) \nonumber \\
+ 2 \int d\omega k^2 \int dx W_x(\omega, x, x) \frac{\partial G_i^*(\omega, x, x_0)}{\partial z} G_m(\omega, x_0, x_0)
\]

(11)

Note that the amplitude functions in (12) are different from the exploding-reflector modeling. In the latter case, (12) is further simplified to

\[
R(x, x_0) = -2 \int d\omega k^2 \int dx W_x(\omega, x, x) G_i^*(\omega, x, x_0) \times \\
\times G_m(\omega, x_0; x) \frac{\partial G_i^*(\omega, x, x_0)}{\partial z} G_m(\omega, x_0; x_0)
\]

(12)

The above derived resolution matrix (operator) is in fact the spatial resolution matrix for the whole acquisition and migration process (the imaging system). The matrix element \(R(x_0, x)\) is called the resolving kernel of the resolution operator (Backus and Gilbert, 1970; Tarantola, 1987), which is the point spreading function of the imaging system. In this way the point spreading function (impulse response) is defined under the guidance of general inversion theory.

For zero-offset (zero receiver aperture) acquisition, or monostatic measurement in radar terminology, the above defined resolution matrix is degenerated to

\[
R(x, x_0) = -2 \int d\omega k^2 \int dx W_x(\omega, x, x) G_i^*(\omega, x, x_0) \times \\
\times G_m(\omega, x_0; x) \frac{\partial G_i^*(\omega, x, x_0)}{\partial z} G_m(\omega, x_0; x_0)
\]

(13)

The above derived resolution matrix (operator) is in fact the spatial resolution matrix for the whole acquisition and migration process (the imaging system). The matrix element \(R(x_0, x)\) is called the resolving kernel of the resolution operator (Backus and Gilbert, 1970; Tarantola, 1987), which is the point spreading function of the imaging system. In this way the point spreading function (impulse response) is defined under the guidance of general inversion theory.

If we set the scatterer’s distribution \(s(x') = \delta(x' - x_0)\) in equation (6) and substitute into equation (7), we see the equivalence of the imaging process to the calculation of resolution matrix. This gives us the numerical procedure of calculating the resolution matrix or PSF for any imaging system (including acquisition and migration)/inversion). In the same way the PSF’s of

Figure 1. Schematic diagrams showing the modeling (data acquisition) and imaging (inversion) processes.
zero-offset imaging process or imaging with exploding-reflector modeling defined in (12) and (13) respectively, can be calculated by numerical simulations.

Angular-spectral representation of point spreading function (PSF)

Angular-spectral representation of resolution of PSF is more intuitive. We can see directly the information coverage in local angle-domain. We perform local 3D Fourier transform on $R(x, \chi_o)$ with respect to $x$ with coordinate center at $\chi_o$:

$$R(K, \chi_o) = \int dR(x) e^{-iK \cdot z} W(x, \chi_o) R(x, \chi_o)$$  \hspace{1cm} (14)

where $K$ is the 3D wavenumber vector and $W(x, \chi_o)$ is a 3D window function to localize $R(x, \chi_o)$. For simplicity, we set the weighting functions $W_s$ and $W_g$ in migration as unity.

Substituting (11) into (14) we obtain

$$R(K, \chi_o) = - \int dK_x \int_{A_f} d\omega k^2$$

$$\times \int_{A_f} dx_s G^*_s(\omega, x_o, K - K_x; x_s) G_m(\omega, x_s; x)$$

$$\times 2 \int_{A_f} dx_s \frac{\partial G^*_s(\omega, x_o, K_x; x_s)}{\partial \omega} G_m(\omega, x_s; x_o)$$

where $G(\omega, x_o, K; x')$ is the beamlet decomposition (or local plane-wave decomposition) of the Green’s function around $x_o$, and $x'$ is the source or receiver position. Beamlet decomposition decomposes the field of a Green’s function into beamlets. Each beamlet has a center location $x_o$ with certain width, and a lobe in the specified direction. In (15) $A_f$ is the frequency aperture (band), and the integration over $K_x$ is a convolution integral in $K$ domain and $K_x = K_s \hat{\theta}_s$, with $\hat{\theta}_s$ as the unit angular vector to the receiver direction. We see also that $K_x = K_s \hat{\theta}_s = k \hat{\theta}_s$, with $\hat{\theta}_s$ as the unit vector in the source direction. From (15), $G^*_s(\omega, x_o, K_s; x_s) \cdot G_m(\omega, x_s; x_o)$ can be considered as a point source at $x_o$ radiating wavefield to $x_s$, and then backpropagated (phase conjugate) again to $x_o$, with the local plane wave directions $\hat{\theta}_s$. If we use the high-frequency asymptotic approximation of Green’s function, then the incident wave direction will be determined by the ray direction. For the more general case of wave propagation, the local plane wave decomposition is more appropriate (Wu and Chen, 2002; Xie and Wu, 2002; Wu et al. 2004). In a similar way we can analyze $[\partial G^*_s(\omega, x_o, K_s; x_s) / \partial \omega] G_m(\omega, x_s; x_o)$ The effect of data aperture (domain of integration), including the frequency band and acquisition spatial aperture, is in the integration apertures $A_f$, $A_s$ and $A_r$.

In the similar way we can write the PSF in angular spectrum domain for zero-offset and exploding reflectors modeling respectively:
Spatial resolution of seismic imaging

\[
R(K, x_0) = -2 \int dK_x \int d\omega k^2 \int dx_x G^*_i (\omega, x_0, k\hat{\theta}_i ; x_x) \times \]
\[ \times G_M (\omega, x_0, x_x) \frac{\partial G^*_i (\omega, x_0, k\hat{\theta}_i ; x_x)}{\partial z} G_M (\omega, x_0, x_x) \]

\[
R(K, x_0) = -2 \int d\omega k^2 \int dx_x \times \]
\[ \times \frac{\partial G^*_i (\omega, x_0, k\hat{\theta}_i ; x_x)}{\partial z} G_M (\omega, x_0, x_x) \]

Under asymptotic approximation, we can put \( \hat{\theta}_i = \hat{\theta}_s \) and omit the integration over \( K_x \) in (17).

![Figure 2. Theoretical PSF (point spreading function) in angular spectrum domain at some points for the SEG/EAGE salt data acquisition system.](image)

Theoretical resolution of an acquisition system and the real resolution for the imaging system

If we assume that an exact Green’s function \( G_i \) is used as the inverse propagator \( G_i \), then the effect of imperfect propagators can be eliminated, and the resolution is totally determined by the data aperture. From (15) we see that the phase information will be accurate around \( x_0 \), and therefore it is sufficient to investigate only the amplitude spectra in the resolution analysis. The resolution derived this way is a theoretical limit of the acquisition system similar to the resolution studied by previous investigations (Beylkin et al., 1986; Gelius et al., 2002; Gibson, R.L. Jr. and C. Tzimeas, 2002). Figure 2 shows the spectral coverage of the acquisition system for the case of SEG/EAGE salt model. In this case, the propagator is approximated by a one-way dual domain propagator (generalized screen propagator). Although the amplitude may be different from the case of using the actual propagator in data acquisition, the general shapes of the spectral coverage should be close to the theoretical limit of the acquisition system. We see
that for the point in the smooth background (the upper left point) the resolution spectral coverage are quite broad. On the other hand, the spectral coverages of the two points in the subsalt region are distorted in different degrees due to the influence of the salt body to the angular coverage.

To show the difference between the real resolution for an imaging system and the theoretical resolution, we give an example of imaging using exploding reflector data in random media. A ray-Kirchhoff approximation is used as the migration operator (backpropagator). Since the medium has random heterogeneities with scale comparable to the wavelength of the central frequency, the ray approximation of the Green’s function can produce large errors for long range propagation. Figure 3 shows the comparison between the angular spectra of the theoretical resolution (acquisition resolution) (on the left) and the actual resolution in the migrated image (image resolution) (on the right). We see the distortion and shrink of the spectrum due to the inaccurate operator. Fehler et al. (2005) gives the image in space domain showing deteriorated PSF in this case.

![Figure 3. Comparison of the theoretical resolution (acquisition resolution) and the actual resolution of image: On the left is the angular spectrum of the PSF calculated from acquisition resolution; on the right is the actual PSF calculated from the image resolution (equation 18).](image)

Conclusions

The resolution of an imaging system defined in this paper based on the inverse theory and the local angle domain decomposition of Green’s functions, includes both the effects of acquisition system and imaging (migration) process. It represents and quantifies the actual resolution we observed in the migrated images. If we neglect the factors such as the errors in backpropagation (migration), and consider only the influence of the acquisition system (frequency band and spatial aperture), the resolution obtained is the resolution limit of the system, the best resolution an acquisition system can get. Theoretical analysis and numerical examples have shown the importance of propagator accuracy to the evaluation of resolution in complex media.

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Seismic resolution and illumination: A wave-equation-based analysis

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SUMMARY

We develop a method to evaluate the resolution of seismic images using a wave-equation-based operator. The one-way wave-equation-based propagator is used to extrapolate the wavefields from both the source and receivers to a target region. The plane wave analysis method is used to decompose the wavefields and calculate the wavenumber domain resolution function. The method can be applied to complex models without smoothing velocity. It can also handle irregular acquisition geometry and finite frequency band. The resulting resolution function can be used to estimate image quality or correct the image distortion caused by the uneven illumination.

INTRODUCTION

The effect of acquisition on resolution of a seismic image can be evaluated using the coverage of scattering wavenumbers in the target Fourier space (Beylkin, et al., 1985). This coverage is affected by the acquisition configuration, the background velocity model and the frequency band of the signal. Due to the irregular acquisition geometry and complex velocity model involved in the seismic migration imaging processes, the resolution analysis is usually conducted using simplified model geometries or using the ray-based high-frequency asymptotic methods (e.g., Schuster and Hu, 2000; Gibson and Tzimeas, 2002; Yu and Schuster, 2003). The seismic illumination analysis shares many common basis with the resolution analysis (Muerdter and Ratcliff, 2001ab; Muerdter et al., 2001; Berkhout et al., 2001; and Volker et al., 2001). However, traditionally illumination analysis mostly focuses on how the model space is covered by seismic energy and there is no emphasis on image distortion caused by illumination.

Recently, progresses have been made in several related areas. The angle related information is emphasized in the illumination analysis and the relationship between the illumination and resolution being investigated (Gelius, et al., 2002; Lecomte, et al. 2003). The local angle related information has been extracted from the wave-equation-based propagators and applied to the illumination analysis (Xie and Wu, 2002; Wu and Chen, 2002, 2003; Wu et al. 2003; Xie et al. 2003, 2004).

In this paper, we investigate the relationship between the resolution and illumination using the wave-equation-based propagator. The resolution function can be calculated in both wavenumber and space domains. The new method has the advantage of handling wave motion phenomena. The complex models can be treated without smoothing velocity. Finite frequency band can be used in the calculation. This method can also handle irregular acquisition

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geometries. To demonstrate potential applications of this method, numerical examples are calculated using the 2D SEG/EAGE salt model.

This analysis neglects the influence of different approximations of migration operators and variations of the model from the true structure to the resolution. For a full theory of imaging resolution including the influence of imperfect propagators, and the related numerical examples, see our other studies (Wu, et al., 2005; Fehler, et al., 2005).

Figure 1. Coordinate system used in the analysis.

FORMULATION

Consider using a survey system composed of a source located at \( r_s \) and a receiver located at \( r_g \) to investigate the subsurface target within a small region \( V_r \) neighboring location \( r \) (see Figure 1). With one-way Green’s functions for heterogeneous background velocity model, the reflection seismic data can be expressed as

\[
\begin{align*}
\mathbf{u}(r, r_s, r_g) &= 2k_0^3 \int_{V_r} m(r') G(r'; r_s) G(r'; r_g) \, dv', \\
\end{align*}
\]

where \( m(r) = \delta c/c(r) \) is the velocity perturbation, \( k_0 = \omega/c_0(r) \) is the background wavenumber and \( c_0 \) is a local background velocity. Equation (1) and the following equations are in frequency domain but we omit the apparent frequency variable. The prestack depth image \( I \) at location \( r' \) within \( V \) can be expressed as

\[
\begin{align*}
I(r, r', r, r_g) &= G^* (r^*; r_s) G^* (r^*; r_g) \mathbf{u}(r, r_s, r_g) \\
&= 2k_0^3 G^* (r^*; r_s) G^* (r^*; r_g) \int_{V_r} m(r') G(r'; r_s) G(r'; r_g) \, dv'. \\
\end{align*}
\]

The Green’s functions \( G \) in above equations can be decomposed into plane waves within \( V \)

\[
G(r^*; r_s, r_g) = \int G(k, r, r_s, r_g) e^{ik \cdot r'} \, dk.
\]

Substituting equation (3) into equation (2), we obtain
\[I(\mathbf{r}, \mathbf{r}', \mathbf{r}_s, \mathbf{r}_g) = \int \mathcal{A}(\mathbf{r}, \mathbf{k}_s, \mathbf{k}_g; \mathbf{r}_s, \mathbf{r}_g) m(\mathbf{r}, \mathbf{k}_s + \mathbf{k}_g) e^{i(\mathbf{k}_s + \mathbf{k}_g) \cdot \mathbf{r}'} d\mathbf{k}_s d\mathbf{k}_g,\]  

(4)

Figure 2. Upper panel: 2-D SEG/EAGE salt velocity model; Middle panel: Wavenumber domain resolution functions; and Bottom panel: Enlarged resolution functions for selected locations.

where

\[m(\mathbf{r}, \mathbf{k}_s + \mathbf{k}_g) = \int m(\mathbf{r}') e^{i(\mathbf{k}_s + \mathbf{k}_g) \cdot \mathbf{r}'} d\mathbf{r}'.\]  

(5)

\[\mathcal{A}(\mathbf{r}, \mathbf{k}_s, \mathbf{k}_g; \mathbf{r}_s, \mathbf{r}_g) = 2k_0^3 G^2(\mathbf{k}_s, \mathbf{r}; \mathbf{r}_s) G(\mathbf{k}_g, \mathbf{r}; \mathbf{r}_g) \times G^2(\mathbf{k}_g, \mathbf{r}; \mathbf{r}_g) G(\mathbf{k}_g, \mathbf{r}; \mathbf{r}_g).\]  

(6)

In above equations, \(\mathbf{k}_s\) and \(\mathbf{k}_g\) are local transforms with respect to \(\mathbf{r}'\) or \(\mathbf{r}''\) (not \(\mathbf{r}_s\) and \(\mathbf{r}_g\)), subscripts \(s\) and \(g\) are for source- and receiver-side wavefields, respectively. In equation (4), introducing transforms \(\mathbf{k}_d = \mathbf{k}_g + \mathbf{k}_s\) and \(\mathbf{k}_e = \mathbf{k}_g - \mathbf{k}_s\), integrating \(\mathcal{A}(\mathbf{r}, \mathbf{k}_d, \mathbf{r}_s, \mathbf{r}_g)\) with respect to \(\mathbf{k}_s\) and recovering the frequency variable \(\omega\), we have

\[I(\omega, \mathbf{r}, \mathbf{r}', \mathbf{r}_s, \mathbf{r}_g) = s(\omega) \int \mathcal{A}(\omega, \mathbf{r}, \mathbf{k}_d; \mathbf{r}_s, \mathbf{r}_g) m(\mathbf{r}, \mathbf{k}_d) e^{i\mathbf{k}_d \cdot \mathbf{r}'} d\mathbf{k}_d,\]  

(7)

where \(s(\omega)\) is the source spectrum, \(\mathcal{A}(\omega, \mathbf{r}, \mathbf{k}_d, \mathbf{r}_s, \mathbf{r}_g)\) is the wavenumber domain illumination function given by Xie, et al. (2004). For an acquisition system composed of multiple sources and receivers and within a finite frequency band, the image can be expressed as

\[I(\mathbf{r}, \mathbf{k}_d) = R(\mathbf{r}, \mathbf{k}_d) m(\mathbf{r}, \mathbf{k}_d),\]  

(8)
where

\[ R(\mathbf{r}, \mathbf{k}_d) = \int s(\omega) \sum_{\mathbf{r}_s, \mathbf{r}_r} A(\omega, \mathbf{r}, \mathbf{k}_d, \mathbf{r}_s, \mathbf{r}_r) d\omega. \]  

Equation (8) can be expressed in the space domain as well

\[ I(\mathbf{r}, \mathbf{r}^*) = R(\mathbf{r}, \mathbf{r}^*) \ast m(\mathbf{r}, \mathbf{r}^*), \]  

where “\( \ast \)” stands for the convolution with respect to \( \mathbf{r}^* \) and

\[ R(\mathbf{r}, \mathbf{r}^*) = \int R(\mathbf{r}, \mathbf{k}_d) e^{i\mathbf{k}_d \cdot \mathbf{r}^*} d\mathbf{k}_d. \]  

Figure 3. Upper panel: Point scatters used to test the resolution function. Lower panel: Point spreading functions in the SEG/EAGE salt model. The center frequency of the Ricker wavelet is 15 Hz.
Figure 4. Upper panel: Small 40-degree dipping structures used to test the resolution function. Lower panel: The migration images of the dipping structures in the SEG/EAGE salt model. The center frequency of the Ricker wavelet is 15 Hz.

Figure 5. The process of generating a synthetic image from the velocity model. Shown in the upper panel is velocity model, and in the lower panel are localized operations from velocity model to synthetic image. The dashed square indicates the region under processing.
Equations (9) and (11) give the point spreading function in wavenumber and space domains. They are related by the Fourier transform. Equation (9) also shows the relationship between the illumination and resolution. We see from equations (8) and (10) that the image \( I \) is a distorted version of the model \( m \). These equations show that, given the resolution function \( R \), one can generate a synthetic “migration image” by convolving \( R \) with the model \( m \) (Lecomte, et al. 2003), or correct the illumination caused image distortion by deconvolving \( R \) from the migration images (Sjoeborg and Lecomte, 2003).

**NUMERICAL EXAMPLES**

To demonstrate the resolution analysis using a wave-equation propagator, a group of numerical examples are calculated using the 2-D SEG/EAGE salt model and its acquisition configuration. The wavefield is extrapolated using a wide-angle one-way propagator (Xie and Wu, 1998). Shown in Figure 2 are the velocity model and the wavenumber domain resolution function using equation (9). The enlarged resolution functions are shown for selected locations in the bottom panel of Figure 2. In the sub-salt region, the resolution function is weak and distorted, resulting in poor-quality images for structures oriented in certain directions.

Two examples are calculated to show the responses of structures to the background velocity model and acquisition system. The upper panel of Figure 3 shows an array of point scatters embedded in the SEG/EAGE salt model. The resulting images in the lower panel of Figure 3 simulate the point spreading functions at different locations in the model. In Figure 4, an array of 40-degree dipping reflectors is used. The migration image reveals the illumination to the specific dipping angle.
Figure 7. The process of illumination correction for the prestack depth image. Upper panel: Prestack image of the salt model; Lower panel: localized operations for image correction. The dashed square indicates the region under processing.

Figure 8. Upper panel: The original prestack depth image for the salt model. Lower panel: Resolution deconvolved depth image.

Figure 5 explains the process of generating a synthetic image. The upper panel is the velocity model. The region within the dashed square is chosen to demonstrate the process. In the lower panel and from left to right, the velocity model is Fourier transformed to the wavenumber domain and filtered by the wavenumber domain resolution function. Then, the filtered model is transformed back to the space domain. Repeating this process for the entire model generates the
synthetic image. Shown in Figure 6 are the reflectors and synthetic image generated using this technique.

Figure 7 explains the process of image correction. The upper panel is the prestack depth image. The region within the dashed square is chosen to demonstrate the processing. In the Lower panel and from left to right, the depth image is Fourier transformed to the wavenumber domain and the resolution function is deconvolved from the image. The process is actually conducted in the wavenumber domain by division. Proper water level is used to maintain the stability. The modified spectrum is transformed back to the space domain and forms the modified image. Repeating this process for the entire model generates the resolution deconvolved image. Figure 8 shows the original depth image and the resolution deconvolved image. The images of the subsalt structures, particularly some of the steeply dipping structures, are improved.

CONCLUSIONS

In this research, we developed a method for resolution and illumination analysis. Using the wave-equation-based propagator and plane wave decomposition, the method can be applied to complex models without smoothing velocity. Different acquisition configurations can be adopted in the calculation. The resolution function can be used to simulate a synthetic migration image, correct the depth image, and conduct illumination analysis for specific acquisition systems and velocity models.

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Seismic image resolution: numerical investigation of role of migration imaging operator

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Summary

The resolution of seismic migration imaging is often given as some function of the data aperture, the size of the Fresnel zone at the image location, the frequency of the data used in the migration and the illumination of the target region. Traditional resolution analysis is based on calculation of a Point Spreading Function (PSF) in the wavenumber domain from the scattering wavenumbers at the image point. This spectral representation can be converted into a space-domain PSF by Fourier Transform. However, other issues influence image resolution or distort migrated images. For example, the velocity model is generally not reliably known and variations of the model from the true structure can have a significant impact on image focus, thus reducing resolution. Another factor that limits resolution is the type and accuracy of the propagator used in imaging. To compare the resolution of images obtained when using ray-based Kirchhoff migration with that obtained from wave equation migration, we developed a 2D heterogeneous model that can be numerically simulated. Poststack migrations of exploding reflector data obtained for the model show that the PSF are influenced by the migration operator and are not necessarily the same as expected from the range of scattering wavenumbers estimated using a simple analysis approach.

Introduction

The resolution of seismic migration imaging is an important factor to consider in both acquisition design, data processing, and in image interpretation. Beylkin (1985) presented a formulation based on the generalized Radon transform and Born scattering that has provided a basis for most subsequent investigations of the resolution of seismic imaging. The formulation allows the estimation of seismic resolution at each point within an image based on survey design including source-receiver geometry and Earth model. Lecomte and Gelius (1998) and Gelius et al (2002) provide an informative discussion of the Beylkin approach and show that the predicted resolution, or Point Spreading Function (PSF) is a function of the range of scattering wavenumber at an image point.

\[ R(x, x_0) = \int h_{x_0} e^{j(k_0 x - k_0 x_0)} R \cdot B \, dk, \]

where \( R \) is the resolution as a function of \( x \) for point \( x_0 \) and \( k_0 \) is the scattering wavenumber. Note that the integration is only over the range of scattering wavenumbers resulting from the acquisition geometry and frequency content of the data. Scattering wavenumber, also referred to as exchange wavenumber, is the difference between the wavenumber of the incident wave and the scattered wave at the image point. While this formulation is useful, it does not account for practical effects such as the signal-to-noise ratio of data or fold, which strongly influence image quality and hence resolution (Gibson and Tzannes, 2002). Nevertheless, it has provided a useful tool for survey design (von Seggern, 1991; Gibson and Tzannes, 2002, Lecomte et al. 2003) and deconvolving the image blur caused by limited resolution (Sjoeborg et al., 2003).

Figure 1. Heterogeneous model used for the resolution investigation. The exploding source point is shown by a star.

Data processing will also have a significant influence on image resolution. For example the use of an incorrect velocity model not only degrades an image but leads to poor image resolution. Use of an unreliable raytracer in ray-based Kirchhoff migration imaging will degrade image resolution. It is expected that using a more reliable migration operator, such as one based on the wave equation rather than ray theory, should lead to better image resolution. Thus, acquisition-based resolution analysis is unlikely to lead to a complete estimation of resolution.

Xie et al (2005) have investigated image resolution of wave-equation imaging using the Beylkin (1985) approach adapted to wave-equation imaging. They use a wave-equation migration operator to recursively backpropagate source and receiver wavefields down to a target depth where a local analysis is done to decompose the wavefield into the wavenumber domain at a given image point. This gives the range of scattering angles at the image point, which can be used to estimate the image resolution.
Wu et al. (2005) provide a new look at image resolution based on inverse theory. They develop a formulation for estimating the image resolution at a given point in an image that is a function of not only data acquisition but also the migration operator. This allows us to take account of how reliably and completely the migration operator reproduces the data. For exploring reflector data, they show that the Point Spread Function (PSF) in the wavenumber domain can be expressed as:

$$R(k_x, k_y) = -2 \int d\omega \int dx \frac{\partial G_{J}(\omega, x, z)}{\partial x} G_{J}(\omega, x, z)$$

(2)

and in the space domain as:

$$R(x, z) = -2 \int d\omega \int dx \frac{\partial G_{J}(\omega, x, z)}{\partial x} G_{J}(\omega, x, z)$$

(3)

where the resolution function is in the wavenumber domain, $\omega$ is angular frequency, $G_J$ is the modeling Green function representing the true earth operator, and $G_J$ is the imaging operator. Note that $R$ is a function of the data and the migration operator in this formulation.

![Surface Receiver Gather](image1)

Figure 2. Numerically-generated trace data obtained for source at position of star in Figure 1. First-arrivals are shown superimposed on traces.

Approach

We wish to investigate the resolution of an image obtained in a medium containing small scale heterogeneity, whose structure is known. We choose a medium whose scale of heterogeneity is the same as the scale of the dominant wavelength of the imaging wavefield to allow us to investigate the effect of the migration propagator accuracy on the image resolution. We thus investigate imaging in one realization of a 2D randomly heterogeneous medium. The medium is described by a Gaussian-type autocorrelation function (Sato and Fehler, 1998). The background (average) velocity of the medium is 3500 m/s, the correlation length is 100 m and the fractional velocity fluctuation is 5%. The medium is shown in Figure 1.

The scalar acoustic finite difference code described in Fehler et al. (2000) was used to generate exploding-reflector synthetic seismograms for an exploding reflector source located at the position of the star in Figure 1. To make the illuminating wavefield have dominant wavelength that is on the same scale as the scale of the medium heterogeneity, we use a 35 Hz Ricker wavelet source. Figure 2 shows the surface receiver gather generated for the lower source point.

The exploding reflector shot gathers were migrated using the ray-based Kirchhoff migration imaging code described by Fehler et al. (2002). Data were migrated using only first arrivals and no corrections were made for ray amplitudes or phases. Traveltimes were calculated using a wavefront construction algorithm. Figure 2 shows traveltimes calculated using the raytracing algorithm in comparison with the trace data. The traveltimes predicted by the raytracing scheme appear to be quite reliable although the trace data contain significant later arrivals that will not be used in first-arrival migration.

![Image for Homogeneous Model](image2)

Figure 3. Migrated image for a point source in a homogeneous medium.

Data were also migrated using a wave-equation migration scheme. We used a 2D version of the globally optimized Fourier finite-difference migration approach of Huang and Fehler (2000). This method is a dual-domain approach that includes a phase shift in the wavenumber domain, a correction in the space domain that is similar to that used in the split-step Fourier migration method and finally a finite difference correction in the space domain to correct the
migrated wavefield for velocity heterogeneity.

Figure 3 shows the migrated image obtained using exploding reflector data generated in a homogeneous medium. The migration was performed using a homogeneous velocity model. The migrations obtained using the wave equation and the Kirchhoff methods are visually identical.

Figure 4 shows the result obtained from using the wave equation imaging approach to image in the heterogeneous medium. The correct velocities were used during the migration. The resolution is comparable to that shown in Figure 3, obtained for the homogeneous medium. Figure 5 shows the image obtained using a first-arrival ray-based Kirchhoff migration approach. The size of the central dark spot in the image is comparable to that obtained by the wave equation approach; however, the diffraction pattern radiating from the image point has significant amplitude at locations well away from the image point.

**Wave Equation Migration Image**

![Wave Equation Migration Image](image)

Figure 4. Wave-equation migration image of exploding reflector data for exploding reflector in the heterogeneous medium shown in Figure 1.

**Kirchhoff Migration Image**

![Kirchhoff Migration Image](image)

Figure 5. Ray-based Kirchhoff migration image of exploding reflector data for the heterogeneous medium. Only first arrivals are used in the migration.

**Point Spread Functions**

Figure 6 shows the range of scattering wavenumbers calculated using raytracing for the homogeneous medium. We assumed that the data have a bandwidth of 17 – 70 Hz, roughly that expected for a 35 Hz Ricker source. This plot is equivalent to the angular spectrum of the PSF for the homogeneous medium that is consistent with Equation (1). The spectrum of scattering wavenumbers is nearly identical to a wavenumber spectrum of the image in Figure 3, which confirms the relation between the wavenumber spectrum of the resolution and the scattering wavenumber spectrum for this simple case.

![Point Spread Functions](image)

Figure 6. Range of scattering wavenumbers calculated for the exploding reflector model in a homogeneous medium. This range is nearly identical to the wavenumber spectrum of the image shown in Figure 3, which shows the equivalence of the scattering wavenumber spectrum and the spectrum of the resolution at the point.

**Angular Spectrum of the PSF**

![Angular Spectrum of the PSF](image)

Figure 7. Angular spectrum of the PSF for wave-equation migration in the heterogeneous medium.
The angular spectra of the PSFs for the images in heterogeneous media can be estimated by taking the 2D Fourier Transform of the images since this was only one source point when generating the exploding reflector dataset.

Figure 7 shows the angular spectrum of the PSF for the wave-equation migration approach. Note that the spectrum is not as rich as the one in Figure 6, which indicates that even wave-equation migration degrades the image resolution compared to what we would expect from that calculated from the acquisition process.

![Angular spectrum of the PSF for Kirchhoff migration in the heterogeneous medium.](image)

Figure 8 shows the angular spectrum of the PSF calculated for the first-arrival ray-based Kirchhoff migration in the heterogeneous medium. The PSF is clearly not as rich or smooth as the one obtained using wave-equation migration. There is some streaking of the angular spectrum of the PSF, which shows up as linear trends passing through the origin, caused by the existence of preferential scattering angles from the image point for rays that reach the surface, which is due to the heterogeneous velocity structure. This limits the resolution and clearly indicates why the image in Figure 5 is inferior to the one in Figure 4.

Conclusions

We have performed a numerical investigation of imaging resolution in a complex medium. We have found that the angular spectrum of the Point Spread Function can be a useful tool in assessing imaging resolution. We have shown that the angular spectrum of the PSF is different in real imaging scenarios than its estimate based on acquisition. We have also shown that the PSF is dependent on the migration operator employed in a given imaging scenario and hence migration imaging resolution is a function of the operator used.

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Acknowledgments

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PART IV

BEAMLET MIGRATION/IMAGING
Basic concepts for beamlet method and comparison with GSP method

Mingqiu Luo and Shengwen Jin

Summary

The beamlet propagator is illustrated and compared with traditional one-way wave equation based dual domain methods, such as SSF and GSP methods. The beamlet methods essentially are “localized SSF methods”, and can take the advantage of the minimal errors that come up with the perturbation approximation. The “localized SSF methods”, however, are implemented completely through the free propagation matrixes in beamlet domain. Propagating waves in beamlet domain is an accurate and efficient way to extrapolate the wave field under the windowed reference velocities. In addition to the similar efficiency as GSP, beamlet methods natively provide local angle information, which can be used for directional illumination, true reflection imaging, acquisition aperture compensation, velocity updating and AVA analysis.

Introduction

The one-way wave equation based propagators can be roughly classified to frequency-space (FX) domain, frequency-wavenumber (FK) domain, and dual domain or frequency-wavenumber-space domain (FKX) based on the implementations of these methods. Generally, the dual domain propagators tend to produce a better image result in complex media, such as SSF (Stoffa et al, 1990), FFD (Ristow et al., 1994), and GSP (Wu, 1996, Huang et al., 1999, de Hoop et al., 1999, Xie and Wu, 1999). In dual domain methods, the heterogeneous velocity is decomposed into a global reference velocity and the corresponding global perturbation that accounts for lateral velocity variations at each depth level. In the presence of strong contrast velocity medium, this global velocity perturbation could be very large at some locations, leading to difficulties in correctly and efficiently propagating wide angle waves.

Steinberg (1993), Steinberg and Birman (1995) derived the phase-space propagators using the windowed Fourier transform (WFT) and a perturbation approach. Jin and Wu (1999) proposed a windowed phase screen migration using WFT. These studies represent the effort of developing localized propagators instead of the traditional global propagator methods. The localized propagators are mainly controlled by the local properties of the heterogeneous media and therefore are much easier to make good approximations than the global ones. WFT, however, is a non-orthogonal basis. Inverse WFT is thus formidably expensive for wave field reconstruction.

The beamlet migration methods have been proposed recently. Wavelet transform, instead of WFT, is used to shuffle the wave field between space and wavenumber domain. Based on the local perturbation theory (Wu et al., 2000), wave field propagates under the local reference velocity. Propagators using Gabor-Daubechies frame (GDF) (Wu and Chen, 2001) and local cosine basis (LCB) (Wang and Wu, 2002) are two major beamlets. The LCB beamlet was further extended to 3D case with affordable efficiency (Wang, et al., 2003; Luo and Wu, 2003).

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The shifted window and average local reference velocity were introduced to improve the image quality of beamlet migration (Luo, et al., 2004; Luo and Wu, 2005). In this paper, we illustrate the physical concepts of beamlet method and compare it with the dual domain methods, such as SSF and GSP in terms of the accuracy and efficiency.

**Theoretical comparison of beamlet, SSF and GSP methods**

For traditional dual domain methods, such as SSF, GSP and FFD methods, a global reference velocity is used for wave field extrapolation at each depth level. Wave field propagates with a phase shift in wavenumber domain under the single global reference velocity medium followed by the phase screen correction that accounts for global velocity perturbation in space domain. An additional wide-angle compensation term is performed for GSP and FFD methods. While for beamlet method, the medium is adaptively partitioned into a number of windows in which the local reference velocity is selected. The wave field is then decomposed into beamlets within the window. Similar to SSF method, the beamlets in each window propagate by a phase shift using the corresponding local reference velocity. The phase screen correction term is accounted for the local velocity perturbation. Therefore, the beamlet method can be regarded as “localized SSF method”, as shown in figure 1.

<table>
<thead>
<tr>
<th>Method</th>
<th>Local Reference Velocity</th>
<th>Phase Screen Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSF</td>
<td>phase shift with global</td>
<td>phase screen correction</td>
</tr>
<tr>
<td></td>
<td>reference velocity</td>
<td>with global perturbation</td>
</tr>
<tr>
<td>GSP</td>
<td>phase shift with global</td>
<td>phase screen correction</td>
</tr>
<tr>
<td>FFD</td>
<td>reference velocity</td>
<td>with global perturbation</td>
</tr>
<tr>
<td>Beamlet</td>
<td>phase shift with local</td>
<td>phase screen correction</td>
</tr>
<tr>
<td></td>
<td>reference velocity</td>
<td>with local perturbation</td>
</tr>
<tr>
<td>LCB</td>
<td>propagation in beamlet</td>
<td></td>
</tr>
<tr>
<td>GDF</td>
<td>domain</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1. Comparison of beamlet, SSF, GSP and FFD methods.

Generally, the local reference velocity provides a better approximation to the original media than the global reference velocity, since the local perturbation could be much smaller than the global perturbation, which can dramatically minimize the errors that come up with the perturbation approximation. We take the 2D SEG/EAGE model as an example to demonstrate this capability. The velocity model is shown in Figure 2(a). The minimum velocity at each depth level is selected as the global reference velocity for SSF and GSP methods as shown in figure 2(b). In this case, the global reference velocity is a function of depth. Figure 2(c) shows the corresponding global perturbation that looks similar to the original velocity model. For beamlet method, the velocity model can be adaptively partitioned into a number of smaller windows. In each window, the minimum velocity is selected as its local reference velocity as shown in figure 2(d), resulting in a small local velocity perturbation as shown in figure 2(e). Under the small local perturbation, beamlet method usually provides accurate wave field extrapolation.
(a) 2D SEG/EAGE velocity model

(b) Global reference velocity for SSF, GSP and FFD methods

(c) Global velocity perturbation for SSF, GSP and FFD methods

(d) Local reference velocity for beamlet method
The essential characteristic of beamlet method is that waves propagate in beamlet domain. The wave field is decomposed into beamlets in each window, and the beamlets propagate from windows to windows. Instead of global FFT or WFT, wavelet transform is applied to local windows. As shown in figure 3, the different colors in the windows denote the local reference velocities, and the arrows denote the beamlets. Figure 3(a) shows all beamlets propagating from depth $z$ to depth $z + \delta z$ across all windows. Figure 3(b) illustrates the propagation of beamlets in one window. Figure 3(c) shows the propagation of one beamlet.
depart from a single window. Unlike the plane waves, the propagation of beamlets is coupled. After propagating for one depth step, a single beamlet with respect to one particular angle turns to be beamlets with all propagation angles and distribute across all windows. Figure 3(c) illustrates such a process.

Comparisons of accuracy and efficiency among beamlet, SSF, GSP methods

The typical implementation of SSF method at each depth step consists of FFT from space to wavenumber domain, free propagation with a phase shift in wavenumber domain, inverse FFT from wavenumber back to space domain, and phase screen correction that accounts for velocity perturbation in space domain. An additional wide-angle compensation term is performed for GSP and FFD. A number of different schemes can be utilized to solve the wide-angle term, FD scheme is used here for GSP migration.

The implementation of beamlet method consists of wavelet transform (transfer space domain wavefield into beamlets), propagating each beamlet and summation (propagation in beamlet domain), inverse wavelet transform (transfer beamlets back into space domain wavefield), and phase screen correction. Propagating each beamlet is the most critical procedure, since each single beamlet with a particular angle turns to be beamlets in all window with different directions and amplitudes after extrapolation of one depth step as shown in figure 3(c). It is difficult to keep all these beamlets in propagation. Most beamlets with very small amplitudes will be neglected. The number of beamlets kept in the single beamlet’s propagation is defined as the non-zero number. The accuracy and efficiency of the beamlet method depends greatly on the non-zero numbers. Smaller non-zero number means less accurate and more efficient.

SSF, GSP, LCB and GDF methods are implemented in the same style to minimize the difference caused by the implementation. As shown in figure 4, post-stack migration results on the 2D SEG/EAGE salt model are compared. In terms of the computational cost and image quality, we see that: 1) SSF method is the fastest method but with worst results; 2) LCB beamlet method can be as efficient as GSP method with comparable results and is about 50% more expensive than SSF; 3) GDF beamlet method is less efficient but produces better images; 4) Using larger non-zero number in beamlet method can improve image quality but with less efficiency. In practical applications, the accuracy and efficiency is model dependent.
(b) GSP method, computing cost is about 1.53

(d) LCB method, computing cost is about 1.55

(e) LCB method, computing cost is about 1.66

(f) GDF method, computing cost is about 3.35
Special features of beamlet method

The beamlet method natively contains local angle information during wave field extrapolation. Wave field is decomposed into beamlets that propagate along the local direction or angle. Figure 5 shows one beamlet’s propagation in a homogeneous medium for LCB and GDF methods. Figure 6 shows the partial depth images of the 2D SEG/EAGE salt model by pre-stack beamlet migration. Figure 6(a) corresponds to the image from the vertical local receiver angles, while figure 6(b) is the partial image of -40 degree local receiver angles. Such local angle information can be further used for directional illumination, amplitude compensation, AVA analysis, velocity analysis, and so on.
Figure 5, beamlets propagation in homogeneous media

Figure 6. Partial images in local angle domain.
Conclusion

The beamlet propagators are illustrated and compared with SSF and GSP methods. Beamlet methods extrapolate the wave field through local reference velocity. Due to the small velocity perturbation in local window, beamlet migration produces high-quality depth images. In addition to the similar efficiency as GSP, beamlet methods natively extract local angle information that can be further used for directional illumination, true reflection imaging, acquisition aperture compensation, velocity updating and AVA analysis.

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Local cosine basis beamlet migration using average window reference velocity

Mingqiu Luo and Ru-Shan Wu

Summary

In this paper, the average window reference velocity is introduced to the local cosine basis (LCB) beamlet migration. In this method, the velocity model is windowed at each depth, and each window has a local reference velocity. The wave field in each window is decomposed into beamlets. Previously, all the beamlets departing from the same window will propagate with the same local window reference velocity, followed by the local perturbation correction. Actually, for the wide angle beamlets, they will cross several windows during the one-step propagation, and these windows may have very different reference velocities, especially for rapidly lateral velocity variations. Therefore, the average velocity between the departing and arriving windows, which is defined as the average window reference velocity, should be used as the reference velocity for the beamlets’ propagation to minimize the errors. Numerical examples on impulse response and Sigsbee2A datasets demonstrate the improvement of the image quality by using average window reference velocity.

Introduction

Wu et al. (2000) proposed beamlet migration methods based on local reference velocity and local perturbation theory. The migration methods with Gabor-Daubechies frame (GDF) and local cosine basis (LCB) have been presented by Wu and Chen (2001), Wang and Wu (2002), Wang et al. (2003), and Luo and Wu (2003), Luo and Jin (2005). As an improvement, Luo et al. (2004) had introduced the shifted windows to suppress the artifacts near steeply inclined interface of large velocity contracts.

In beamlet methods, the velocity model is windowed at each depth, and each window has a local reference velocity. The wave fields are decomposed into beamlets at each window. All the beamlets in a same window will propagate with the local window reference velocity (the reference velocity in the window), and followed by local perturbation correction. However, especially for wide angle beamlets, the wave will cross several windows after one-step propagation. So the average window reference velocity should provide more accurate phase information of the wide-angle waves. In this paper, we first give a brief description for the LCB beamlet method, and then discuss the concept of average window reference velocity. The improvement in image quality will be illustrated through numerical examples.

Wave propagation in beamlet domain

Beamlet one-way propagation consists of three steps, as shown in figure 1, the arrows denote beamlets, and different window colors denote different reference velocities. In the first step, wave field is decomposed into beamlets at depth $z$; In the second step, the beamlets are one-way propagated to the next depth $z + \Delta z$; In the third step, the wave field at depth $z + \Delta z$ is
reconstructed from all the propagated beamlets. The basic theory and method can be summarized as follows.

![Wave propagation in beamlet domain](image)

**Figure 1.** Wave propagation in beamlet domain.

Generally, in frequency-space domain, the scalar equation can be written as,

\[
[\partial_z^2 + \partial_y^2 + \partial_x^2 + \omega^2 / \nu^2 (x, y, z)] u(x, y, z, \omega) = 0
\]

(1)

where \( \omega \) denotes frequency, \( \nu(x, y, z) \) is velocity and \( u(x, y, z, \omega) \) stands the frequency domain wave field. For simplicity, \( u(x, y, z) \) is used instead of \( u(x, y, z, \omega) \) to denote the frequency domain wave field in the following part of the paper. The wave field at depth \( z \) can be decomposed into beamlets with windows along the x-axis and y-axis,

\[
u_z(x, y) = \sum_n \sum_m \sum_p \sum_q < \tilde{u}_z(x, y), \tilde{b}_{\text{mapp}}(x, y) > \tilde{b}_{\text{mapp}}(x, y)
\]

(2)

where \( \tilde{b}_{\text{mapp}}(x, y) \) are the decomposition beamlets, \( \tilde{u}_z(\tilde{x}_n, \tilde{y}_p, m \Delta \xi, q \Delta \gamma) \) are the coefficients of the decomposition beamlets located at space window \( (\tilde{x}_n, \tilde{y}_p) \) and wavenumber window \( (\tilde{m} \Delta \xi, \tilde{q} \Delta \gamma) \), \(< , > \) stands for inner product \( \langle \tilde{u}_z(x, y), \tilde{b}_{\text{mapp}}(x, y) \rangle := \int \int dx dy \tilde{u}_z(x, y) \tilde{b}_{\text{mapp}}(x, y) \).

If the space window and the wavenumber window are evenly divided, there are \( \tilde{x}_n = n \Delta x \), \( \tilde{y}_p = p \Delta y \), \( \tilde{m} = m \Delta \xi \), \( \tilde{q} = q \Delta \gamma \) and \( \Delta x \), \( \Delta y \), \( \Delta \xi \), \( \Delta \gamma \) are the sampling intervals in x-axis, y-axis, wavenumber \( \xi \) and \( \gamma \) respectively.

For the local cosine basis (LCB), the beamlets are

\[
b_{\text{mapp}}(x, y) = \frac{2}{\sqrt{L_n L_p}} b_n(x) b_p(y) \cos[\pi(m + 1/2)(x - \tilde{x}_n)/L_n] \cos[\pi(q + 1/2)(y - \tilde{y}_p)/L_p]
\]

(3)

where \( L_n = \tilde{x}_{n+1} - \tilde{x}_n \), \( L_p = \tilde{y}_{p+1} - \tilde{y}_p \) are the nominal lengths of the windows in x-axis and y-axis, and \( b_n(x) \), \( b_p(y) \) are the bell (window) functions. For the properties and fast algorithms of LCB and local cosine transform see Mallat (1998), and Wickerhauser (1994).

In beamlet domain, the wave field can be propagated with beamlet propagator matrices \( P \). The beamlet domain wave field at depth \( z + \Delta z \) can be obtained as
\[
\hat{u}_{z+\Delta z}(\overline{x}, \overline{y}, \overline{\xi}, \overline{\eta}) = \sum_{n} \sum_{p} \sum_{m} \sum_{q} P(\overline{x}, \overline{y}, \overline{\xi}, \overline{\eta}; \overline{x}_n, \overline{y}_p, \overline{\xi}_m, \overline{\eta}_q) \hat{u}_z(\overline{x}_n, \overline{y}_p, \overline{\xi}_m, \overline{\eta}_q)
\]
\[
= \sum_{n} \sum_{p} \sum_{m} \sum_{q} P_{\text{free, nnag}}(\overline{x}, \overline{y}, \overline{\xi}, \overline{\eta}) \hat{u}_z(\overline{x}_n, \overline{y}_p, \overline{\xi}_m, \overline{\eta}_q)
\]

Here \(P_{\text{free, nnag}}\) are the matrix elements of the 8D beamlet propagator matrix \(P\), which represents the beamlet propagation and cross-coupling.

The wave field at depth \(z + \Delta z\) can be reconstructed from the beamlet domain wave field through

\[
u_{z+\Delta z}(x, y) = \sum_{l} \sum_{j} \sum_{i} \sum_{r} \hat{u}_{z+\Delta z}(\overline{x}_l, \overline{y}_j, \overline{\xi}_i, \overline{\eta}_r) b_{\text{free}}(x, y)
\]

**Beamlet propagate with local reference velocity and local perturbation approximation**

A local perturbation approximation can be applied to getting the beamlet propagator matrix \(P\). It uses a local reference velocity \(v_0(\overline{x}_n, \overline{y}_p, z)\) for each window \((\overline{x}_n, \overline{y}_p)\), and the local perturbation usually is small, so that the first order approximation, i.e. the phase-screen approximation can be adopted for the correction in each window. That is,

\[
\sqrt{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \omega^2/v^2(x, y, z)} = \sqrt{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \omega^2/v_0^2(\overline{x}_n, \overline{y}_p, z) + \Delta k_{\text{pp}}(x, y, z) + \cdots}
\]

where \(\Delta k_{\text{pp}}(x, y, z) = \omega(1/v(x, y, z) - 1/v_0(\overline{x}_n, \overline{y}_p, z))\) denotes the local perturbation.

The propagator matrix \(P\) for beamlets can be decomposed into a free propagator matrix \(P^0\) in beamlet domain and a phase-screen approximation correction in the mixed domain (local space domain). That is

\[
P_{\text{free, nnag}} = e^{z i \Delta k_{\text{pp}}(x, y, z) \Delta z} P^0_{\text{free, nnag}}
\]

with

\[
P^0_{\text{free, nnag}} = e^{z i \Delta k_{\text{pp}}(x, y, z) \Delta z} P^0_{\text{free, nnag}}
\]

where \(P^0_{\text{free, nnag}}\) are the matrix elements of the beamlet free propagator in local background media (Wu et al., 2000; Wang and Wu, 2002; Wang et al., 2003; Luo and Wu, 2003). For the local cosine basis, the free propagator matrix \(P^0\) can be written as

\[
P^0(\overline{x}, \overline{y}, \overline{\xi}, \overline{\eta}, \overline{x}_p, \overline{y}_p, \overline{\xi}_m, \overline{\eta}_m, \overline{\eta}_q) = \frac{1}{4 L_n L_p} \frac{1}{(2\pi)^2} \int d\xi d\eta e^{-i \xi_n \xi_p/2} \xi_{\text{pp}} e^{i \eta_n \eta_p/2} \eta_{\text{pp}} e^{i \xi_m \xi_p/2} \xi_{\text{pm}} e^{i \eta_m \eta_p/2} \eta_{\text{pm}} e^{i \xi_n \xi_p/2} \xi_{\text{mp}} e^{i \eta_n \eta_p/2} \eta_{\text{mp}}
\]

Because of using the local reference velocity, the local perturbation in a window is much smaller than the perturbation for global reference velocity. Therefore, the local perturbation approximation is a better method when compared to SSF (Stoffa, et al., 1990) or phase-screen method for wave propagation in a heterogeneous media.
Formula (8) gives the free propagator matrix based on the local window reference velocity. It means that the same local window reference velocity is used for the propagation of all the beamlets with the same departing window, regardless their arriving windows, as shown in figure 2(a) for multi beamlets, and figure 2(b) for a single beamlet.

Figure 2. Beamlets propagation with the local window reference velocity

Figure 3. Beamlets propagation with the average window reference velocity.
Average window reference velocity

Actually, for the cross-window beamlet propagation, the use of average velocity along its path (may include several windows) can minimize the errors that come with perturbation approximation. That is, for all the cross-window beamlets, if they have the same departing window and the same arrive window, they can use the same average velocity between the departing and arriving window as their reference velocity for propagation with perturbation approximation. For example, as shown in figure 3, the average velocity between window (i, j) and (i-1, j) can be used for all the beamlets propagating from window (i, j) to (i-1, j), regardless the beamlets’ angles.

Let the average window reference velocity between window \((\bar{x}_n, \bar{y}_p)\) and window \((\bar{x}_i, \bar{y}_i)\) to be \(v_0(\bar{x}_i, \bar{y}_i, \bar{x}_n, \bar{y}_p, z)\), then the local perturbation becomes,

\[
\Delta k_{li,npj}(x, y, z) = \omega (1/v(x, y, z) - 1/v_0(\bar{x}_i, \bar{y}_i, \bar{x}_n, \bar{y}_p, z)).
\]  

(9)

The propagation matrix \(\mathbf{P}\) can be rewrite as,

\[
\mathbf{P}_{li,npj} = e^{2i\Delta k_{li,npj}(x, y, z)\Delta t} \mathbf{P}_{li,npj}^0.
\]  

(10)

With the free propagation matrix satisfies,

\[
P^0(\bar{x}_i, \bar{y}_i, \bar{x}_f, \bar{y}_f, \bar{x}_n, \bar{y}_p, \bar{x}_m, \bar{y}_q, \mathbf{P}_q) = \frac{1}{4L_nL_p} \cdot \frac{1}{(2\pi)^2} \int d\xi d\gamma e^{-i\xi_fL_f/2} \hat{b}_0(\xi - \xi_f) + e^{i\xi_fL_f/2} \hat{b}_0(\xi + \xi_f) \cdot \left[ e^{i\xi_mL_m/2} \hat{b}_0(\xi - \xi_m) + e^{-i\xi_mL_m/2} \hat{b}_0(\xi + \xi_m) \right] \cdot \left[ e^{i\gamma_qL_q/2} \hat{b}_0(\gamma - \gamma_q) + e^{-i\gamma_qL_q/2} \hat{b}_0(\gamma + \gamma_q) \right] \cdot \left[ e^{i\xi_lL_l/2} \hat{b}_0(\xi - \xi_l) + e^{-i\xi_lL_l/2} \hat{b}_0(\xi + \xi_l) \right] \cdot e^{i\xi(n_0-m_0)} e^{\Delta \xi^2/\nu_0^2 (\bar{n}, \bar{y}, \bar{x}, \bar{y}, z)} e^{\Delta \gamma^2/\nu_0^2 (\bar{n}, \bar{y}, \bar{x}, \bar{y}, z)} .
\]  

(11)

Numerical results

1. Impulse responses comparison in \(v(x, z)\) media

We applied the one-way LCB propagator to get the impulse response in a linearly variation velocity model, as shown in figure 4(a). The minimum velocity is 2000m/s, linearly variation parameter, \(dv/dx=0.125\), and \(dv/dz=0.25\). The velocity model has 1024 samples in distance and 512 in depth, each has a 10m sample interval. The LCB propagators with local window reference velocity and with average window reference velocity are applied. The impulse responses at time 0.5s, 1.0s and 1.5s are compared, as shown in figure 4(b) and (c). The impulse responses differences between the two methods are shown in figure (d).
We can see figure 4(d) that, for the wide angle waves, there are a little phase improvement for the propagator with average window reference velocity, and the improvement increase with the propagation time (or distance).

2. Prestack image comparison on 2D Sigsbee2A model

The benchmark data Sigsbee2A from the SMAART joint venture is used here to see the effect of the average window reference velocity. The dataset has 500 shots with right-side receivers, the maximum receivers for one shot is 348. The original velocity model has 3201 samples in distance and 1200 samples in depth, both the sample intervals are 25ft, as shown in figure 6.
Figure 6. Sigsbee2A velocity model.

Figure 7. Images of the Sigsbee2A model through LCB methods.

The shot profile LCB method is applied, figure 7(a) is the image result with the local window reference velocity, and figure 7(b) is the image result with average window reference velocity.
Generally, the two propagators provide similar results. However, there is still some noticeable difference in the lower boundary of the salt body, as shown in figure 8. The propagator with average window reference velocity gives more balanced amplitudes and better continuity for the boundary, especially for the steep parts.

![Image](a) image by LCB with the local window’s reference velocity

![Image](b) image by LCB with the average window reference velocity

Figure 8. Part of enlarged images through LCB methods.

Conclusion

For LCB beamlet migration, the beamlets’ propagation may cross several windows. Applying the average window reference velocity instead of the local window reference velocity can minimize the errors that come with the local perturbation approximation. Numerical results demonstrate its improvement on the image quality for complex media.
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References


LCB migration using average window
PART V

SOME SUBMITTED
AND PUBLISHED PAPERS
Measurement of phase fluctuations for transmitted waves in random media

Yingcai Zheng and Ru-Shan Wu

Abstract

Transmission fluctuations of log-amplitude and phase can be used to construct coherence functions to infer the statistical characteristics of crustal and mantle small-scale heterogeneities. Recent numerical simulations of wave propagation in random media have called attention to an apparent discrepancy of phase coherence functions between simulations and theory. In this paper, we show that the reported discrepancy is a result of the inappropriate phase estimates adopted in some literature. Using actual unwrapped phase fluctuations, instead of estimates of the fluctuations obtained by picking first arrivals by the threshold method, we find that the theory and the measurements for finite difference synthetic data produce consistent results. The method of first arrival picking does not yield adequate estimates of phase fluctuation due to the scattering dispersion of waves in the random medium. Phase unwrapping is viable and allows correct inference of medium properties.

Keywords: coherence functions; waves in random media; phase unwrapping;

Introduction

With the advent of modern global seismology [e.g., Lay and Wallace, 1995], the deterministic approach of global seismic tomography [e.g., Grand et al., 1997] revealed large-scale heterogeneities (~ a few hundred kilometers) in Earth’s interior but failed to detect the small-scale heterogeneities (~1 to 100 km) that are important in geochemistry and magma genesis [Anderson, 2004]. Coherence function studies for phase and amplitude fluctuations of transmitted teleseismic waves, on the other hand, provide a statistical approach that can be used to infer the statistical characteristics of the mantle and crustal small-scale heterogeneities. Aki [1973] pioneered the study of this type by employing Chernov’s Transverse Coherence Function (TCF) theory [see Chernov, 1960] of wave propagation in Gaussian random media to study the structure under the seismic array LASA. Subsequently, Flatté and Wu [1988] derived a two-random-layer model for lithosphere heterogeneities using data observed at NORSAR.

By recognizing the non-Gaussian nature of the heterogeneities [Sato, 1979; Wu, 1982; Wu et al., 1994; Jones and Holliger, 1997; Goff and Holliger, 1999], Wu and Flatté [1990] extended Chernov’s TCF theory to general multi-scale random media using the Rytov approximation and they further developed the Angular Coherence Functions (ACFs) and the general theory of Joint Transverse-Angular Coherence Function (JTACF) that used teleseismic waves with different incident angles to characterize the depth-dependent medium spectra. Using the Born approximation, Chen and Aki [1991] derived similar results. Wu and Xie [1991] conducted numerical experiments to obtain the depth-dependent spectra of the random medium using the JTACFs. However, this theory was challenged by some recent numerical simulations for wave propagation in random media [e.g., Line et al., 1998; Hong et al., 2004], which reported
discrepancies between data and theoretical predictions for the phase coherence functions across an array by using wavelet-based or finite difference methods.

These discrepancies motivated us to make a close inspection of the theory and the simulation techniques. We analyzed this problem in the context of well-established TCF theory because it is simpler, but illustrative. Previous theoretical developments regarding wave transmission fluctuation problems have largely been in the context of acoustic waves in random media. Using data derived from teleseismic $P$ waves, acoustic theory still has great applicability in seismology especially for the case of transmission fluctuations, which only use the waveform in the first $P$ arrival window. In this window, the particle motion is primarily compressional and conversion from $P$ to $S$ wave is negligible [Knopoff and Hudson, 1966] at high frequencies.

**Theory and data acquisition techniques in numerical simulations**

In studies of wave propagation in an inhomogeneous medium, it is common practice to separate the velocity field into a constant background velocity and a perturbation field [e.g., Wu, 1989]. For transmission fluctuation problems, only the forward scattering is considered. Due to the weak scattering limitation of the Born approximation, which is unable to treat the phase accumulation for long range propagation, often the Rytov approximation is invoked in theory. In Rytov approximation, the total wavefield $p$ is expressed as $p = \exp(\Psi_t)$, where $\Psi_t$ is complex phase field. In this expression, the effect of forward scattering is represented by a perturbation in the complex phase term $\Psi_t = \Psi_0 + \Psi_s$, where $\Psi_0$ is the background phase field, and $\Psi_s$ is the scattered phase field. Since $p = A(\omega) \exp[i\phi_t(\omega)]$, where $A(\omega)$ and $\phi_t(\omega)$ are amplitude and phase as functions of angular frequency $\omega$. Therefore $\Psi_t = \log A + i\phi_t$ and $\Psi_0 = \log A_0 + i\phi_0$. The perturbed phase field can be written as $\Psi_s = \log(A/A_0) + i(\phi_t - \phi_0)$. Considering a layer of thickness, $L$, with random velocity heterogeneity being excited by a vertically incident harmonic plane wave at the bottom (Figure 1), the log-amplitude $\log A(\omega)$ and the unwrapped phase $\phi_t(\omega)$ can be measured from the total signal at the receivers. The fluctuation functions $u = \log(A/A_0)$ and $\phi = (\phi_t - \phi_0)$ are then used to form the auto- (and cross-) coherence functions to infer the statistical properties of the random media. The TCF (transverse coherence function) theory predicts little variation of the phase TCF as a function of propagation distance at small correlation lags (Figure 2a), which contrasts with the results presented in Hong [2004] (as in Figure 2c).

If the theory and the synthetic data are correct, the only factors that could produce the discrepancy are associated with the phase/amplitude measurement techniques. Obviously, the correct method to obtain $A(\omega)$ is to compute the Fourier spectral amplitude at the desired frequency for the synthetic data around a limited time window that covers the first arrival. The criterion to determine the time window length is discussed below. Phase measurement is more complex in many ways and there are several approaches: waveform cross-correlation [VanDecar and Crosson, 1990], direct first-arrival picking by using a threshold-type method [e.g., Line et al., 1998; Hong et al., 2004] and the phase unwrapping method [e.g., Tribolet, 1977]. The cross-correlation method works best when the first-arrival waveforms are highly similar, however, this is not the case for waves propagating large distances in an inhomogeneous medium whose characteristic scale $a$ is comparable to the wavelength. Strong focusing and defocusing, wave
diffraction and interference are likely to change the waveform of the first arrival. Phase fluctuation estimation from the first arrivals (e.g., \( \phi = \omega \Delta \tau \), \( \Delta \tau \) the traveltime differences between seismograms) using the threshold method applied to the synthetic data by assuming that the phase fluctuation is a linear function of frequency. However, this assumption is not valid in the case of random media. In this case, the traveltime of a first arrival may not be the traveltime in the background medium, but the minimum traveltime along certain path in the random medium (Fermi path). Therefore the traveltime fluctuation does not correspond to the phase fluctuation. Only the measurement of the unwrapped phase is compatible to the theory.

**Numerical example**

We use an acoustic two-dimensional full wave finite difference (FD) method (4\(^{\text{th}}\) and 2\(^{\text{nd}}\) order accuracy in spatial and time derivatives, respectively) to calculate waveforms passing through random media and compare the seismograms computed by the FD method and those by the Rytov approximation. We found that the results by the two numerical methods have very little difference, implying a negligible multiple scattering in this case (i.e., weak and smooth velocity perturbation field). We use a 40km by 40km model with a background velocity of 2.5 km/s. Plane waves impinge at the bottom of the model at \( z = 0 \) km and 10 receiver lines are placed at depths \( z_i = i \times \delta = 2.42 \) km. We use FD method to simulate the wavefield and results are shown here for a random medium with a Gaussian correlation function with root-mean-square velocity perturbations of 1% and characteristic scale length \( a = 0.5 \) km. The upgoing plane wave has a Ricker source time function of central frequency \( f_0 = 4.5 \) Hz. The wavelength and the characteristic scale of the random medium are comparable. Figure 2 (a) shows the theoretical curves for the phase and logA coherence functions at different propagation distances. It is clear that at small spatial correlation lags, the phase coherence functions are independent of propagation distance. This is because the factors that control the correlation length of the phase fluctuation are dominated by the scale of the heterogeneities [e.g., Tartarskii, 1971; Wu and Flatté, 1990]. However, the correlation lengths for logA fluctuation increase with propagation distances due to the increasing Fresnel radius. In the synthetic data, the onset of the first arrival is not difficult to pick because no noise is present.

The estimated TCFs calculated from phase unwrapping and spectral amplitudes (Figure 2 (b)) agree very well with the theory. This supports the validity of the theory as long as multiple scattering is not important. Figure 2 (c) is the estimated TCF using first arrival times for the estimate the phase fluctuations. The discrepancy with 2(a) is typical of that reported in earlier studies, although in this case we are also careful to ensure we are in a valid domain for applying the TCF theory. Because of the body wave scattering dispersion [e.g., McLaughlin and Anderson, 1987], the first-arrival onset inherently indicates the fastest group velocity. The longer the source-receiver distance is, the larger is the discrepancy from the phase velocity. The estimated phase coherence functions in Figure 2(c) thus represent the group velocity fluctuation of the dispersed waveforms, not the phase fluctuation, and this is inconsistent with the basic theory. We conducted many similar simulations that all corroborate these findings. In all cases, use of unwrapped phase rather that arrival onsets yields good agreement with TCF theory.
Conclusions

Examination of a reported discrepancy between theory and numerical simulations for wave propagation in random media indicates that the discrepancy originates from the assumption that first-arrival traveltimes provide adequate estimates of phase for the early part in the waveform. We demonstrated that a method of phase unwrapping is required to obtain valid phase fluctuations. The correct TCF’s estimation enables the applications of this approach to the estimation of crustal and mantle random heterogeneity, as long as digital waveforms are available for analysis.

Acknowledgement

We thank Xiaobi Xie for providing us the finite difference code and helpful discussions. Professor Thorne Lay generously provided much valuable comment to have improved the paper greatly. This is contribution xxxxxx of the Center for the Study of Imaging and Dynamics of the Earth, IGPP, UCSC. We acknowledge the support by the fund from DOE/Basic Sciences.
References


**Figure 1.** Schematic showing a vertically incident plane wave propagating through a layer of thickness, L, with random inhomogeneities. The receivers (inverted triangles) are at the surface.
Figure 2. Theoretical predictions and the measured from numerical simulations of phase (left panels) and logA (right panels) TCFs from a Gaussian random medium. Each panel shows curves for increasing vertical propagation distance, $\delta$, in the inhomogeneous medium (a) TCF theory; (b) results obtained from the phase estimates based on unwrapped phase; and (c) results obtained using phase fluctuations derived from the first arrival’s traveltimes. In (b) and (c), the logA measurement methods are the same.
A one-way dual-domain propagator for scalar qP-waves in VTI media

Qiyu Han and Ru-Shan Wu

ABSTRACT

In this paper, we present an anisotropic one-way propagator for modeling and imaging quasi-P (qP) waves in transversely isotropic media with a vertically symmetric axis (VTI media). We derive the dispersion relation for a scalar qP-wave using elastic wave equations for anisotropic media. By applying a rational approximation to the dispersion relation, we obtain a one-way, dual-domain, scalar qP-wave propagator for heterogeneous VTI media. The propagator includes a phase-shift term and both phase-screen and large-angle correction terms. The phase-shift term is implemented in the wavenumber domain, while the other terms are implemented in the space domain. Fourier transformations are used to shuttle the wavefield between the two domains. This propagator can be used to propagate qP-wavefields within an isotropic or a VTI medium, with either medium containing lateral heterogeneities. Error analysis of the impulse response and dispersion relations demonstrates that the propagator is accurate and stable and has a wide-angle capability. The application of the propagator to the imaging of qP-wave data with VTI models which contain complex structures and large perturbations of velocity and anisotropy results in excellent image quality. This demonstrates the potential value of the propagator for use in modeling and imaging qP-wavefields within strongly heterogeneous VTI media.

INTRODUCTION

Isotropic algorithms applied to modeling and imaging in transversely isotropic (TI) media produce mispositioning of structures (Martin et al., 1992; Larner and Cohen, 1993; Isaac and Lawton, 1999). In response to this shortcoming, many imaging methods for TI media have been developed (Meadows et al., 1987; Uren et al., 1990; Gonzalez et al., 1991; Kitchen, 1991; Sena and Toksoz, 1993; Meadows and Abriel, 1994; Uzcategui, 1995; Le Rousseu, 1997; Le Rousseu and de Hoop, 2001; Ferguson and Margrave, 2002). Anisotropic models commonly use transversely isotropic media with a vertically symmetric axis (VTI media) (Key and Helbig, 1956; Thomas, 1986; Schoenberg and de Hoop, 2000), and much effort has been devoted to developing one-way wave propagation methods for these media (Uren et al., 1990; Kitchen, 1991; Meadows and Abriel, 1994; Uzcategui, 1995; Le Rousseu, 1997; Ristow and Ruhl, 1997; Le Rousseu and de Hoop, 2001; Ferguson and Margrave, 2002).

For wave equation-based methods, a one-way propagator is usually derived from the exact dispersion relation by use of a rational approximation. The dispersion relation in anisotropic media can be expressed as a function of phase angle and anisotropy parameters. This type of dispersion relation can be implemented in the wavenumber domain. Lateral heterogeneity can be treated using phase shift plus interpolation (PSPI) (Le Rousseu, 1997). This method requires substantial computation time, especially with anisotropic media. As an alternative, the dispersion relation for a one-way propagator can be expressed in the dual domain (wavenumber–space domain) and successfully applied to weakly heterogeneous VTI media with a dual-domain implementation (Le Rousseu and de Hoop, 2001). Accurate one-way propagators are preferable with strongly heterogeneous VTI media.

In this paper, we first derive the dispersion relation for a scalar qP-wave using the generalized screen approximation. We then develop the one-way propagator for VTI media. The angular dependence of the propagator is analyzed using the dispersion relation and impulse responses. Finally, we demonstrate the utility of the propagator by applying it to synthetic data sets simulated using complex VTI models.

THE qP-WAVE DISPERSION RELATION FOR 2D VTI MEDIA

Ray-tracing and wave equation methods are frequently used for wave propagation within anisotropic media.
Ray-tracing methods require the use of either slowness or velocity as a basic parameter. With the asymptotic expansion of the ray series for elastic-wave propagation in anisotropic media, the velocities of qP-, quasi-SV (qSV), and quasi-SH (qSH) waves can be obtained from the characteristic equation of the elastic-wave equation (Cerveny, 1972; Daley and Hron, 1977).

For one-way wave equation methods, rational representation of the relation between the vertical and horizontal wavenumbers is necessary. In this section, we derive the scalar qP-wave dispersion relation in the angular frequency-wavenumber (ω-k) domain from the full elastic wave equation for a general anisotropic medium.

To obtain the dispersion relation of the pure qP-wave, we introduce the wave potentials to the elastic-wave equation. This decouples the qP-wave from the qSV- and qSH-waves. In a homogeneous VTI medium, the potential of the qSH-mode is independent of the qP- and qSV-potentials. If the z-direction is taken as the unique axis, the equations for qP- and qSV-waves are represented in terms of the potential functions as

\[
\begin{align*}
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} & = c_{44} \phi + (c_{13} + c_{44}) \psi = \rho \frac{\partial^2 \phi}{\partial t^2}, \\
\frac{\partial^2 \phi}{\partial z^2} & = (c_{13} + c_{44}) \phi + c_{33} \frac{\partial^2 \psi}{\partial x^2} = \rho \frac{\partial^2 \psi}{\partial t^2},
\end{align*}
\]

where \( \phi \) is the potential of the qP-wave and \( \psi \) is the potential of the qSV-wave. The coefficients in these equations represent the density and anisotropic elasticity of the medium (Daley and Hron, 1977; Thomsen, 1986).

In an infinite homogeneous VTI solid, scalar qP- and qSV-wave equations in the \( \omega-k \) domain are obtained by applying Fourier transforms to equation 1. The vector equation of plane-wave motion in the \( \omega-k \) domain is

\[
du = 0,
\]

with

\[
u = \begin{pmatrix} \phi \\ \psi \end{pmatrix},
\]

\[
d = \begin{pmatrix} k_x^2 c_{11} + k_y^2 c_{44} - \rho \omega^2 & k_z^2 (c_{13} + c_{44}) \\ k_z^2 (c_{13} + c_{44}) & k_z^2 c_{33} + k_\perp^2 - \rho \omega^2 \end{pmatrix},
\]

where \( \phi \) and \( \psi \) are the qP- and qSV-wave potentials after Fourier transformation and where \( k_x \) and \( k_z \) are the wavenumbers along the x- and z-directions. Setting the determinant of the Christoffel matrix to zero gives the dispersion equation

\[
a k_x^2 + a k_z^2 + a_0 = 0,
\]

where

\[
a_0 = c_{11} c_{44} k_x^4 - \rho \omega^2 (c_{11} + c_{44}) k_z^2 + \rho^2 \omega^4,
\]

\[
a_2 = (c_{11} c_{33} - c_{13}^2 - 2 c_{13} c_{44}) k_z^2 - \rho \omega^2 (c_{33} + c_{44}),
\]

\[
a_4 = c_{33} c_{44}.
\]

The elastic constants in formulae 5 may be written in terms of the isotropic compressional velocity \( c \), the shear velocity \( \beta \), and the Thomsen parameters \( \delta \) and \( \epsilon \) (Thomsen, 1986). The coefficients of equation 4 then become

\[
a_0 = \rho^2 \beta^2 (1 + 2 \epsilon) \beta^2 \left( \beta^2 + (1 + 2 \epsilon) \omega^2 \right) k_z^2 + \rho^2 \omega^4,
\]

\[
a_2 = (1 + 2 \epsilon) \rho^2 \beta^2 - 2 \rho^2 \beta^2 \chi - \rho^2 \omega^2 (\alpha^2 + \beta^2),
\]

\[
a_4 = \rho^2 \alpha^2 \beta^2,
\]

where

\[
\chi = \sqrt{(\alpha^2 - \beta^2)(1 + 2 \epsilon) \omega^2 - \beta^2},
\]

\[
\epsilon = \frac{c_{11} - c_{33}}{2 c_{33}},
\]

\[
\delta = \frac{(c_{11} + c_{44})^2 - (c_{33} + c_{44})^2}{2 c_{33} (c_{33} - c_{44})}.
\]

Generally, equation 4 has four vertical wavenumber solutions corresponding to forward and backward qP-waves \( \pm (1/4 a_2) \left[ -2 a_2 (a_2 + (a_2^2 - 4 a_0 a_4))^{1/2} \right] \) and forward and backward qSV-waves \( \pm (1/4 a_4) \left[ -2 a_4 (a_2 - (a_2^2 - 4 a_0 a_4))^{1/2} \right] \). In homogeneous isotropic media, it is well known that the P-wave and S-wave vertical wavenumbers are independent of each other. The above solutions show that in homogeneous VTI media, the qP- and qSV-waves are coupled. Vertical wavenumber \( k_z \) of the qP-mode is a function of the qSV-velocity, and \( k_z \) of the qSV-mode is a function of the qP-velocity. Decoupled dispersion relations for the qP- and qSV-modes can be derived by mathematical methods such as rational approximation (Schoenberg and de Hoop, 2000). However, our numerical experiments show that the qSV-velocity makes an insignificant contribution to the vertical wavenumber for the qP-mode.

From the above solutions, we also note that qP- and qSV-waves do not exist when the qSV-wave velocity is set equal to zero. To obtain the vertical wavenumber of the qP-mode, we set the qSV-wave velocity equal to zero (Alkhalifah, 1998) in equation 4. The qP-wave dispersion relation is represented as

\[
\begin{align*}
\dot{a}_0 k_z^2 + \dot{a}_0 &= 0, \\
\dot{a}_0 &= \rho^2 \omega^4 - \alpha^2 \beta^2 \omega^2 (1 + 2 \epsilon) k_z^2, \\
\dot{a}_2 &= (\alpha^2 \beta^2 (1 + 2 \epsilon) - \alpha^4 \beta^2 (1 + 2 \epsilon)) k_z^2 - \rho^2 \alpha^2 \omega^2.
\end{align*}
\]

The vertical wavenumber in dispersion relation 8 is

\[
k_z = \pm \left( \frac{\dot{a}_0 \omega^4 - \alpha^2 \beta^2 s^2 \xi k_z^2}{s^2 \omega^2 - \eta k_z^2} \right)^{1/2},
\]

with \( s = 1 + 2 \epsilon, n = 2 (\epsilon - \delta), \) and \( \eta = 1/\alpha \). Equation 10 gives the vertical wavenumber for the qP-mode in general homogeneous VTI media. For elliptically anisotropic media, \( \xi \neq 1 \) and \( \eta = 0 \). For isotropic media, \( \xi = 1 \) and \( \eta = 0 \).

**ONE-WAY PROPAGATOR IN VTI MEDIA**

An accurate one-way propagator is required for high-quality seismic modeling and imaging in VTI media. As one step in the derivation of this propagator, we will apply the GSP (generalized screen propagator) approximation to the
square root operator for one-way wave propagation. One-way propagation of the wavefield in the wavenumber domain from depth $z_i$ to $z_{i+1}$ in a homogeneous VTI thin slab, normal to the $z$-direction, can be written as

$$\hat{\phi}_{i+1}(k_z, z) = \hat{p}_i(k_z, z)\hat{\phi}_i(k_z, z),$$  \hspace{1cm} (11)

where $\hat{\phi}_{i+1}(k_z, z)$ is the wavefield at $z_{i+1}$, $\hat{\phi}_i(k_z, z)$ is the wavefield at $z_i$, and $\hat{p}_i(k_z, z)$ is a propagator, such that

$$\hat{p}_i(k_z, z) = A(k_z, z)\exp[i\Gamma_i(k_z, z)\Delta z],$$  \hspace{1cm} (12)

with $\Gamma_i(k_z, z)$ representing the approximate vertical wavenumber, $A(k_z, z)$ representing the WKBJ amplitude, $i = \sqrt{-1}$, and $\Delta z = z_{i+1} - z_i$. Positive values of $\Gamma_i(k_z, z)$ describe forward wave movement, while negative values correspond to backward wave movement. We decompose the VTI medium into a homogeneous background medium and perturbation components. We also decompose the vertical wavenumber into a wavenumber for the background medium and a wavenumber correction, so that

$$\Gamma_i(k_z, z) = \Gamma_{b}(k_z, z) + \Gamma_{c}(k_z, z),$$  \hspace{1cm} (13)

where $\Gamma_{b}(k_z, z)$ is the vertical wavenumber for the homogeneous background and $\Gamma_{c}(k_z, z)$ is the vertical wavenumber correction for the perturbation components. The wavenumber correction plays a central role in improving the large-angle accuracy of the wavefield. The propagator $\hat{p}_i(k_z, z)$ is then represented with two subpropagators as

$$\hat{p}_i(k_z, z) = \hat{p}_b(k_z, z)\hat{p}_c(k_z, z),$$  \hspace{1cm} (14)

where

$$\hat{p}_b(k_z, z) = A(k_z, z)\exp[i\Gamma_{b}(k_z, z)\Delta z]$$  \hspace{1cm} (15)

is the subpropagator for the background wavefield and

$$\hat{p}_c(k_z, z) = \exp[i\Gamma_{c}(k_z, z)\Delta z]$$  \hspace{1cm} (16)

is the subpropagator for the deviation component of the wavefield, which is a wavenumber correction for perturbations of the medium. In the wavenumber domain, the WKBJ amplitude is written as $A(k_z, z) = \sqrt{k_{c0}(k_z, z)/k_{b0}(k_z, z)}$, with $k_{c0}(k_z, z)$ representing the vertical wavenumber for the homogeneous background medium.

By expanding the square root of the dispersion relation 10 using the GSP approximation (Wu, 1994, 2003; Xie and Wu, 1998), the vertical wavenumbers $\Gamma_{b}(k_z, z) \text{ and } \Gamma_{c}(k_z, z)$ in equation 13 can be represented as (see Appendix A)

$$\Gamma_{b}(k_z, z) = k_{b0}(k_z, z)$$  \hspace{1cm} (17)

and

$$\Gamma_{c}(k_z, z) = w_0 + w_1k_z^2 + \frac{w_2k_z^4}{w_3 + w_4k_z^2},$$  \hspace{1cm} (18)

where the coefficients $w_l$, ($l = 0, 1, 2, 3, \text{ or } 4$), are functions of the perturbations.

The subpropagators $\hat{p}_b(k_z, z)$ and $\hat{p}_c(k_z, z)$ propagate the wavefield from one thin-slab to another. According to equations 11 and 14, the equation for wave propagation in the homogeneous medium is then written as

$$\hat{\phi}_{i+1}(k_z, z) = \hat{p}_b(k_z, z)\hat{\phi}_i(k_z, z),$$  \hspace{1cm} (19)

with $\hat{\phi}_{i+1}(k_z, z) = \hat{p}_b(k_z, z)\hat{\phi}_b(k_z, z)$. In the wavenumber domain, the propagation operation is global. For each wavenumber, the wavefield data in the thin slab must be processed together. If the medium is heterogeneous, the expression of the propagator will be complex, resulting in low computational efficiency. This deficiency is removed in the spatial domain. Applying the inverse Fourier transform to equation 18, we represent $\Gamma_i(k_z, z)$ in the spatial domain as

$$\Gamma_i(x, z) = \Gamma_{b}(x, z) + \Gamma_{c}(x, z),$$  \hspace{1cm} (20)

where $\Gamma_{b}(x, z)$ is the phase-screen correction and $\Gamma_{c}(x, z) = w_0$ and where $\Gamma_{b}(x, z)$ is the finite-difference correction and $\Gamma_{c}(x, z) = w_1\hat{\phi}^2 + w_2\hat{\phi}^4/(w_3 + w_4\hat{\phi}^2)$. The phase-screen correction for the phase-shifted wavefield is represented as

$$\phi_{i+1}(x, z) = \phi_{i}(x, z)\phi_{i+1}(x, z),$$  \hspace{1cm} (21)

where $\phi_{i}(x, z) = \exp[i\Gamma_{b}(x, z)\Delta x]$ and $\phi_{i+1}(x, z) = \mathcal{F}^{-1}\hat{\phi}_{i+1}(k_z, z)$. The term $\mathcal{F}^{-1}(\cdot)$ is the 1D inverse Fourier transform in the thin slab. The finite-difference correction for the phase-screen-corrected wavefield is written as

$$\phi_{i+1}(x, z) = \Gamma_{c}(x, z)\phi_{i+1}(x, z).$$  \hspace{1cm} (22)

Note that expressions 21 and 22 are derived by assuming a homogeneous medium. The variations of their coefficients in local regions should be small in comparison with the variations of the wavefield. Since they perform local corrections in the spatial domain, these expressions are appropriate within such regions of heterogeneous media.

If perturbations exist, the propagator includes both the wavenumber- and spatial-domain subpropagators; if no perturbations exist, the propagator includes only the wavenumber-domain subpropagator $\hat{p}_b(k_z, z)$, which is the well-known phase-shift propagator for homogeneous VTI media (Gonzalez et al., 1991; Kitchenside, 1991; Meadows and Abnel, 1994). The spatial domain subpropagators correct the local wavefield for local perturbations. Therefore, the localized operation works even if the thin slab is heterogeneous, i.e., the perturbations are not equal at all locations within the thin slab. The propagator can be used for complex VTI solids. For example, if $\Delta s \neq 0, \delta_0 = 0, \delta_1 = 1, \Delta h = \Delta k = 0$, then the wave propagates in an isotropic background with isotropic velocity perturbations; if $\Delta s = 0, \delta_0 = 0, \delta_1 = 1, \Delta h = 0$, the wave propagates in an isotropic background with isotropic velocity perturbation and anisotropy perturbations. The most general case is $\Delta s \neq 0, \delta_0 \neq 0, \delta_1 \neq 1, \Delta h \neq 0, \Delta k \neq 0, \Delta k \neq 0$, in which case the wave propagates in a VTI background with slowness and anisotropy perturbations.

**ERROR ANALYSIS**

In this section, the accuracy of the propagator is discussed. We first calculate the error caused by the dispersion relation approximation using different anisotropy and perturbation values in the $w-k$ domain. We then calculate the impulse response using the dual-domain propagator with different background media and perturbation components and compare it to an accurate impulse response calculated with the phase-shift method for true homogeneous VTI media.
Accuracy of dispersion relation

To test the approximate dispersion relation for different anisotropy and velocity perturbations, we consider four VTI models as shown in Table 1. We use very strong anisotropy (the third and fourth models, which might be unrealistic) to show the mathematical flexibility of equation 13.

Figure 1 shows the approximate and exact dispersion relations for a frequency of 20 Hz. The solid curves, computed with equation 10, are exact dispersion relations in VTI media; the short-dashed curves, computed with equation 13, are their corresponding approximations. To see the degree of anisotropy, we also show the exact isotropic dispersion relations (the dotted curves). The percentage perturbation is defined as $\Delta \nu / \nu_0 \times 100\%$, with $\nu_0$ representing either $\delta_0$, $e_0$, or $a_0$ and with $\Delta \nu$ representing either $\Delta \delta$, $\Delta e$, or $\Delta a$.

Figure 1a describes the dispersion for weak anisotropy and moderate constant perturbation (100%). The parameters of the background medium are $\delta_0 = 0.025$, $e_0 = 0.0005$, and $a_0 = 2500$ m/s. The approximate dispersion relation has nearly perfect agreement with the accurate dispersion. Figure 1b is the dispersion in a strong VTI medium ($\delta_0 = 0.100$, $e_0 = 0.075$, and $a_0 = 1250$ m/s), and perturbations are 300%. This shows that the propagator has fairly good accuracy. Figures 1c and 1d show two cases of very strong anisotropy with very large perturbations ($\delta_0 = 0.043$, $e_0 = 0.114$, $a_0 = 714$ m/s and $\delta_0 = 0.082$, $e_0 = 0.064$, $a_0 = 455$ m/s and perturbations are 600% and 1000%). The approximations have a nearly perfect match with the exact dispersion relations, even for strong perturbations of the anisotropy and velocity.

Figure 2 presents corresponding relative errors for the four cases in Figure 1 at different propagation angles. The relative error is defined in the $\omega-k$ domain as $E_\delta(\theta) = |k_r(\theta) - \Gamma(\theta)| / k_r(\theta) \times 100$, where $\theta$ is a propagation angle calculated according to the accurate dispersion relation. From Figure 2 we see that the dispersion relation is exact in the vertical direction ($\theta = 0^\circ$), which is the feature of phase-screen approximation. The error is usually less than 1% for propagation angles less than 45°, and the error is less than 10% at the largest propagation angle. This proves that dispersion equation 13 closely approximates the exact dispersion relation 10 in the $\omega-k$ domain.

Accuracy of impulse response

The propagator for wavefield modeling and imaging is implemented in the $\omega-k$ domain and in the angular frequency-spatial ($\omega-x$) domain. The phase shift is implemented in $\omega-k$, and the phase-screen and large-angle corrections are in $\omega-x$. The large-angle corrections are implemented with implicit finite-difference procedures. In contrast to the computation of the dispersion relation, the implementation in the dual domain introduces artifacts due to the use of the finite-difference method. Figure 3a shows artifacts occurring at large angles. These artifacts are suppressed by tapering in the $\omega-k$ domain, although the procedure causes some distortion of

<table>
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<tr>
<th>$\delta_0$</th>
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<th>$a_0$ (m/s)</th>
<th>Perturbation (%)</th>
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<tr>
<td>0.025</td>
<td>0.0005</td>
<td>2500</td>
<td>100</td>
</tr>
<tr>
<td>0.100</td>
<td>0.075</td>
<td>1250</td>
<td>300</td>
</tr>
<tr>
<td>0.043</td>
<td>0.114</td>
<td>714</td>
<td>600</td>
</tr>
<tr>
<td>0.082</td>
<td>0.064</td>
<td>455</td>
<td>1000</td>
</tr>
</tbody>
</table>

Figure 1. Approximate and exact dispersion relations for homogeneous media with different background media and perturbation values shown in Table 1. The solid curves show the exact dispersion relations, the short-dashed curves show their corresponding approximations, and the dotted curves show their corresponding exact isotropic dispersion relations. Perturbations are (a) 100%, (b) 300%, (c) 600%, and (d) 1000%.
amplitudes at very large angles. Figure 3b is the impulse response after tapering.

To analyze the accuracy of the propagator, we made tests several times with different perturbation values. The qP-wave velocity of the model is 5000 m/s and $\delta = 0.3$, $\epsilon = 0.2$. The perturbations are 100%, 200%, 300%, 400%, respectively. Spatial samples are 280 in the x-direction and 140 in the z-direction, and $dz = dx = 15$ m. The number of time samples is 512, and the sample interval is 4 ms. The peak frequency of the Ricker wavelet is 20 Hz. Figure 4 shows corresponding responses at 300 ms computed with the propagator. The responses with solid lines are computed with the phase-shift method in actual media, while the responses with dotted lines are computed with the approximate propagator. The propagation at large angles is tapered for the approximate propagator, while

![Graph](image)

Figure 2. The relative errors of the propagator for different angles in a weakly anisotropic medium with moderate constant perturbations (dotted curve), a strongly anisotropic medium with large constant perturbations (dot-dashed curve), and two very strongly anisotropic media with two larger perturbation values (dashed curve and solid curve).

![Images](images)

Figure 3. Large-angle corrections introduce artifacts (a), which can be suppressed by tapering in the $\omega$-$k$ domain (b).

![Images](images)

Figure 4. Responses in a homogeneous VTI medium. The responses with solid lines are computed with a phase-shift method in actual media. The responses with dotted lines are computed with the approximate propagator that propagates in a VTI background with constant percentage perturbations of (a) 100%, (B) 200%, (c) 300%, and (d) 400%.
we do not apply any taper for the phase-shift propagator. As shown in the last section, in a medium without any horizontal perturbations, the wavefield computed with the phase-shift propagator is accurate.

Figure 4 demonstrates that the results from our propagator in media with different perturbations agree closely with those from the phase-shift method in the true media. The propagator has a good degree of accuracy, even for large perturbations and at large propagation angles.

Figure 5 describes relative errors corresponding to the approximate responses in Figure 4. The errors of the dispersion relation and those of the dual-domain propagator exhibit similar behavior. From Figure 4 we see that with increasing levels of perturbation, the relative error seems to increase slowly at large angles.

**EXAMPLES OF APPLICATION TO IMAGING**

This propagator can be used for both seismic modeling and imaging. In this paper, we test its imaging performance with two synthetic data sets. One data set is for testing its ability to image steeply inclined reflectors embedded in a VTI medium with strong perturbations. Another data set is for testing its ability to image highly heterogeneous VTI media with complex structures and faults.

**Imaging reflectors with different angles**

As shown in Figure 6, the model has reflectors with dips of 0°, 15°, 30°, 45°, 60°, 75°, and 90°. The grid dimensions of the model are 100 gridlines in depth and 700 gridlines in the horizontal, with \( dx = dz = 12.19 \) m. A similar model is used by Le Roux, et al. (1997). The anisotropy of the model is characterized by \( \epsilon = 0.2 \) and \( \delta = 0.3 \). The qP-wave velocity model is composed of the background velocity of 701.04 m/s and vertical and lateral gradients of \( \beta, \alpha = 0.6 \) s \(^{-1} \) and \( \beta, \alpha = 1.0 \) s \(^{-1} \). The largest velocity perturbation at the model surface (the first thin slab) is 730%; the largest perturbation at the model bottom (the last thin slab) is 267%. A Kirchhoff modeling method for a factorized TTI medium (Alkhalifah, 1995) is used to generate a zero-offset section with the central frequency of 8.5 Hz. As shown in Figure 7, the synthetic section has 700 traces and 401 samples per trace. The sampling interval is 10 ms.

Imaging results are shown in Figure 8. Figure 8a shows that when the anisotropy of the model is ignored, the isotropic propagator gives a poor image. The steep reflectors are seriously undermigrated, and the energy at the diffraction points of the structures is unfocused. The reflectors are well imaged when the propagator takes the anisotropy into account (Figure 8b). This example demonstrates that the one-way propagator in VTI media developed in this paper has excellent performance for imaging steep reflectors in strongly heterogeneous VTI media.

**Imaging a modified SEG/EAGE salt model**

We modified the SEG/EAGE salt model, which has strong velocity perturbations, complex structure, and faults. First, we transformed it into an anisotropic model by adding anisotropic
components to the original isotropic representation. Then we compared the imaging results with the Kirchhoff imaging method for anisotropic media based on a zero-offset section.

The grid dimensions of the SEG/EAGE model are 300 gridlines in depth and 1290 gridlines in the horizontal, with spacing \( dx = dz = 12.19 \) m. Figure 9 shows the compressional velocity distribution of the original model. While keeping the original distribution of compressional velocities, we introduce Thomsen’s parameters to the model to construct a VTI model. We modify the surrounding sediment and faults into a VTI representation. The anisotropy parameters of the model are linear functions of perturbation of the compressional velocity, i.e., \( \delta(x, z) = 0.80 (\alpha(x, z) - \alpha_{\text{ref}}) / \alpha_{\text{ref}} + 0.005 \), \( \epsilon(x, z) = 0.64(\alpha(x, z) - \alpha_{\text{ref}}) / \alpha_{\text{ref}} + 0.005 \). The maximum values of \( \epsilon \) and \( \delta \) are 0.32 and 0.40, respectively, while their minimum values are each 0.005. It might be more reasonable to keep the salt body isotropic by separating it from its surroundings. In this case, traveltime continuation would be necessary for the traveltime table computation with the anisotropic Kirchhoff method. Unfortunately, it would be difficult for such an irregular boundary. In this paper, for simplicity, we make the salt body weakly anisotropic (\( \epsilon = 0.002, \, \delta = 0.001 \)) to avoid explicitly dealing with the irregular boundary of the salt body.

The ray-tracing method based on a finite-element technique (Han and Shen, 2002a, b) was applied for the traveltime table computation. This ray-tracing method is efficient, especially for complex models with shadow zones.

A zero-offset section generated by exploding-reflector modeling with our anisotropic propagator is shown in Figure 10. The peak frequency of the Ricker wavelet is 8 Hz. The sampling interval is 8 ms, and the number of samples per trace is 626. The CMP number is 1290, and the CMP spacing is 12.19 m.

Figure 11 shows poststack depth imaging results using both the one-way propagator and the anisotropic Kirchhoff imaging method. Figure 11a is the image migrated with an isotropic propagator, in which the anisotropy parameters of the propagator in equation 14 were ignored. Compared with the model in Figure 9, the reflections in Figure 11a are not well imaged by the isotropic propagator because the anisotropy is ignored. Although the energy is well focused, the image has artifacts and the steep structures are mispositioned. Figure 11b is imaged with a true VTI propagator. In contrast to Figure 11a, this image has fewer artifacts, and the structures and faults are imaged correctly. Figure 11c is the imaging result using the Kirchhoff method. Shallow reflections, including the upper boundary of the salt body, are well imaged. However, artifacts

![Figure 8. Depth images of the section shown in Figure 7 migrated with (a) an isotropic propagator and with (b) the true anisotropic propagator.](image)

![Figure 9. Compressional velocity distribution of the modified anisotropic SEG/EAGE salt model.](image)

![Figure 10. Zero-offset section for the modified anisotropic SEG/EAGE salt model. Peak frequency of the Ricker wavelet is 8 Hz. The sampling interval is 8 ms. The number of traces is 1290, and the number of samples per trace is 626.](image)
One-way propagator for VTI media

and the wide-angle correction, is implemented in the space domain. The propagator can be used for complex VTI media. Results show that the propagator is accurate for strong perturbations of velocity and anisotropy parameters even at large propagation angles. The propagator can be used for qP-wave field simulation and imaging/migration. In this paper we only derived the propagator for a scalar qP-wave in 2D VTI media; the propagator for 3D media may be derived easily from the 2D case. With successive application in several directions, the 2D propagator may be used for a 3D medium. This application of the 2D propagator substantially reduces computation time and memory requirement over use of the 3D propagator. We intend to describe this procedure in the future.

ACKNOWLEDGMENTS

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APPENDIX A

CONSTRUCTION OF THE APPROXIMATE ONE-WAY PROPAGATOR

The one-way phase-shift propagator is accurate in horizontally homogeneous VTI media (Meadows et al., 1987; Kitchenside, 1991; Meadows and Abriel, 1994). This propagator is inaccurate at large angles in laterally heterogeneous media, especially for large perturbations of anisotropy and velocity. Extensions such as PSPI (Le Rouxseau, 1997) and GSP series (Le Rousseau and de Hoop, 2001) have been developed to accommodate lateral variations of the model parameters. The finite-difference technique is an efficient method for the application of large-angle corrections in isotropic media (Nielson and Ruhl, 1994; Jin et al., 1998; Xie and Wu, 1998; Wu, 2003). In this paper, we develop a correction with finite-difference terms using the generalized screen (GS) approximation.

To improve the wide-angle accuracy of the GS method in the presence of a large velocity contrast, a two-term correction is often used to reduce the angle-dependent propagation error from the dispersion relation approximations (Jin et al., 1998; Xie and Wu, 1998, 1999; Wu, 2003). Our numerical experiments demonstrate that wave propagation in the presence of anisotropy requires additional correction. This occurs because in VTI media the angle-dependent errors resulting from approximation of the dispersion relation have similar effects to variations in anisotropy. If the heterogeneity of the medium is strong enough, we cannot distinguish angle-dependent errors from the effect of anisotropy when we simulate, image, or analyze the anisotropic wavefield with this propagator. For example, we may misinterpret the approximation errors as anisotropy effects when we make the migration-based anisotropy analysis. Here, we derive a three-term correction for the dispersion relation approximation, which is more accurate than the two-term correction.
According to the dispersion relation 10, the vertical wavenumber of the correction term is

$$
\Gamma_c(k_x, z) = k_x - k_0 \approx \omega \left[ 1 + \frac{\left( \kappa - \kappa_0 \right) k_x^2}{\kappa_0^2 a^2} \right]^{1/2} - \omega_0 \left[ 1 + \frac{\left( \kappa - \kappa_0 \right) k_x^2}{\kappa_0^2 a^2} \right]^{1/2}.
$$

(A-1)

Using Taylor expansions for $\kappa_x$ and $\kappa_0$ and keeping up to the sixth-order term of the series, we obtain the approximate phase correction

$$
\Gamma_c(k_x, z) = \gamma_0 + \gamma_1 k_x^2 + \gamma_2 k_x^4 + \gamma_3 k_x^6,
$$

(A-2)

where the coefficients are defined with perturbations of the media, $\Delta \kappa_k (t = 0, 1, 3, 4)$, and $\gamma_0 = -\omega_0 \kappa_0, \gamma_1 = (1/2\omega_0)\Delta \kappa_1, \gamma_2 = (1/\omega_0^2)\Delta \kappa_2, \gamma_3 = (1/\omega_0^3)\Delta \kappa_3$. The perturbations of the media are defined as $\Delta \kappa_0 = \kappa_0 - \kappa_0, \Delta \kappa_1 = \kappa_1 - \kappa_0, \Delta \kappa_2 = \kappa_2 - \kappa_0, \Delta \kappa_3 = \kappa_3 - \kappa_0$. The parameters with a bar represent the homogeneous background terms, and the parameters without a bar pertain to the true media. These parameters are written as $k_0 = \kappa_0, k_0 = \kappa_0, k_1 = (\kappa - \kappa_0)/\kappa_0, k_2 = (\kappa - \kappa_0)(\kappa^2 - \kappa_0^2)/\kappa_0^2, k_3 = (\kappa - \kappa_0)(\kappa^3 - \kappa_0^3)/\kappa_0^3, k_4 = (\kappa - \kappa_0)(\kappa^4 - \kappa_0^4)/\kappa_0^4)$, where $k_0$ and $\kappa_0$ are the anisotropy parameters of the true VTI medium, and $\kappa$ and $\gamma$ are the anisotropy parameters of the true VTI medium, which are defined using the Thomson parameters $\kappa_0$ and $\kappa_0$ (Thommen et al., 1986), so that $\kappa_0 = 2(k_0 - \kappa_0), \kappa_0 = 1 + 2\alpha_0, \kappa_0 = 2(\kappa - \kappa_0)$, and $\kappa_0 = 1 + 2\alpha_0$.

We now construct rational terms $w_1 w_2 k_x^2 + w_2 w_3 k_x^4 / (w_3 + w_4 k_x^2)$, where the unknown coefficients $w_1, w_2, w_3, w_4$ (1, 2, 3, 4), are determined through A-1. With a Taylor expansion of $w_3 k_x^2 / (w_3 + w_4 k_x^2)$ up to the sixth order in $k_x$, we compare the coefficients of A-1 and obtain the five coefficients $w_1 = \gamma_0, w_2 = (\gamma_1 - \gamma_2^2), w_3 = \gamma_2, w_4 = \gamma_3, \kappa_1 = \kappa_0 \gamma_0 \gamma_1$. We may then rewrite equation A-2 in the form

$$
\Gamma_c(k_x, z) = w_1 + w_2 k_x^2 + w_3 k_x^4 / (w_3 + w_4 k_x^2).
$$

(A-3)

Note that the first term of equation A-3, $w_1 z_2$, is the phase-screen term in a GS expansion, while $w_2 k_x^2$ and $w_3 k_x^4 / (w_3 + w_4 k_x^2)$ are the wide-angle correction terms.

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Investigating explosion source energy partitioning and $L_g$-wave excitation using a finite-difference plus slowness analysis method

Xiao-Bi Xie, Zengxi Ge and Thorne Lay

Abstract   A finite-difference modeling plus slowness analysis method is developed to investigate near-source explosion energy partitioning and $L_g$-wave excitation. The finite-difference method is used to calculate seismic wave excitation and propagation and a local slowness analysis is used for quantifying how energy will be partitioned into the long-range propagation regime. Due to its high efficiency, the method can simulate near-source processes using very fine structures. A large number of source and model parameters can be examined for broad frequency ranges. As examples, $P$-$pS$-$L_g$ conversion and $S^*$-to-$L_g$ excitation in the presence of near-source scattering are tested as mechanisms for $L_g$-wave excitation. The numerical results reveal that the depth of the source and the depth of the scattering process have strong effects on the $P$-to-$S$ conversion and partitioning of energy into trapped or leaking signals. The $L_g$-wave excitation spectra from these mechanisms are also investigated. The modeling shows that $S^*$-to-$L_g$ excitation is generally stronger for low-frequencies and shallow source depths while $P$-to-$L_g$ scattering is stronger for high-frequencies.

Introduction

With the current emphasis on global monitoring for low-yield nuclear tests, regional seismic phases such as $L_g$ and $L_g$ coda have become very important for magnitude and yield estimation of underground nuclear tests. (e.g., Nuttli, 1986; Xie, et al., 1996; Patton, 2001). In addition, various $P/S$-type amplitude ratios for high frequency regional phases (e.g., $Pn/Sn$, $Pn/Lg$, $Pg/Lg$, $Pg/Sn$) have become important for event discrimination (e.g., Taylor et al., 1989; Kim et al., 1993, 1997; Walter et al., 1995; Fisk et al., 1996; Taylor, 1996; Taylor and Hartse, 1997; Hartse et al., 1997; Fan and Lay, 1998a-c). The applications of regional phases for yield estimation and event discrimination are largely based on empirical approaches, and while very promising in many cases, there are major questions about the nature of excitation of $S$-wave dominated phases such as $L_g$, and there are similar questions regarding the relative excitation effects for $P/S$-type ratios in regional phases, particularly given the huge scatter observed in both earthquake and explosion data populations.

There are now many observational and theoretical studies addressing the regional phase energy partitioning issue in both the near source environment and the propagation path environment. Along the propagation path, the existence of small-scale heterogeneities in the crust and the associated seismic wave scattering has long been addressed by seismologists (Wu and Aki, 1988; Sato and Fehler, 1998), and its effect on the long-range energy partitioning has been documented (Wu, et al., 2000a, b). In the source region, several possible near-source energy excitation mechanisms have proposed, including $P$-to-$L_g$ scattering, $pS$-to-$L_g$ conversion at the free surface, $Rg$-to-$L_g$ coupling, $S^*$-to-$L_g$ conversion, spall excitation of $S$, tectonic release and rock-damage (e.g., Day and Mclaughlin, 1991; Gupta et al., 1992, 1997; Wallace, 1991;
Gutowski, et al., 1984; Xie and Lay, 1994; Vogfjord, 1997; Johnson and Sammis, 2001). There is continuing controversy about which mechanism(s) dominate the explosion source energy partitioning processes.

Due to the complex excitation and energy partitioning processes associated with regional phases, it is difficult to empirically separate the contribution of individual energy partitioning mechanisms by analysis of the data. Numerical modeling approaches are thus of great importance for investigating the excitation and propagation of regional phases. Early numerical simulations investigated the propagation and excitation characteristics of regional phases in horizontally stratified crustal models (e.g., Bouchon, 1982; Campillo et al., 1984; Campillo and Paul, 1992). However, with a 1D model, phenomena associated with laterally varying crustal heterogeneity cannot be generated, so the results were very limited. Kennett and Mykkeltveit (1984) and Kennett (1989) used the coupled mode method to calculate \( L_g \)-wave propagation in crustal waveguides with weak lateral heterogeneities. The method is suitable for low frequency waves. Using full wave numerical techniques such as finite-difference (FD) (Frankel 1986; Vidale and Helmberger, 1988; Hayashi, et al., 2001) or a pseudo spectrum method (Kosloff and Baysal, 1982; Tessmer and Kosloff , 1994; Orrey, et al., 2003) to explore seismic wave propagation has the advantage that complicated two- or three-dimensional structures can be included in the model and the complete wave field can be calculated. These numerical methods are also useful for investigating \( L_g \)-wave excitation and propagation in complicated crustal waveguides. Xie and Lay (1994) investigated \( L_g \)-wave excitation using the FD method. Jih (1995, 1996) investigated \( R_g \)-to-\( L_g \) coupling as a possible \( L_g \) excitation mechanism. With an anelastic FD method, Bradley and Jones (1998, 1999) investigated \( L_g \) propagation and attenuation in Western China and India. Using the 2D and 3D general Fourier methods, Bonner, et al. (2003) investigated \( R_g \) and \( L_g \) generation, and partially reproduced the observed spectrum from the Depth of Burial Experiment. Stevens et al. (2003) investigated the physical basis of explosion generated \( S \)-waves using a 2D nonlinear FD method, which handles axisymmetric near-source effects including spall, cracking, and nonlinear deformation.

The main disadvantages of these numerical methods, which provide complete synthetic seismograms, are their low computation efficiency and huge computer memory requirement, especially when applied to investigate the characteristics of broadband \( L_g \) excitation. For the purpose of small nuclear test monitoring, the range of interest for \( L_g \)-wave simulation involves a broad frequency band (0.2 to 10 Hz) and long propagation distances (up to 1000 km or more). At the same time, factors that control the source energy partitioning depend on the detailed source mechanism and fine near-source velocity structure. In addition, there are multiple mechanisms that may potentially contribute to the energy partitioning process. Numerous parameters need to be tested to investigate the characteristics of these mechanisms, especially their contributions to the frequency dependent features of observable discriminants. If random heterogeneities are to be considered, as is likely to be important for high frequency signals, the results have to be calculated statistically from simulations using a large number of realizations. This limits the approach of complete FD synthesis for real recording geometries.

Although there are continuing controversies about the dominant \( P \)-to-\( S \) transfer mechanisms affecting regional phases, most investigators agree that appreciable energy from explosion sources is converted to \( S \)-waves in the near-source region (e.g., Myers, et al., 2003).
The physical processes by which an explosion source generates regional phases can be described as energy partitioning taking place in the near-source region. The partitioned energy subsequently propagates through a long waveguide, where secondary energy partitioning effects may occur, but these are less affected by the type of source involved. The combined processes in the near-source region and along the path naturally separate the wave field energy into groups of observable regional phases according to their slowness and group velocity. If the propagation effect is not of primary interest, it is desirable to avoid calculating the immensely time-consuming long-distance propagation part of the problem by focusing on the near-source energy partitioning processes. To achieve this, we developed a method based on the finite-difference simulation and local slowness analysis to investigate the near-source energy partitioning of an explosion source. This method investigates the partitioning process right at the source region, but quantifies how energy will transfer into the long range propagation path, which is critical for comparisons with data. The localized analysis thereby provides uncontaminated information isolating the physical processes controlling the energy partitioning. In the following sections, we first present the method with numerical calculations used to demonstrate its validity. Then, as examples, we use this method to investigate a number of potential $Lg$-wave excitation mechanisms and their contributions to explosion source energy partitioning.

Methodology

We limit our 2D finite-difference simulation to a relatively small model and analyze the wavefield within the model to investigate the source energy partitioning and the excitation of trapped regional phases such as $Lg$. Many authors (e.g. Frankel, 1989; Xie and Lay, 1994; Vogfjord, 1997) have pointed out that for S-wave energy to be trapped in the waveguide, reverberating to generate the $Lg$-wave, it must propagate with post critical angle at the Moho discontinuity. However, in the waveguide and especially in the near-source region, the wavefield is highly complex. It is impractical to trace each phase in the spatial-time domain. An alternate, but equally valid way of tracking the wave energy is in the slowness domain. Multiply-reflected waves may arrive simultaneously in time, but in the slowness domain their energy distribution gives clear information about the wave intensity, slowness, and propagation direction. Several methods can be used to transfer spatial-time domain data into slowness (or equivalently wavenumber) domain information, for example, FK analysis or slant stacking. Here we employ a local slant stacking method to conduct slowness analysis, working simultaneously in both the space and slowness domains. We call this the Finite-Difference Slowness Analysis (FDSA) method.

Local Slowness Analysis

Two-dimensional local slant stacking in the horizontal and vertical directions can be expressed as

$$\mathbf{u}(\mathbf{r}, \mathbf{p}, t, \omega) = \frac{1}{C_1} \sum_r W_h(\mathbf{r}' - \mathbf{r}) u[\mathbf{r}', t - \mathbf{p} \cdot (\mathbf{r}' - \mathbf{r}), \omega],$$  \hspace{1cm} (1)

where $\mathbf{r} = \hat{x} r_x + \hat{z} r_z$ is the 2D position vector, $\hat{x}$ and $\hat{z}$ are unit vectors in the $x$ and $z$ direction, $\mathbf{r}'$ is the location of the receiver, $t$ is time, $\mathbf{u}(\mathbf{r}, t, \omega)$ is the bandpass filtered synthetic seismogram with central frequency $\omega$, $W_h(\mathbf{r}' - \mathbf{r})$ is a two-dimensional space window centered at $\mathbf{r}$, $\mathbf{p} = \hat{p} \hat{p}$ is
the slowness vector, \( \hat{e}_p \) is the unit vector of the slowness direction, \( p = v^{-1} \) is the wave slowness, \( v = v_p \) or \( v = v_s \) is \( P \)- or \( S \)-wave velocity, \( C_i \) is a normalization factor determined by the size of the space window and bandwidth of the frequency filter. The space window is for a small vertical seismic array with size related to both space resolution and slowness resolution. A larger array gives better slowness resolution but tends to smear the spatial resolution, while a small array gives better spatial resolution but less accurate slowness calculation. A proper trade-off between space and slowness resolution is required. The receiver interval should be small enough to avoid spatial aliasing. The average energy density of the wavefield as a function of space, time, slowness and frequency can be obtained as

\[
D(r, p, t, \omega) = \frac{1}{2} \rho \hat{u}^2(r, p, t, \omega),
\]

and the energy flux related to slowness vector \( p \) can be calculated as

\[
J = vD(r, p, t, \omega)\hat{e}_p,
\]

where \( u = |\hat{u}| \) is the amplitude of the stacked seismogram, \( \rho \) is the density. Similarly, the one-dimensional horizontal slant stack can be calculated as

\[
u(r, p_x, t, \omega) = \frac{1}{C_2} \sum_{x'} W_h(x' - x, z) u[x', z, t - p_s(x' - x), \omega],
\]

where \( p_s = p\hat{e}_p \cdot \hat{e}_x \) is the horizontal slowness, \( W_h(x' - x, z) \) is a one-dimensional space window, and \( C_2 \) is a normalization factor similar to \( C_1 \) in equation (1). The average energy density as a function of \( p_x \) can be expressed as

\[
D(r, p_x, t, \omega) = \frac{1}{2} \rho \hat{u}^2(r, p_x, t, \omega).
\]

The pure horizontal energy flux related to horizontal slowness \( p_x \) can be obtained from equation (3)

\[
J_h(r, p_x, t, \omega) = \int J(r, p, t, \omega) \cdot \hat{e}_p \, dp_x.
\]

Alternatively, \( J_h \) can be calculated from horizontal slant stacking, i.e., equations (4) and (5)

\[
J_h(r, p_x, t, \omega) = vD(r, p_x, t, \omega)\hat{e}_p \cdot \hat{e}_x
\]

where \( \hat{e}_p \cdot \hat{e}_x = v p_x = \sin i \) and \( i \) is the wave incident angle (relative to the vertical direction). In the waveguide, the energy passing through a surface \( S \) within a frequency range \( \Omega \), a time window \( T \) and a slowness band \( P \) can be calculated as

\[
E(S, P, T, \Omega) = \int_S \int_T \int_{\Omega} J \hat{e}_n \, d\omega \, dt \, d\mathbf{p} \, ds,
\]

where \( \hat{e}_n \) is the unit normal vector of the surface element \( ds \), and \( d\mathbf{p} = dp_x dp_z \). When choosing a vertical intersection as the surface, equation (8) becomes

\[
E(S, P, T, \Omega) = \int_S \int_T \int_{\Omega} J_h \, d\omega \, dt \, d\mathbf{p} \, dz.
\]

Equations (8) and (9) provide the basis for extracting energy from joint domains. To investigate the near-source energy partitioning and regional phase excitation, the energy generated from specific mechanisms is decomposed into multiple domains and analyzed based on its dynamic and kinematic properties. The condition for energy to be trapped in the crustal waveguide, i.e., \( p_x \geq 1/v_{S\text{-mantle}} \) (where \( v_{S\text{-mantle}} \) is the upper mantle \( S \) velocity) is applied. We then use the joint window \( (S, P, T, \Omega) \) to sort the trapped energy and estimate the contribution of specific mechanisms to regional phases such as \( Lg \). Equations (8) or (9) can
be partially integrated, which allows the energy to be projected onto different domains. The analysis within multiple domains provides additional information to characterize the contributions from different mechanisms. This method has the flexibility that we can either intercept the entire waveguide energy flux or just monitor the energy from specific phases or mechanisms. The calculation using two-dimensional slowness analysis has the advantage that energy is fully expanded in the entire slowness domain. The slowness vector \( \mathbf{p} \) for both \( P \)- and \( S \)-waves can be obtained independently, giving us more information to investigate complicated near-source processes. The calculation based on one-dimensional slowness analysis gives the energy as a function of horizontal slowness \( p_x \). It provides the necessary information to separate the trapped and leaking energy. Although it does not directly give the full slowness domain information, by combining dynamic and kinematic characteristics, we can resolve the near-source phenomena in most cases without ambiguity.

Equations (8) and (9) are expressed with energy, since energy can be directly summed. However, regional phase observations are usually taken from amplitudes. To compare the numerical prediction with observations, we calculate the normalized square root of the energy

\[
A = \left( \frac{E}{E_0} \right)^{1/2},
\]

where \( E_0 \) is a normalization factor which can be obtained by calculating the response of a unit source. The unit source is located at unit distance in an infinitely homogeneous model. It has a unit intensity, the same source time function as that used in the simulation and its response passes through the same frequency filter. Normalized square root energy \( A \) is consistent with the conventional \( Lg \)-wave measurement based on the RMS amplitude of the waveforms. Another advantage of using \( A \) is that the variation of amplitude distribution is smoother than the variation of energy. For these reasons, throughout this paper, we will use the normalized square root energy in all figures although we sometimes simply call it “energy”.

The FDSA process can be briefly described as the following steps. (1) Use the finite-difference method to calculate the synthetic wavefield for near-source velocity model. (2) Apply the space window to the synthetic wavefield and locate the area to be investigated. (3) A series of bandpass filters with different central frequencies are applied to the synthetic seismograms. (4) Apply time windows to the seismograms to locate specific sections that contain the phases of interest. (5) Slant stack the windowed synthetic seismograms for a range of slownesses. (6) Calculate energy from stacked seismograms which gives the wavefield energy as a function of space, time, frequency and slowness. (7) Based on the dynamic and kinematic properties of specific phases as well as the criterion for energy to be trapped in the waveguide, sort the energy in the multiple domains to estimate their contributions to the regional phases.

Figure 1 shows examples of one- and two-dimensional slowness analyses. The energy can either be expressed as a function in 2D slowness domain, or as a function in mixed slowness-depth domain. For references, the upper mantle \( S \)-slowness \( 1/v_{S\text{-mantle}} \), crustal \( P \)-slowness \( 1/v_p \) and \( S \)-slowness \( 1/v_s \) are labeled in the figure, with \( v_{S\text{-mantle}} = 4.57 \text{ km/s}, \ v_p = 6.0 \text{ km/s} \) and \( v_s = 3.5 \text{ km/s} \). All \( S \)-wave energy with horizontal slowness larger than the upper mantle \( S \)-slowness, whether directly radiated from the source or generated as secondary phases, will be trapped in the crustal waveguide and will contribute to the guided regional phases unless
subsequent scattering causes it to leak out. The trapped $S$-wave energy determined within the near-source region depends on two factors, the intensity of the $S$-wave energy radiated or converted in the near-source region and the percentage of the energy that can intrinsically be trapped in the waveguide. Figure 2 is a sketch showing the configuration of the FDSA method for investigating the near-source processes. The model uses an explosion source, a fine scale near-source velocity model and a short distance receiver array to provide synthetic seismograms for the ensuing slowness analysis. Typically, the size of the model used by the FDSA method is much smaller than that required for a full-scale regional wave simulation. This allows a variety of very detailed near-source velocity models to be used and broadband synthetic seismograms to be calculated.

Figure 3 gives an example of slowness analysis. Shown on the top is the synthetic seismogram at the center of the mini array. In the middle are energy distributions for $P$-coda, $Lg$ and $Rg$-waves in 2D slowness domain. These energy distributions clearly show that the $P$-coda is composed of $P$- and reflected $pS$-waves. Both of these have relatively small horizontal slowness. The $Lg$-wave is composed of multiply reflected $S$-waves. Its energy falls on the $S$-wave slowness circle, and part of the energy stays on the right of the upper mantle $S$-slowness and forms trapped phases. The $Rg$-wave is also a trapped mode with horizontal slowness larger than the $S$-slowness. Shown at the bottom of Figure 3 is the energy isolated by the slowness analysis. The horizontal axis is time or, equivalently, the inverse of the group velocity. The vertical coordinate is the horizontal slowness or, equivalently, the apparent horizontal phase velocity. The solid circles are energy measured in the slowness domain with their sizes being proportional to the amount of energy. The horizontal dashed line marks the upper mantle $S$-slowness that divides the trapped and leaky energy.

Testing the Validity of the Method

Before using the FDSA method to investigate different $Lg$-wave excitation mechanisms, we will check the validity of the method by comparing the energy partitioning predicted at short distances using slowness analysis with the energy measured from long distance surface receivers using a conventional method. For all numerical examples calculated in this paper, unless otherwise indicated, we use the horizontally layered Eastern Kazakh (EK) model (Priestley et al., 1988) as the background and modify it by adding random velocity fluctuations at different locations. The EK model (Table 1) has a high velocity top layer with $v_p = 5.05$ km/s and an upper mantle $S$-wave velocity $v_{s-mantle} = 4.57$ km/s. For this layered model, $P$-waves radiated from an explosive source cannot be effectively converted to trapped $S$-waves. The random velocity perturbations added have an exponential power spectrum with horizontal and vertical correlation lengths equal to 0.5 km. We call the part of the model with velocity perturbations the “random patch”. Within the patch, both $P$- and $S$-wave velocities have the same relative RMS perturbation and the relative density perturbation is 50% of the relative velocity perturbation. To eliminate the possible effect of sharp edges of these random patches, a space window with smoothed edges is applied to the patch. The sizes, locations and the RMS perturbations of these patches will vary depending on the purpose of the investigation.

We first test models with scattering at different depths by varying the location of random patches within the EK model. These patches are 2.5 km in vertical extent, extend horizontally
from 5 to 25 km and are located at different depths. Within the patches, the $P$- and $S$-wave velocities have 10% RMS fluctuations. An explosion source is located at depth 0.5 km. Figure 4 shows the slowness analysis results at a distance of 180 km in the crustal waveguide. Columns (a) to (c) are for frequencies 0.3-1.5 Hz, 0.8-2.0 Hz and 2.0-5.0 Hz, respectively. Each panel is similar to the example shown in Figure 3. The top row is for the background velocity model and the other rows are calculated for models with near-source random velocity perturbations. The depths of the random patches are labeled in the figure. We first focus on the low-frequency results in column (a). For the background velocity model, there is a strong $Rg$ phase but very little energy within the $Lg$ group velocity window of 3.0-3.5 km/s, which is typical for a crustal model with a high velocity top layer. For models with random velocity patches, compared with the background model, considerable energy is transferred to the $Lg$-wave through scattering, while the $Rg$-wave is weakened. The tendency is that the shallower the random patch, the more energy is scattered into $Lg$. For the EK model, which has a thick high speed crust, the distance for $Pn$ to cross $Pg$ at the free surface is about 200 km. At 180 km distance, there is no prominent $Pn$ energy shown in this analysis.

For high frequency results in column (c), from the top panel we see strong $Pg$ energy. There is also energy within the $Lg$ group velocity window. However, this energy has a similar horizontal slowness to the $Pg$-wave, implying that it is generated from $Pg$ through $P$-to-$P$ and $P$-to-$S$ reflections on the free surface and interfaces such as the Moho discontinuity. Although the energy exists at short distances, the steep incident angle causes energy to gradually leak to the upper mantle through multiple reflections and it cannot form trapped regional phases. In the other panels, after adding random velocity perturbations in the near-source region, part of the $P$ energy transfers to the $Lg$ wave, i.e., energy falls into the proper group velocity and slowness windows. In general, the scattering affects $Rg$-to-$Lg$ energy at low frequencies and affects $P$-to-$Lg$ coupling at high frequencies. Summing up energy through the waveguide cross section and within the proper time (group velocity) windows allows the energy for the related wave types to be obtained. Specifically, summing up the energy located within the $Lg$ group velocity window and above the upper mantle $S$-slowness allows $Lg$ energy at long distances to be predicted. It is necessary to confirm this calculation.

Figure 5 compares the waveguide energy obtained using different methods. The left panel shows the square root energy passing through the waveguide cross section at 180 km calculated from the slowness analysis shown in Figure 4 and the right panel shows the RMS amplitudes at 450 km calculated from surface measurements using a conventional processing technique. The same velocity model used in Figure 4 is adopted here and a full-scale finite-difference simulation is computed to provide data at the two distances. For simplicity, we label the predicted trapped energy as “$Lg$” energy. The relative energy changes of $Pg$-, $Lg$- and $Rg$-waves, as functions of the depth of the random velocity patches are shown in Figure 5. The dashed lines indicate the energy level for the layered background velocity model. Short bars indicate the energy measured for each patch position. The calculations for different phases demonstrate that the relative change of energy obtained in the waveguide slowness analysis corresponds closely to that obtained on the free surface, even for dramatic changes such as the strong scattering of $Rg$. The results confirm that the slowness analysis within the waveguide correctly predicts the surface regional observations at greater distance.
To examine the energy flux measurements obtained at even shorter distances, we compare the slowness analysis measurements at 50 km with those at 100 km. For these two distances, Figure 6 gives examples of horizontal slowness analyses, which show quite different features in the slowness-depth domain due to evolution of the wavefield with range. Figure 7 compares the corresponding waveguide energy measured at these two distances for different frequency bands. The vertical and horizontal coordinates are for measurements at the two distances and dots represent results for different source depths and velocity models. Although a wide range of near-source structures and source depths are used to generate these measurements, the results show a general linear relationship for all frequency bands. This further verifies that we can use a small model to investigate the near-source energy partitioning robustly.

Investigating Regional Phase Excitation

\( P-pS-Lg \) and \( P-Lg \) Conversion

We first investigate the \( P-pS-Lg \) coupling caused by the near-source lateral velocity variations and assess its effect on the explosion \( S \)-wave energy budget. In a horizontally layered model with overburden \( P \)-wave velocity larger than the upper mantle \( S \)-wave velocity, the free surface reflected \( pS \)-wave has a steep incidence angle and cannot be trapped in the crustal waveguide to form \( Lg \). In this case, the energy transfer through \( P-pS-Lg \) coupling is almost zero. Although it is generally agreed that the existence of near-source lateral velocity variation can increase the \( P \)-to-\( Lg \) energy exchange, the detailed mechanism underlying this process is still not fully understood. The lateral velocity variation can affect the \( P-pS \) conversion through different mechanisms. First, while entering or leaving the free surface, both \( P \) and \( pS \) waves can hit the heterogeneities and the wavefront can be distorted due to scattering. Depending on the scale length of the heterogeneities and the wavelengths of incoming waves, the scattered waves may carry energy which satisfies the slowness criterion \( p_x \geq p_{S\text{-mantle}} \). Another mechanism involves both heterogeneities and the free surface. When a high speed \( P \)-wave grazes along the free surface, each point at the surface can be seen as a secondary \( P \)- and \( S \)-wave source. The smoothness of the free surface and wavefront guarantees the seamless generation of a \( pS \) wavefront through constructive interference. If the system is disturbed by either a distorted incoming wavefront (e.g., waves already passing through a heterogeneous area) or an imperfect free surface (the existence of near-surface heterogeneities or uneven topography), the perfect constructive interference cannot be maintained and the residual wavefield generated by the uncancelled free-surface scattered waves will appear as spherical waves radiated from the free surface. Their broad slowness range makes this an effective mechanism for generating trapped phases. The free surface, which forces the \( P \)-wave to \( S \)-wave conversion, plays an important role in this process. The slowness and frequency content of scattered waves will depend on the characteristic scales of the wavefield and velocity model.

Figure 8 compares the simulated \( P-pS-Lg \) coupling in models with and without near surface lateral velocity variations. Figure 8a is for the EK-model. A shallow explosion source located at depth 0.5 km generates \( P \), \( pS \) and \( Rg \) waves. Two-dimensional slowness analysis is conducted for selected phases in the wavefield and the results are shown together with the wavefield snapshot. The synthetic seismograms were bandpass filtered between 2.0 and 6.0 Hz before the slowness analysis. As can be seen from the result, the \( P \)-wave leads the wavefield and
has a distinct slowness. Reverberations within the uppermost crust causes multiply parallel $pS$ wavefronts with their horizontal slowness approximately equal to the overburden $P$-slowness. The $pS$ energy stays to the left of the upper-mantle $S$-slowness and there is no energy transferred from $P$ to $Lg$. In Figure 8b, a shallow random velocity patch is added to the EK-model to test the effect of near surface scattering. The random patch has a 5% RMS velocity fluctuation and is located at distances 5 to 15 km and depths 0 to 2.5 km (shown in the snapshot as a shaded area). The slowness analyses are conducted for $P$, $pS$ and $pS$-coda. Although $P$- and early $pS$-waves are barely affected, the $pS$-coda clearly contains some scattered energy with horizontal slowness to the right of the upper-mantle $S$-slowness.

To investigate further the scattering from a shallow random patch, horizontal slowness analysis is conducted at a distance of 20 km and depth 0-12.5 km. Figure 9 shows the energy distribution in the slowness-depth domain with arrival times and major phases labeled in the frames. The two prominent down-going phases are the $P$-wave and the free-surface reflected $pS$-wave. The $Rg$ energy enters the array at 6.0 s with its depth close to the surface and slowness beyond the $S$-slowness. Shown in row (a) is the result using the EK-model. Due to the nearly horizontal propagation of the $P$-wave at the free surface, the $pS$-wave energy has a horizontal slowness which is similar to the overburden $P$-slowness and the energy falls to the left of the upper mantle $S$-slowness. In rows (b) and (c), shallow random velocity patches with RMS velocity fluctuations 3% and 5% are added to the EK-model at distances 5 to 15 km and depths 0 to 2.5 km (the same position as in Figure 8b). As can be seen in the figure, scattering causes part of the $pS$ energy to cross the upper mantle $S$-slowness and build up in the dashed rectangles. At shallow depth, the slowness of the scattered energy approaches the $S$-slowness. With an increase in depth, the slowness of this energy gradually merges with the $P$-slowness. The slowness behavior is consistent with scattering of waves at shallow depth, increasing as shallow heterogeneity increases.

Figure 10 investigates scattering taking place at deeper depths. The configuration of the source and model is similar to that used in Figure 8b, except the random patch with 3% RMS velocity fluctuation is added to the EK-model between distances 5 and 15 km and depths 2.5 and 10 km (shown in the snapshot as a shaded area). The 2D slowness analysis is conducted for selected phases in the wavefield and the results are presented in the figure. After passing through the random region, there is $P$-coda composed of scattered $P$- and $S$-waves generated from the direct $P$-wave. Although the early part of the $pS$-wave does not contribute to the trapped energy, its later part contains energy located to the right of the upper-mantle $S$-slowness which therefore will contribute to the trapped regional phases. Figure 11 gives the energy distribution in the slowness-depth domain for different models where row (a) is for the EK-model and rows (b) and (c) are for the EK-model with 3% and 5% RMS fluctuations in a random patch like that used in Figure 10. The slowness analysis is conducted at a distance of 20 km and for depths between 0 and 12.5 km. As expected, with the EK-model no energy is seen beyond the upper mantle $S$-slowness, but after the lateral velocity variations are introduced, energy starts to build up to the right of the upper-mantle $S$-slowness. Two types of scattered energy can be found in the slowness-depth domain: weak but widely distributed $S$ energy (indicated by the dashed ellipses) and scattered energy linked to the $pS$-wave (indicated by the dashed rectangles). Both types of energy satisfy the criterion $p_s \geq p_{s\text{-mantle}}$ and will contribute to the $Lg$-wave. The widely spread scattered $S$-wave is generated by the $P$-$Lg$ coupling through volumetric scattering. The scattering
process redistributes the angle spectrum of the original incident waves. Since there is no lateral heterogeneity at the top of the crust, scattered $pS$-waves are generated by the interaction between distorted incident $P$-waves and a smooth free surface. Comparing Figure 11 to Figure 9, the scattered $pS$-wave generated from a deeper random patch appears later than that from a shallow random patch, which implies in the former case the wave must first be distorted before it can generate the scattered $pS$-wave. Both volumetric scattering and scattering near the free-surface affect the general $P$-to-$Lg$ conversion.

Contributions from the $S^*$-Wave

For shallow explosion sources, the $S^*$-wave may become a significant contributor to $Lg$ (Gutowski, et al., 1984; Xie and Lay, 1994; Vogfjord, 1997). The amplitude of $S^*$ can be large if the source depth is within a fraction of a wavelength from an interface. This makes its excitation highly dependent on the source depth and frequency. Figure 12 shows snapshots for explosion sources at 0.5 km and 3.0 km, respectively. The result clearly shows that a shallow source generates larger $S^*$- and $Rg$-waves. We investigate the contribution of the $S^*$-wave within the EK model. Figure 13 shows horizontal slowness analyses at a distance 35 km and for depths between 0 and 30 km. The time window is chosen between 11 to 13 s after the direct $P$-wave passes the receiver array. The synthetic seismograms are bandpass filtered between 1.0 - 5.0 Hz. The four rows from top to the bottom correspond to source depths 0.25 km, 0.5 km, 1.0 km, and 2.0 km, respectively. The major arrival is the down-going free surface reflected $pS$-wave, which has a horizontal slowness similar to the overburden $P$-slowness. As can be expected from a horizontally layered model, the $pS$ energy stays to the left of the upper mantle $S$-slowness and has no contribution to the trapped regional phases. For shallow sources, the $Rg$-wave enters the array at about 12 s and its energy concentrates between 0 to 3 km, as can be seen on the upper right corners in the slowness-depth domain. For source depth of 2.0 km, the $Rg$-wave is very weak. The $S^*$-wave enters the array from a shallow depth and gradually merges with the $pS$-wave (also refer to Figure 12). The $S^*$-wave is strong for shallow sources and its amplitude decreases with increasing source depth. Very little $S^*$ energy can be observed for source depths below 2 km. In the joint domains, the $S^*$-energy can be isolated and quantified even within a complicated wavefield, which is very difficult using remote surface synthetics. The dashed rectangles are the time-slowness-depth window used to locate the $S^*$ energy. The time window is chosen after the arrival of direct $P$-wave, and a variable depth range is chosen to avoid contamination from the $Rg$-wave. The slowness range is chosen between 0.23 and 0.34 s/km. The energy from successive windows can be summed together to give the contribution of $S^*$ to the trapped regional phases.

The Frequency Dependent $Lg$ Excitation Function

The frequency dependence of $Lg$-wave excitation is rooted in the underlying physical processes and is usually controlled by different characteristic scales. For example, the excitation of $Lg$ by $S^*$, $Rg$-to-$S$ scattering and spall are all highly source depth dependent. The excitation spectra from individual or joint mechanisms contributing to regional phases depict the frequency dependence of these processes. Frequency dependent $P/S$ ratios will depend on the excitation functions of multiple phases. We use FDSA to quantify $Lg$-wave excitation spectra from $S^*$- and $pS$-waves. Figure 14 gives the $S^*$-to-$Lg$ excitation spectra as functions of source depth and
frequency. The slowness analysis and multi-domain window used to pick the trapped energy are similar to that shown in Figure 13. A series of bandpass filters is used to give responses at different frequencies. The vertical coordinate is the normalized relative energy $(E/E_0)^{1/2}$. The results clearly show that the $S^\ast$-to-$L_g$ excitation is generally enhanced for lower frequency and shallow source depth. The major contribution comes from sources located above 1 km. For sources at depths below 1 km, only low frequency energy below 1 Hz has an essential contribution to $L_g$-wave excitation. However, the responses are also model dependent. For a model with a homogeneous crust (Figure 14a), the distribution has simple monotonic tendencies in both source depth and frequency. For the EK model (Figure 14b), the excitation spectrum has a maximum at depth 1 km and a more complicated frequency dependence. This may reflect the fact that the EK model has an interface at 1 km depth. The $S^\ast$-waves generated or reflected from multiple interfaces may interfere with each other and give a complicated frequency spectrum.

To investigate the combined effect for $S^\ast$-wave and near-source scattering, we add shallow random velocity patches to the EK model. The random patch extends between distances of 5 to 25 km and depths of 0 to 2.5 km. Shown in Figures 15a and b are excitation spectra for random patches with RMS velocity fluctuations of 3% and 5%, respectively. The most prominent feature is the build up of high frequency energy. The scattered energy increases with increase of the RMS velocity fluctuations. Figures 15c and d isolate the scattered energy by subtracting the excitation spectrum of the EK-model from the spectra for models with random velocity patches. Two types of energy can be identified within the frequency-depth domain. The high-frequency energy results from $P$-$pS$-$L_g$ and $P$-to-$L_g$ scattering. This energy is especially important for deeper sources to generate $L_g$-waves, since a deeper source generates little trapped energy in a horizontally layered model. The low-frequency energy concentrated at shallow source depths comes from $R_g$-to-$L_g$ scattering. Figure 16 gives the excitation spectra for the EK-model with deeper random patches. The random patch is located between distances 5 to 25 km and depths 7.5 to 10.0 km. Figures 16a and b give excitation spectra for random patches with RMS velocity fluctuations of 3% and 5%, respectively. Figures 16c and d give the isolated scattered energy. The scattered energy from the deeper random patches has little low-frequency content, which supports the interpretation that the low-frequency energy comes from the $R_g$-to-$L_g$ scattering. The frequency dependent excitation spectra establish the relationship between the observations and the characteristics of sources and near-source structures. They provide the basis for evaluating the dominant mechanisms for $L_g$-wave excitation.

Discussion and Conclusions

A finite-difference modeling plus slowness analysis (FDSA) method has been developed to investigate near-source energy partitioning and $L_g$-wave excitation of explosive sources. The method has two major advantages. First, it allows us to study the near-source processes in multiple domains including space, time, slowness and frequency. This provides an opportunity to isolate different mechanisms within the complex near-source environment. Second, the FDSA method can be applied at a close range, well before the $L_g$-wave is actually formed. It provides us with uncontaminated near-source information by calculating a relatively small velocity model with very fine near-source structures. Since this is a very efficient method, we can use it to investigate a broad frequency band and to test a large number of source-model parameters. As
examples, we investigated the contributions of $P$-$pS$-$Lg$ conversion and $S^*$-wave excitation to the generation of trapped regional phase $Lg$ using models with near-source random velocity fluctuations. The contribution of $S^*$-to-$Lg$ is concentrated at low frequencies and occurs for very shallow source depths. The contribution of $P$-$pS$-$Lg$ coupling to $Lg$ in the presence of near-source small-scale random heterogeneities is concentrated at high-frequencies. The excitation spectra of these mechanisms were calculated.

There are other potential $Lg$ excitation mechanisms (e.g., the $Rg$-to-$Lg$ scattering, spall, and tectonic release), which have not been systematically investigated in this study. The ability to handle broad frequency band makes the FDSA an ideal tool to investigate excitation spectra and $P/S$ type spectrum ratios for different mechanisms and source-model parameters. These spectra and spectrum ratios form the basis of most regional identification discriminants. The numerical modeling can establish a link between the physical model and observable frequency dependent features and place regional event identification on a sound physical basis. Scattering from uneven topographies should be included in future studies. Although most $Lg$ observations are taken from the vertical component of seismograms, the tangential component often has as much energy as the vertical component. Since 2D geometry decouples the $P$-$SV$ problem from the $SH$ problem, it does not provide any information on the coupling between the source and the $SH$ component. The future development of a 3D FDSA method is thus motivated.

Acknowledgments

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References


Lg-wave excitation


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Figure 1. Slowness domain display with (a) energy distribution in 2D slowness domain, and (b) energy distribution in mixed horizontal slowness and depth domain. Vertical lines indicate the upper-mantle S-slowness. Circles and dashed lines denote crustal P- and S-wave slowness. Energy falling to the right of upper-mantle S-slowness can be trapped in the waveguide and contribute to $Lg$.

Figure 2. Configuration for using the FDSA method to investigate near-source processes.
Figure 3. Example of slowness analysis at 180 km distance.

Figure 4. Slowness analysis calculated for the EK model and EK model with random patches at different depths. Column (a) is for frequency band 0.3-1.5 Hz, (b) is for 0.8-2.0 Hz and (c) is for 2.0-5.0 Hz. The top row is for background velocity model and the lower rows are for models with random patches. The depths of random patches are labeled in the panels. Details see the text.
Figure 5. Comparison between waveguide energy flux at 180 km (left column) and the wave energy on the surface at 450 km (right column) for $P_g$, $L_g$ and $R_g$ windows. The frequency range is 0.3-1.2 Hz. Shown in each panel is relative energy versus the depth of random patches. Dashed lines indicate the energy level for background model and short bars indicate the energy changes due to the near-source scattering. If the FDSA method works, the two columns should be consistent.
Figure 6. Examples showing the slowness analyses at distances (a) 50 km and (b) 100 km. The energy that can be trapped in the crustal waveguide is indicated in the figure.

Figure 7. Comparison between trapped waveguide energy measured at 50 km (horizontal coordinate) and 100 km (vertical coordinate) for frequency bands 0.5-1.5 Hz, 1.0-3.0 Hz and 2.0-4.0 Hz. Different dots are results from different velocity models and source depths. The results show a general linear relationship for all frequencies.
Figure 8. $P$-$pS$-$Lg$ conversion due to shallow scattering. (a) is for the EK-model and (b) is for the EK-model with a shallow random patch. The random patch has a 5% RMS velocity fluctuation and is shown as a shaded area between horizontal distances 5 to 15 km and depths 0 to 2.5 km. The snapshots and results of slowness analyses of $P$, $pS$ and $pS$-coda are shown in the figure. Details are given in the text.
Figure 9. Horizontal slowness analyses for investigating the $P$-$pS$-$Lg$ coupling with (a) the EK-model, (b) the EK-model plus a 3% shallow random patch and (c) the EK-model plus a 5% random patch. The slowness analyses are conducted at distance 20 km and for depth range 0 to 12.5 km. The configuration of the source and model is the same as that used in Figure 8. Major phases are labeled in the figure and energy circled by dashed rectangles is scattered $pS$-wave.
Figure 10. The $P$-$pS$-$Lg$ conversion due to a deeper random patch. The random patch has a 3% RMS velocity fluctuation and is shown as a shaded region between distances 5 and 15 km and depths 2.5 and 10 km. The slowness analysis for $P$, $P$-coda, $pS$ and $pS$-coda are also shown in the figure.
Figure 11. Energy distribution in depth and horizontal slowness domain for (a) the EK-model, (b) EK-model plus a 3% random patch and (c) EK-model plus a 5% random patch. The configuration of the source and model is same as that used in Figure 10. Energy circled by dashed rectangles is $P-pS-Lg$ scattering and energy circled by dashed ellipses is $P-Lg$ scattering.
Figure 12. Wavefield snapshots for explosion sources at depths (a) 0.5 km and (b) 3.0 km. Note that a shallower explosion is a more efficient source for generating $S^*$ and $Rg$-waves.

Figure 13. Slowness analysis for investigating $S^*$-to-$Lg$ conversion. Different rows are for different source depths. Dashed rectangles indicate the time-space-slowness windows used to pick the $S^*$ energy.
Figure 14. Normalized $L_g$ excitation spectra for sources in different velocity models and at different depths with (a) a model with a homogeneous crust and (b) the EK-model.
Figure 15. Normalized $Lg$ excitation spectra for sources in the EK-model with shallow random patches, for (a) the EK-model with a 3% shallow random patch, (b) the EK-model with a 5% shallow random patch, (c) and (d) the isolated scattered energy in (a) and (b) respectively due to the random patches found by removing the energy for the layered models. Note different vertical scales are used for scattered energy.
Figure 16. Normalized $Lg$ excitation spectra for sources in the EK-model with deep random patches, for (a) the EK-model with a 3% deep random patch, (b) EK-model with a 5% deep random patch, (c) and (d) the isolated scattered energy in (a) and (b) due to the random patches.
P-SV wavefield connection technique
for regional wave propagation simulation

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ABSTRACT
A boundary element (BE) method is developed to calculate the two-dimensional P-SV elastic response for crustal waveguides with irregular topographic features. In order to simulate long-range propagation of regional waves, a connection technique is proposed to avoid large matrix inversions which become formidable for long range, high-frequency problems. By using this technique, a long crustal waveguide can be divided into relatively shorter sections, and the BE method can be used section by section to model the effects of rough topography on wave propagation at extended regional distances. The validity of the technique is tested by comparison with a direct calculation. Numerical simulations with this scheme show that rough topography can scatter the $P$ and Rayleigh waves and attenuate the energy propagating in the waveguide. This method can be used in computing the site effects on sites such as canyons, mountains, and valleys. The connection technique expands this method to deal with large earth models with irregular topography.

Key words: boundary element method, regional wave, topographic effects, scattering, wavefield connection technique

INTRODUCTION
Regional phases have long been recognized as an important issue in the study of large-scale crustal structures, small-scale crustal heterogeneities, seismic sources, and analysis of underground explosions and earthquakes. To improve path corrections as well as verify empirical observations, numerical simulations are needed to estimate the path effects on regional wave propagation. A number of regional numerical modeling methods have been developed to model $Lg$ propagation behavior and path effects, depending on the complexity of the crust heterogeneity in the region considered. Finite-difference (FD) and finite-element (FE) methods are universal numerical techniques for simulation in general heterogeneous media. Because of the computational intensity, these full-waveform methods are often limited to the case of low frequencies for $Lg$ simulation at regional distances. To reduce computation time, some alternative and flexible approaches, for example, ray diagrams (e.g., Bostock and Kennett, 1990; Keers et al., 1996) and dynamic-ray-tracing method (Gibson and Campillo, 1994), have been used to obtain intuitive understanding of path effects on $Lg$ propagation. For long range $Lg$ wave propagation, Wu et al. (1996, 2000) introduced a half-space GSP (Generalized Screen Propagator) for modeling the main characteristics of $Lg$ (2-D SH case) in heterogeneous crustal waveguides. The method was extended to include the case of irregular surface topographies (Wu and Wu, 2001)

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Based on observations and numerical experiments, large-scale crustal structures with variations in free surface and Moho topography control the principal characteristics of $L_g$ propagation. In contrast to the FD and FE methods, the boundary integral methods are more suitable for modeling complex reflection and transmission effects across large-scale crustal structures with rugged topography. For instance, the boundary element (BE) method provides a geometrically accurate description of irregular interfaces. Because the BE method is formulated in terms of integrals along boundaries, the traction-free condition for a rugged free surface is easily and naturally treated. However, for regional waveguides up to thousands of kilometers, the BE method in its original form leads to very large matrices to be inverted and its implementation is prohibitively expensive. In a previous paper (Fu and Wu, 2001), a $SH$ wavefield connection technique was developed by which the BE method can be used section by section for an event at far regional distance up to several thousand kilometers; the method takes the output of the previous section as the input of the next section in order to complete the entire crustal waveguide computation. Full-wave BE modeling is implemented within each section. The wavefield connection technique is used to couple the fields calculated in two adjoining sections. The division of sections is based on the complexity of crustal structures with a criterion of minimizing the possible multiple backscattering between sections. This approach for long regional waveguides leads to significant computational savings in time and memory compared with the whole waveguide Boundary Element method. The section-by-section approach also leads to a hybrid modeling scheme of BE and GSP. The BE method is implemented in the frequency domain and has a kernel function compatible with the GSP method. In the hybrid scheme, the time-consuming BE method can be used to handle the sections with complicated boundary structures and severe surface topographies. Subsequently the output will be used as the input to the GSP method for modeling sections with a large volume of moderately heterogeneous media and mild topographies. The hybrid method has been applied to two crustal waveguide models from the Tibet region, one with $L_g$ blockage and another without blockage (Fu and Wu, 2001).

The object of this paper is to develop a $P$-$SV$ wavefield connection technique for simulating elastic wave propagation in regional crustal waveguides. We first introduce an integral equation representation for the crustal waveguide problems. We then briefly describe the principle of the elastic BE method and validate the computation programs using previously published results. We develop the $P$-$SV$ wavefield connection technique and test it using numerical experiments. Finally we apply the section-by-section approach to $L_g$ propagation simulations in long regional waveguides.

**BOUNDARY INTEGRAL EQUATION FOR CRUSTAL WAVEGUIDE**

Consider 2-D steady-state elastic wave propagation in a simplified crustal waveguide $\Omega_1$ bounded by a rough free surface $\Gamma_1$ and an irregular Moho interface $\Gamma_2$. Figure 1 depicts the geometry of the problem. The waveguide medium is isotropic and homogeneous, described by the Lame constants ($\lambda_1$ and $\mu_1$) and density ($\rho_1$). The displacement vector $u(r)$ at a location $r(x, z)$ satisfies the following elastic wave equation

$$
\mu \nabla^2 u(r) + (\lambda + \mu) \nabla \cdot u(r) + \rho \omega^2 u(r) = -f(r, \omega)
$$

where $f(r, \omega)$ is the body force occupying a region $\Omega$. $u(r)$ also satisfies the traction-free boundary condition on $\Gamma_1$ and the continuity condition of displacement and traction across the
Moho. The medium in the mantle $\Omega_2$ is described by $\lambda_2$, $\mu_2$, and $\rho_2$. We add two artificial boundaries $\Gamma_\infty$ at the two truncated edges of the waveguide to form a closed solution domain $\Gamma = \Gamma_1 + \Gamma_2 + \Gamma_\infty$. Based on the representation theorem (e.g., Aki and Richards, 1980), equation (1) can be transformed into the following integral formulation:

$$C(r)u(r) + \int_{\Gamma_0} [u(r')\Sigma(r,r') - t(r')G(r,r')]d\Gamma(r') = \int_{\Omega} f(r',\omega)G(r,r')d\Omega(r')$$

(2)

where $t(r)$ is the traction vector, the coefficient $C(r)$ generally depends on the local geometry at $r$, and $G(r,r')$ and $\Sigma(r,r')$ are the fundamental solutions (Green’s tensors) for displacements and tractions, respectively. $C(r), \Sigma(r,r')$ and $G(r,r')$ are $2 \times 2$ matrices for 2-D problems. In this equation, $r$ is the position of observation points and $r'$ is the position of scattering points. For simplicity, the source distribution consists of a point source at $r_0$ located inside $\Omega_1$. The source integral over $\Omega_1$ in the right side of equation Error! Reference source not found. can be reduced to

$$\int_{\Omega_1} G(r,r')f(r',\omega)d\Omega' = G(r,r_0)f(\omega)$$

(3)

where $f(\omega)$ is the source spectrum vector.

The displacement Green’s tensor $G(r,r')$ satisfies:

$$([\lambda + \mu]\nabla \cdot + \mu \nabla^2)G(r,r') + \rho \omega^2 G(r,r') = -\delta(r - r')$$

(4)

with the solution given by Morse and Feshbach (1953). The traction Green’s tensor $\Sigma(r,r')$ can be derived from $G(r,r')$ using Hooke’s law,

$$\Sigma(r,r') = \lambda I (\nabla \cdot G(r,r')) + \mu (\nabla G(r,r') + G(r,r')\nabla),$$

(5)

where $I$ is the unit dyadic and $G(r,r')\nabla$ is the transpose of $\nabla G(r,r')$ with respect to the corresponding coordinates, see Wu (1994, Appendix). For analytic expressions of $G$ and $\Sigma$ in an isotropic homogeneous elastic medium, these Green’s tensors for 2-D problems can be expressed as (Fu, 1996)

$$G_y = \frac{i}{4\mu} \left[ \psi \delta_{ij} - \phi \gamma_{ij} \right]$$

$$\Sigma_y = \frac{i}{4} \left[ \left( \frac{\partial \psi}{\partial r} - \phi \frac{1}{r} \right) \delta_{ij} \gamma n_i + \phi \gamma n_i \right] - \frac{2\phi}{r} \gamma_{ij} n_i - 2 \phi \gamma_{ij} n_i - \frac{2 \phi}{r} \gamma_{ij} n_i + \left( \frac{\lambda}{\mu} \left[ \frac{\partial \psi}{\partial r} \right] - \frac{\partial \phi}{\partial r} \right) \gamma_{ij}$$

where $i, j = 1, 2$, $n$ is the outward normal of the boundary $\Gamma$, and

$$\psi = \left[ H_0^{(1)}(k_1 r) - \frac{1}{k_1 r} \left( H_1^{(1)}(k_1 r) - \frac{\beta}{\alpha} H_1^{(1)}(k_p r) \right) \right]$$

$$\phi = \left[ \frac{\beta^2}{\alpha^2} H_2^{(1)}(k_p r) - H_2^{(1)}(k_1 r) \right]$$

with $k_1 = \omega / \beta$, $k_p = \omega / \alpha$, $r = |r - r'|$ and $\gamma_j = (r_j - r_j) / r$

The boundary integral representation for wave propagation naturally satisfies the Sommerfeld radiation boundary conditions that are imposed on the far field behavior at infinity. No waves come back to $\Omega_1$ through $\Gamma_\infty$, that is, the following integral on $\Gamma_\infty$ for the interior problem vanishes:
For numerical calculations, truncating the waveguide is necessary and an infinite boundary element absorbing boundary technique (Fu and Wu, 2000) has to be applied to the elements at the truncating points on $\Gamma_1$ and $\Gamma_2$. Considering equations Error! Reference source not found. and Error! Reference source not found., and applying the traction-free condition to the free surface $\Gamma_1$, equation (2) for the interior problem in the crust (Region $\Omega_1$) is simplified to

$$C(r)u(r) + \int_{\Gamma_1} \left[ \Sigma(r, r')u(r') - G(r, r')t(r') \right]dr' = G(r, r_j)f(\omega) \quad (7)$$

Equation Error! Reference source not found. is a starting point for numerical implementation for wave propagation simulation. In order to solve $u(r)$ and $t(r)$ on $\Gamma_2$ we must build the corresponding boundary integral equation in $\Omega_2$. The following integral formulation can be established for $r \in \Omega_2$ bounded by a closed surface $\Gamma = \Gamma_2 + \Gamma_\infty$:

$$C(r)u(r) + \int_{\Gamma_2} \left[ \Sigma(r, r')u(r') - G(r, r')t(r') \right]dr' = 0 \quad (8)$$

Similarly, the integration over the transparent artificial boundary $\Gamma_\infty$ vanishes for the interior problem, simplifying equation (8) as

$$C(r)u(r) + \int_{\Gamma_2} \left[ \Sigma(r, r')u(r') - G(r, r')t(r') \right]dr' = 0 \quad (9)$$

Equations (8) and Error! Reference source not found. provide a description of the field through the crustal waveguide, making possible the simultaneous evaluation of the unknowns ($u(r)$ on $\Gamma_1$ and $u(r)$ and $t(r)$ on $\Gamma_2$) by using the continuity of displacement and traction across $\Gamma_2$.

**BOUNDARY ELEMENT METHOD FOR ELASTIC WAVE SIMULATION**

The boundary integral representations described above can be used to calculate the field at any point inside $\Omega_1$ once $u(r)$ and $t(r)$ are known on the boundaries. We use the BE method to solve the values of $u(r)$ and $t(r)$ on the boundaries. We discretize $\Gamma_1$ into $L_1$ elements and $N_1$ nodes, and $\Gamma_2$ into $L_2$ elements and $N_2$ nodes. The total node number is $N$. By using the linear interpolation shape functions $\phi(\xi)$ in an element between the nodes $I_1$ and $I_2$, the variables $u(r)$ and $t(r)$ are approximated by the linear combination of their node values over the element, for example, (see Figure 1)

$$u(\xi) = \sum_{l=I_1}^{I_2} u(r_l)\phi_l(\xi), \quad (10)$$

where $\xi$ and $l$ denote the local coordinate and local node index of an element. Using the following Kronecker delta function notation relating the local node code $l$ of an element to the global node code $j$

$$\delta_{lj} = \begin{cases} 0, & l \neq j \\ 1, & l = j \end{cases}, \quad (11)$$

we have
\[
\mathbf{u}(\xi) = \sum_{j=1}^{N} \sum_{l=1}^{L} \mathbf{u}(r_j) \phi_j(\xi) \delta_j,
\]
(12)

Letting \( \mathbf{C}_i = \mathbf{C}(r_i) \) and \( \mathbf{u}_i = \mathbf{u}(r_i) \), equation (7) for \( i = 1 \) to \( N \) is discretized into
\[
\mathbf{C}_i \mathbf{u}_i + \sum_{j=1}^{N} \left\{ \sum_{e=1}^{I} \sum_{l=1}^{L} \left[ \int_{\Gamma_{r,e}} \Sigma(r,e,r'(\xi)) \phi_j(\xi) dr'(\xi) \right] \delta_j \mathbf{u}_j + \sum_{e=1}^{I} \sum_{l=1}^{L} \left[ \int_{\Gamma_{r,e}} \Sigma(r,e,r'(\xi)) \phi_j(\xi) dr'(\xi) \right] \delta_j \mathbf{t}_j \right\} = \mathbf{G}(r_i,r_n) f(\omega)
\]
(1)

which can be further rewritten as
\[
\sum_{j=1}^{N} \left[ \mathbf{h}_{ij}^{(1)} \mathbf{u}_j^{(1)} + \mathbf{h}_{ij}^{(2)} \mathbf{u}_j^{(2)} - \mathbf{g}_{ij}^{(2)} \mathbf{t}_j^{(2)} \right] = \mathbf{G}(r_i,r_n) f(\omega),
\]
(14)

with
\[
\mathbf{h}_{ij}^{(1)} = \sum_{e=1}^{I} \sum_{l=1}^{L} \left[ \int_{\Gamma_{r,e}} \Sigma(r,e,r'(\xi)) \phi_j(\xi) dr'(\xi) \right] \delta_j + \mathbf{C}_j^{(1)} \delta_j.
\]
(15)
\[
\mathbf{h}_{ij}^{(2)} = \sum_{e=1}^{I} \sum_{l=1}^{L} \left[ \int_{\Gamma_{r,e}} \Sigma(r,e,r'(\xi)) \phi_j(\xi) dr'(\xi) \right] \delta_j + \mathbf{C}_j^{(2)} \delta_j.
\]
(2)
\[
\mathbf{g}_{ij}^{(2)} = \sum_{e=1}^{I} \sum_{l=1}^{L} \left[ \int_{\Gamma_{r,e}} \mathbf{G}(r,e,r'(\xi)) \phi_j(\xi) dr'(\xi) \right] \delta_j,
\]
(3)

In this expression \( \mathbf{u}_j^{(1)} \) is the displacement vector on the free surface \( \Gamma_1 \) and \( \mathbf{u}_j^{(2)} \) and \( \mathbf{t}_j^{(2)} \) are the displacement and traction vectors on the Moho \( \Gamma_2 \). The coefficient matrices \( \mathbf{h}_{ij}^{(1)} \), \( \mathbf{h}_{ij}^{(2)} \) and \( \mathbf{g}_{ij}^{(2)} \), obtained by numerically integrating the product of the Green’s tensors with interpolation shape functions over elements denotes a concentrated force generated at the \( j \)th scattering point on \( \Gamma \) and applied at the \( i \)th observation point. For \( i = 1 \) to \( N \), equation

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can be further compacted as a matrix equation:
\[
\mathbf{H}^{(1)} \mathbf{u}^{(1)} + \mathbf{H}^{(2)} \mathbf{u}^{(2)} - \mathbf{G}^{(2)} \mathbf{t}^{(2)} = \mathbf{F},
\]
(4)

where \( \mathbf{F} = \rho \mathbf{G}(r_i,r_n) f(\omega) \). Similarly, equation (9) for \( i = 1 \) to \( N \) can be discretized and compacted as
\[
\tilde{\mathbf{H}}^{(2)} \mathbf{u}^{(2)} - \tilde{\mathbf{g}}^{(2)} \mathbf{t}^{(2)} = 0,
\]
(5)

where the coefficient matrices, \( \tilde{\mathbf{H}}^{(2)} \) and \( \tilde{\mathbf{g}}^{(2)} \), are calculated using the medium properties of the mantle \( \Omega_2 \). A simultaneous system of matrix equations (4) and (5) can be assembled using the continuity of displacement and traction across \( \Gamma_2 \),
\[
\begin{cases}
\mathbf{u}^{(2)} = \tilde{\mathbf{u}}^{(2)} \\
\mathbf{t}^{(2)} = -\tilde{\mathbf{t}}^{(2)}
\end{cases}
\]
(6)

by which the unknowns (\( \mathbf{u}(r) \) on \( \Gamma_1 \) and \( \mathbf{u}(r) \) and \( \mathbf{t}(r) \) on \( \Gamma_2 \)) can be found.

These matrices in equations (18) and (19) are full with complex coefficients which are functions of frequency, material property and geometry. The BE method described above can be directly extended to complex geological structures with multiple regions (Fu, 1996) for exploration-oriented seismic modeling. Alvarez-Rubio et al. (2004) applied the BE method to site
effects assessment of laterally varying layered media. Since a large number of matrix operations are involved and the matrix for each frequency component must be inverted, the BE method is computationally intensive at high frequencies for far regional waveguides. To improve computation speed, a frequency-dependant element dimension technique can be adopted in the program implementation (Fu and Mu, 1994). Bouchon et al. (1995) suggested a threshold criterion approach to make the coefficient matrices sparser by removing very small entries in the BE coefficient matrices. An efficient modification to the BE method can be made using the section-by-section approach with the wavefield connection technique developed in this study.

The computation program of the elastic wave BE method described above is tested by dimensionless frequency responses of a semi-circular canyon of radius $a$. Previously published results (Sánchez-Sesma et al., 1985; Sánchez-Sesma and Campillo, 1991) for this typical topographic structure are used for comparison. The two sharp edges at $x = \pm a$ provide a crucial target to validate various numerical methods. Figure 2 shows the comparison with good agreement in both horizontal and vertical amplitudes between our results (solid lines) and the Sánchez-Sesma and Campillo’s solutions for vertically incident plane $P$ waves and for various normalized frequencies $\eta$ defined as $\eta = \frac{\nu}{l}$, where $l$ is the wavelength. Poisson’s ratio is assumed to be 1/3. We can see some minor departures because of different element approximations used by these two numerical methods. Fewer elements per wavelength will reduce the size of the resultant coefficient matrices. To determine an applicable element number per wavelength, the comparisons for $0^\circ$ incidence and $\eta = 1.0$ are given in Figure 3 for different discretization rates of points per wavelength to discretize the arc of the semicircular canyon. We see that a sampling at three points per wavelength could be used for general applications.

The comparisons above confirm the validity of our formulation and computation code. Later we will use this method to validate the connection technique for a larger model. We now demonstrate the applicability of our program by synthesizing wave propagation through a single-layer crustal waveguide with flat free surface. The homogeneous waveguide shown in Figure 4 is 100 km long and 32 km thick, overlaying a flat mantle half-space. The point source ($P$-wave explosive source) is located on the left boundary at 2.0-km depth. Figure 5 shows the synthetic seismograms calculated in the frequency range of 0-4.5 Hz, with receivers at 1-km spacing along a vertical profile at the distance of 80 km from the source. We see that for elastic wave propagation this simple waveguide with both the flat topography and flat Moho leads to complex superposition of various waves, demonstrating the development of converted waves as well as the formation of guided waves as repetitive reflections at both the free surface and Moho. In terms of propagation paths (see Figure 4), we can identify three major systems of waves generated in the waveguide. As shown in Figure 5, the direct $P$-wave from the source to receivers carries a major part of the source energy in this flat boundary waveguide. The $P$-wave incident upon the free surface and the Moho at oblique angles generates two systems of waves, one set off the free surface and another off the Moho. These two systems of waves and their converted waves bounce back and forth between the free surface and the Moho. Since no scattering mechanism is present in the waveguide, the constructive interference of the repeatedly reflected waves presents a checkerboard-like pattern (Jih, 1996) that adequately explains the formation of crustal guided waves either as multiple reflections or as higher modes.

**P-SV WAVEFIELD CONNECTION TECHNIQUE**
We aim to develop a section-by-section approach for simulating wave propagation in regional waveguides to reduce the computation cost of extremely large matrix operations. The wavefield connection technique couples the fields between two adjacent sections. The connection configuration is illustrated in Figure 6(a). An artificial boundary $\Gamma_{AB}$ is introduced as a wavefield connection boundary to a crustal waveguide consisting of an irregular free surface $\Gamma_1$ and an interface $\Gamma_2$. The waveguide is divided into four subdomains, $\Omega_1$, $\Omega_2$, $\Omega_1'$, and $\Omega_2'$. The left boundaries of the first section $\Omega_1$ and $\Omega_1'$, and the right boundaries of the second section $\Omega_2$ and $\Omega_2'$ are assumed to extend to infinity at left and right respectively. The fields in $\Omega_1$ and $\Omega_1'$ are calculated from and the output wavefield $\mathbf{u}_0(\mathbf{r})$ on the connection boundary $\Gamma_{AB}$ will be used to satisfy the boundary condition across $\Gamma_{AB}$ when the BE method is used to calculate wave propagation in $\Omega_2$ and $\Omega_2'$. The output fields are received along the next connection interface $\Gamma_{CD}$, and will be used as the input to the next propagation.

With the initial field $\mathbf{u}_0(\mathbf{r})$ known on the connection boundary $\Gamma_{AB}$, we analyze wave propagation in $\Omega_2$, $\Omega_2'$ by building boundary integral equations for wave propagation in these sections. Apparently, all the space points in $\Omega_2$, $\Omega_2'$ except those on $\Gamma_{AB}$ only receive the scattered field. Therefore, the total field $\mathbf{u}(\mathbf{r})$ on the boundaries of $\Omega_2$ and $\Omega_2'$ is composed of

$$\mathbf{u}(\mathbf{r}) = \begin{cases} \mathbf{u}_0(\mathbf{r}) + \mathbf{u}_s(\mathbf{r}) & \mathbf{r} \in \Gamma_{AB} \\ \mathbf{u}_s(\mathbf{r}) & \mathbf{r} \notin \Gamma_{AB} \end{cases}$$

(7)

Applying the traction-free condition to the free surface $\Gamma_1$, the boundary integral equation can be built for $\mathbf{r} \in \Omega_2$ and $\mathbf{r} \notin \Gamma_{AB}$

$$\mathbf{C}(\mathbf{r})\mathbf{u}_s(\mathbf{r}) + \int_{\Gamma_1 + \Gamma_2} \Sigma(\mathbf{r}, \mathbf{r}')\mathbf{u}_s(\mathbf{r}')d\mathbf{r}' + \int_{\Gamma_{AB}} \Sigma(\mathbf{r}, \mathbf{r}')[\mathbf{u}_0(\mathbf{r}') + \mathbf{u}_s(\mathbf{r}')]d\mathbf{r}' + \int_{\Gamma_{CD}} \Sigma(\mathbf{r}, \mathbf{r}')\mathbf{u}_s(\mathbf{r}')d\mathbf{r}' = \mathbf{G}(\mathbf{r}, \mathbf{r}')\mathbf{t}_s(\mathbf{r}')$$

(8)

The artificial boundaries $\Gamma_{AB}$ and $\Gamma_{CD}$ are assumed to be transparent, implying the outward radiation of energy across $\Gamma_{AB}$ and $\Gamma_{CD}$ always should be in the outward direction with no reflection returning to $\Omega_2$. Therefore, there is no energy contribution scattering from $\Gamma_{AB}$ and $\Gamma_{CD}$, that is,

$$\int_{\Gamma_{AB}} [\Sigma(\mathbf{r}, \mathbf{r}')\mathbf{u}_s(\mathbf{r}') - \mathbf{G}(\mathbf{r}, \mathbf{r}')\mathbf{t}_s(\mathbf{r}')]d\mathbf{r}' = 0 \quad \mathbf{r} \in \Omega_2$$

(9)

and

$$\int_{\Gamma_{CD}} [\Sigma(\mathbf{r}, \mathbf{r}')\mathbf{u}_s(\mathbf{r}') - \mathbf{G}(\mathbf{r}, \mathbf{r}')\mathbf{t}_s(\mathbf{r}')]d\mathbf{r}' = 0 \quad \mathbf{r} \in \Omega_2$$

(10)

Scattering from the artificial truncated points, A, B, C, and D, can be handled using an infinite element absorbing boundary technique (Fu and Wu, 2000). Therefore, equation (22) is reduced to

$$\mathbf{C}(\mathbf{r})\mathbf{u}_s(\mathbf{r}) + \int_{\Gamma_1} \Sigma(\mathbf{r}, \mathbf{r}')\mathbf{u}_s(\mathbf{r}')d\mathbf{r}' + \int_{\Gamma_2} [\Sigma(\mathbf{r}, \mathbf{r}')\mathbf{u}_s(\mathbf{r}') - \mathbf{G}(\mathbf{r}, \mathbf{r}')\mathbf{t}_s(\mathbf{r}')]d\mathbf{r}'$$

$$= \int_{\Gamma_{AB}} [(\mathbf{G}(\mathbf{r}, \mathbf{r}')\mathbf{t}_s(\mathbf{r}') - \Sigma(\mathbf{r}, \mathbf{r}')\mathbf{u}_0(\mathbf{r}')]d\mathbf{r}'$$

(11)

In equation (25), both the initial displacement and traction on the connection boundary are assumed to be known. $\mathbf{u}_0(\mathbf{r})$ can be calculated from the previous section, incident traction
Boundary element simulation

t_0(\mathbf{r}) can also be calculated from the previous section. Alternatively, we can use the elastic Rayleigh integrals, \( (Wu, 1989) \) which contain only either the displacement or the traction field. We can reduce the surface integral in the right side of equation (25) to the elastic wave Rayleigh integral that only contains the initial displacement \( u_0(\mathbf{r}) \):

\[
C(\mathbf{r})u_s(\mathbf{r}) + \int_{\Gamma} \Sigma(\mathbf{r}, \mathbf{r}')u_s(\mathbf{r}')d\mathbf{r}' + \int_{\Gamma} |\Sigma(\mathbf{r}, \mathbf{r}')u_s(\mathbf{r}') - G(\mathbf{r}, \mathbf{r}')t_s(\mathbf{r}')|d\mathbf{r}' = -2\int_{\Gamma} \Sigma(\mathbf{r}, \mathbf{r}')u_0(\mathbf{r}')d\mathbf{r}'.
\]

(12)

We see that the unknowns in equation (12) are \( u_s(\mathbf{r}) \) on \( \Gamma_1 \) and \( u_s(\mathbf{r}) \) on \( \Gamma_2 \). In order to solve for \( u_s(\mathbf{r}) \) and \( t_s(\mathbf{r}) \), we must build the corresponding boundary integral equation in sub-domain \( \Omega_2 \):

\[
C(\mathbf{r})u_s(\mathbf{r}) + \int_{\Gamma} \Sigma(\mathbf{r}, \mathbf{r}')u_s(\mathbf{r}')d\mathbf{r}' + \int_{\Gamma} \Sigma(\mathbf{r}, \mathbf{r}')|(u_0(\mathbf{r}') + u_s(\mathbf{r}'))|d\mathbf{r}' + \int_{\Gamma} \Sigma(\mathbf{r}, \mathbf{r}')u_0(\mathbf{r}')d\mathbf{r}' = \int_{\Gamma} G(\mathbf{r}, \mathbf{r}')t_s(\mathbf{r}')d\mathbf{r}' + \int_{\Gamma} G(\mathbf{r}, \mathbf{r}')t_0(\mathbf{r}')d\mathbf{r}'.
\]

(13)

where a sufficiently long boundary \( \Gamma_{BB} \) is used with its end set as an infinite element. Similarly, since there is no discontinuity across \( \Gamma_{BB} \) and \( \Gamma_{DD} \), we can assume \( \Gamma_{BB} \) and \( \Gamma_{DD} \) to be transparent and reduce equation (13) to

\[
C(\mathbf{r})u_s(\mathbf{r}) + \int_{\Gamma} |\Sigma(\mathbf{r}, \mathbf{r}')u_s(\mathbf{r}') - G(\mathbf{r}, \mathbf{r}')t_s(\mathbf{r}')|d\mathbf{r}' = \int_{\Gamma} |G(\mathbf{r}, \mathbf{r}')t_0(\mathbf{r}') - \Sigma(\mathbf{r}, \mathbf{r}')u_0(\mathbf{r}')|d\mathbf{r}'.
\]

(14)

The integration term containing the initial traction \( t_0(\mathbf{r}) \) in the right side of equation (14) can be removed using the elastic wave Rayleigh integral representation, yielding

\[
C(\mathbf{r})u_s(\mathbf{r}) + \int_{\Gamma} |\Sigma(\mathbf{r}, \mathbf{r}')u_s(\mathbf{r}') - G(\mathbf{r}, \mathbf{r}')t_s(\mathbf{r}')|d\mathbf{r}' = -2\int_{\Gamma} \Sigma(\mathbf{r}, \mathbf{r}')u_0(\mathbf{r}')d\mathbf{r}'.
\]

(15)

The continuity of the displacement and traction across interface \( \Gamma_{BD} \) is employed when equations (12) and (15) are combined to solve the problem. By solving the joint boundary integral equations of \( \Omega_2 \) and \( \Omega_2 \), we can obtain the wavefields \( u_s(\mathbf{r}) \) on \( \Gamma_1 \), \( u_s(\mathbf{r}) \) and \( t_s(\mathbf{r}) \) on \( \Gamma_2 \). The observed field along \( \Gamma_{CD} \) is calculated explicitly from the fields on the boundaries.

Similar to equations (18) and (19), equations (12) and (15) can be expressed in the matrix form

\[
H^{(1)}u^{(1)} + H^{(2)}u^{(2)} - G^{(2)}t^{(2)} = F,
\]

(16)

and

\[
\tilde{H}^{(2)}\tilde{u}^{(2)} - \tilde{G}^{(2)}\tilde{t}^{(2)} = \tilde{F},
\]

(17)

with the continuous condition across \( \Gamma_2 \)

\[
\begin{cases}
    u^{(2)} = \tilde{u}^{(2)} \\
    t^{(2)} = -\tilde{t}^{(2)}
\end{cases}
\]

(18)

By using this technique, wave propagation in a long regional waveguide can be partitioned into a number of section contributions for great saving in both CPU time and memory. For example, if we divide a large model into \( N \) sections, we can reduce the size of matrix by \( N \) times. Thus the total compute time will be reduced by \( N^2(1/N)^3 = 1/N^2 \) times.
VALIDATION FOR P-SV CONNECTION TECHNIQUE

To validate the connection technique, we present a comparison between the wavefield obtained using the BE method to directly calculate wave propagation from the source to the observation surface $\Gamma_{CD}'$ and the wavefield calculated by the connection scheme (Figure 6). In both cases the source time function are the same, with a dominant frequency of 1Hz. First the intermediate wave field $u_0(r)$ on $\Gamma_{AB}'$ (shown Figure 6 (b)) calculated from the source is used as the incident field for wave propagation in $\Omega_1$ and $\Omega_2'$. The dominant arrivals for the incident field at $\Gamma_{AB}'$ consist of direct P wave, pS, pP, and multi-reflection between the two layers. Then the BE method is used to calculate wave propagation from $\Gamma_{AB}'$ to $\Gamma_{CD}'$. More multiply reflected waves between the free surface and the interface can be clearly seen in the seismograms at $\Gamma_{CD}'$. The excellent agreement between the wavefield calculated by the two methods shown in Figure 6(c) and 6(d) confirms the validity of the connection technique.

Although this model is only designed to test the validity of the connection technique, we can see computation savings. For a single-processor Pentium IV 2.0Hz computer, it takes about 2 hours to calculate the wavefield along $\Gamma_{CD}'$ directly from the source, while it takes only 45 mins to compute the wavefield along $\Gamma_{CD}'$ using the connection technique. The memory requirement is also reduced by 4 times.

NUMERICAL EXAMPLES AND APPLICATIONS

In this section, we use the BE connection technique given above to simulate the wave fields for several models with rough topographies. The first model shown in the top panel of Figure 7 is a two-layer crustal model with an irregular topography. There are two discontinuous interfaces located at the depths of 15km and 37km, respectively, with the properties listed in Table 1. The correlation length of the topographic fluctuation is 5km and the RMS amplitude is 0.5km. The receivers are along the surface. An explosive source is located at the depth of 2km, and the source time function is Ricker wavelet with a dominant frequency of 1Hz. Figure 7 shows the synthetic seismogram received along the rough surface. Comparing to the wavefields (Figure 8) for the model with a flat surface, we can see that the amplitudes of the direct P and Rayleigh waves are diminished due to the scattering effects of the rough topography. Both the forward-scattering and backward-scattering waves can be seen in Figure 7.

The top panel of Figure 9 shows four different topographic curves used with the crustal model of Figure 7. The synthetic seismograms are calculated for the exponential and Gaussian random topographies respectively, with each including two RMS amplitudes of 0.5km and 0.25km. The energy attenuation curves corresponding to these topographies are calculated with increasing propagation distance, and are shown in the lower panel of Figure 9. These energy attenuation curves manifest different degrees of topography scattering attenuation. We can see that the exponential topographies lead to more effective energy attenuation than Gaussian topographies, and also the larger the topographic fluctuation is, the stronger is the topographic scattering.

Next we apply the connection technique to regional wave propagation simulations in another synthetic model. As shown in Figure 10(a), the model is a laterally varying crustal waveguide, 600km long and 32km thick, overlaying a mantle half-space. The model is divided into three 200km-long segments. The first segment contains a Gaussian hill which is given by
\[ h(x) = -h_0 \exp\left(-\frac{(x-x_0)^2}{2\sigma^2}\right) \]

where \( x_0 = 62 \text{km}, \ h_0 = 4 \text{km}, \ \sigma = 9.129 \text{km}. \) The second segment presents a thinned waveguide with the 7km-high step. The third segment contains an exponential random topography with correlation length of 5km and RMS height of 0.6km. The point source is located at 8 km depth. The receivers are along both the free surface and vertical profiles that slice these waveguides. The synthetic seismograms are calculated for the frequency range of 0-2.5Hz. We first compute wave propagation from the source to produce the incident wave field on the first connection boundary \( \Gamma_1 \) shown in Figure 10(b), where the multiply-reflected waves, converted waves, head waves and Rayleigh waves can all be clearly seen. Subsequently the BE method is used to calculate wave propagation in the second section and obtain the wave field on the second connection boundary \( \Gamma_2 \) shown in Figure 10(c). Finally we use the wavefield on \( \Gamma_2 \) as the incident field and calculate the wavefield in the third section. Figure 10(d) shows the horizontal wavefield on \( \Gamma_3 \), where the scattering effect of the random topography is obvious in that the Rayleigh waves are mostly scattered and some coda waves appear. The wavefields received respectively along the free surfaces of the three sections are put together and shown in Figure 10(e). Figure 10(f) is the energy distribution curve along the entire free surface where a relatively low energy appears around the hill and the attenuation curve varies dramatically along the random topography. The topography has a great effect on the energy attenuation in the waveguide.

CONCLUSION AND DISCUSSIONS

We present the boundary element method for the 2-D P-SV elastic wave problem. To model the effects of irregular topography on regional wave propagation, we further developed a connection technique to simulate long-range wave propagation section by section. Numerical comparisons with independent methods indicated that the presented method including the connection technique is accurate for regional wave simulation. Numerical results show that irregular topography can attenuate the total wave energy propagating in the crustal waveguide. The boundary element method can be used in computing the site effects on sites such as canyons, mountains, and valleys. The connection technique expands this method to deal with large earth models with irregular topography.

To extend the Boundary Element Method to 3-D case, several changes have to be made. First, the Green’s function of displacement and tractions of 3-D case should be used. Second, because the singularity of the 3-D Green’s function is \( 1/r \) and \( 1/r^2 \), while in 2-D case those are \( 1/r^{1/2} \) and \( 1/r \), the integral should be treated more carefully. Third, the size of matrices to solve the problems will be squared to the 2-D case, because this method includes the matrix inversion, the requirement of memory and computation time will be enlarged. Extension to 3-D cases will be conducted in the future.

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Table 1: Properties of Crust Model

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<th>Vs (km/s)</th>
<th>( \rho (g/cm^3) )</th>
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<td>3.30</td>
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</table>
Figure Captions:

Figure 1: Geometry of a simplified crustal waveguide with irregular topography and Moho interface.

Figure 2: The horizontal and vertical amplitude responses by our method (dotted line) and Sanchez-Sesma and Campillo (1991) (solid line) of a semicircular canyon topography to vertically incident P waves for various dimensionless frequencies $\eta$. 
Figure 3: The horizontal and vertical amplitude responses of a semicircular canyon topography to vertically incident P waves for $\eta = 1.0$ with the solid lines denote the solutions with 21 points per wavelength and the dots denote the specified sampling rates.
Figure 4: A single-layer flat crustal waveguide with an explosive source at 2km depth. The waves generated in the waveguide are grouped into three systems: the first directly from the source to the receivers (solid line), the second off the free surface (dash line) and the third off the Moho (dotted line).

Figure 5: Synthetic seismograms for receivers at 1-km spacing along a vertical profile at 80km for a point source. The horizontal (left panel) and vertical (right panel) components are computed in the frequency range 0-3.5Hz.
Figure 6: The boundary connection technique. (a) The diagram shows the connection formulation. (b) The vertical incident wavefield along the connecting boundary ABB'. (c) Comparison of the horizontal component of the wavefield along CDD' calculated directly from the source (thick lines) and those calculated using the connection technique (thin lines). (d) Comparison of the vertical components.
Figure 7: Synthetic seismogram for a two-layered model with random topography. The source is located at the depth of 2km. The topographic fluctuation has an exponential correlation function with a RMS of 0.5km and a correlation length of 5km. We can see both forward-scattering and backscattering in the seismogram.
Figure 8: Synthetic seismogram for the model shown in Figure 7 but with flat surface. Multiply-reflected waves and Rayleigh waves can be clearly seen in the figure.
Figure 9: Energy attenuation for earth models with different topographies. On the top are the topographies used in our calculation, the thick solid line is a Gaussian random topography with a RMS of 0.5km, thin solid line is a Gaussian random topography with RMS of 0.25km, the thick dash line is exponential random topography with RMS 0.5km and the thin dash line is exponential random topography with RMS 0.25km, while the reference model (flat surface) is shown by the dotted line. On the bottom are the energy attenuation curves for the whole time window, the lines have the same meaning as those in the top figure.
Figure 10. Application to a large model. (a) The model. (b) Horizontal wavefield on $\Gamma_1$. (c) Horizontal wavefield on $\Gamma_2$. (d) Horizontal wavefield on $\Gamma_3$. (e) Horizontal wavefield along the surface. (f) Energy distribution along the surface.
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